NAG Library Routine Document

S07AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S07AAF returns the value of the circular tangent, $\tan x$, via the function name.

2 Specification

```
FUNCTION S07AAF (X, IFAIL)
REAL (KIND=nag_wp) S07AAF
INTEGER            IFAIL
REAL (KIND=nag_wp) X
```

3 Description

S07AAF calculates an approximate value for the circular tangent of its argument, $\tan x$. It is based on the Chebyshev expansion

$$\tan\theta = \theta y(t) = \theta \sum_{r=0} c_r T_r(t)$$

where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$ and $-1 < t < +1, \quad t = 2\left(\dfrac{4\theta}{\pi}\right)^2 - 1$.

The reduction to the standard range is accomplished by taking

$$x = N\pi/2 + \theta$$

where N is an integer and $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$,

i.e., $\theta = x - \left(\dfrac{2x}{\pi}\right)\frac{\pi}{2}$ where $N = \left[\dfrac{2x}{\pi}\right] =$ the nearest integer to $\dfrac{2x}{\pi}$.

From the properties of $\tan x$ it follows that

$$\tan x = \left\{ \begin{matrix} \tan\theta, & N\text{even} \\ -1/\tan\theta, & N\text{odd} \end{matrix} \right\}$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: X – REAL (KIND=nag_wp) *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then

the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

The routine has been called with an argument that is larger in magnitude than F; the default result returned is zero. The value of F is given in the Users' Note for your implementation.

IFAIL $= 2$

The routine has been called with an argument that is too close (as determined using the relative tolerance F) to an odd multiple of $\pi/2$, at which the function is infinite; the routine returns a value with the correct sign but a more or less arbitrary but large magnitude (see Section 7). The value of F is given in the Users' Note for your implementation.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$\epsilon \geq \frac{2x}{\sin 2x}\delta.$$

That is a relative error in the argument, x, is amplified by at least a factor $2x/\sin 2x$ in the result.

Similarly if E is the absolute error in the result this is given by

$$E \geq \frac{x}{\cos^2 x}\delta.$$

The equalities should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but if δ is simply the round-off error in the machine it is possible that internal calculation rounding will lose an extra figure.

The graphs below show the behaviour of these amplification factors.

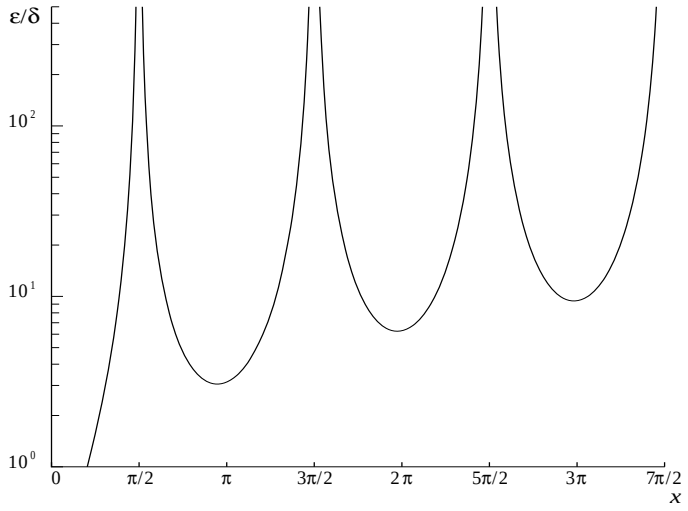

Figure 1

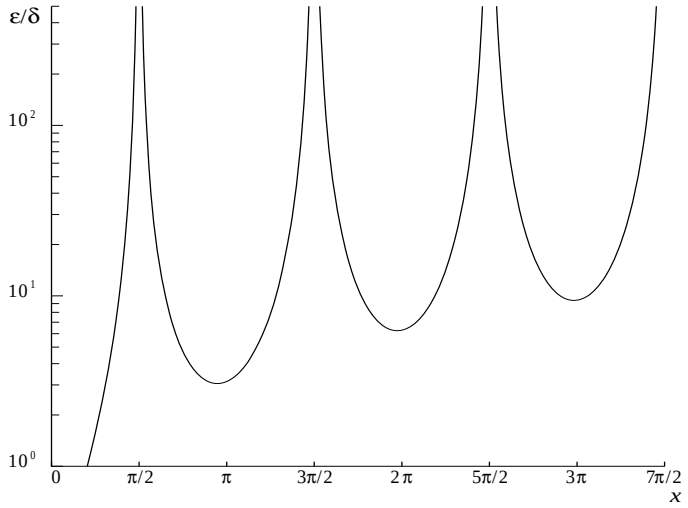

Figure 2

In the principal range it is possible to preserve relative accuracy even near the zero of $\tan x$ at $x = 0$ but at the other zeros only absolute accuracy is possible. Near the infinities of $\tan x$ both the relative and absolute errors become infinite and the routine must fail (error 2).

If N is odd and $|\theta| \leq xF_2$ the routine could not return better than two figures and in all probability would produce a result that was in error in its most significant figure. Therefore the routine fails and it returns the value

$$-\operatorname{sign}\theta\left(\frac{1}{|xF_2|}\right) \simeq -\operatorname{sign}\theta\tan\left(\frac{\pi}{2} - |xF_2|\right)$$

which is the value of the tangent at the nearest argument for which a valid call could be made.

Accuracy is also unavoidably lost if the routine is called with a large argument. If $|x| > F_1$ the routine fails (error 1) and returns zero. (See the Users' Note for your implementation for specific values of F_1 and F_2.)

8 Parallelism and Performance

S07AAF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
    Program s07aafe

!     S07AAF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s07aaf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: x, y
      Integer                          :: ifail, ioerr
!     .. Executable Statements ..
      Write (nout,*) 'S07AAF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Write (nout,*)
      Write (nout,*) '     X           Y'
      Write (nout,*)

data: Do
        Read (nin,*,Iostat=ioerr) x

        If (ioerr<0) Then
          Exit data
        End If

        ifail = -1
        y = s07aaf(x,ifail)

        If (ifail<0) Then
          Exit data
        End If

        Write (nout,99999) x, y
      End Do data

99999 Format (1X,1P,2E12.3)
    End Program s07aafe
```

10.2 Program Data

```
S07AAF Example Program Data
            -2.0
            -0.5
             1.0
             3.0
             1.5708
```

10.3 Program Results

```
 S07AAF Example Program Results

      X           Y

   -2.000E+00   2.185E+00
   -5.000E-01  -5.463E-01
    1.000E+00   1.557E+00
    3.000E+00  -1.425E-01
    1.571E+00  -2.722E+05
```