

# NAG Library Routine Document

## F07PVF (ZHPRFS)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

F07PVF (ZHPRFS) returns error bounds for the solution of a complex Hermitian indefinite system of linear equations with multiple right-hand sides,  $AX = B$ , using packed storage. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

### 2 Specification

SUBROUTINE F07PVF (UPLO, N, NRHS, AP, AFP, IPIV, B, LDB, X, LDX, FERR, &  
BERR, WORK, RWORK, INFO)

INTEGER N, NRHS, IPIV(\*), LDB, LDX, INFO  
REAL (KIND=nag\_wp) FERR(NRHS), BERR(NRHS), RWORK(N)  
COMPLEX (KIND=nag\_wp) AP(\*), AFP(\*), B(LDB,\*), X(LDX,\*), WORK(2\*N)  
CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zhprfs*.

### 3 Description

F07PVF (ZHPRFS) returns the backward errors and estimated bounds on the forward errors for the solution of a complex Hermitian indefinite system of linear equations with multiple right-hand sides  $AX = B$ , using packed storage. The routine handles each right-hand side vector (stored as a column of the matrix  $B$ ) independently, so we describe the function of F07PVF (ZHPRFS) in terms of a single right-hand side  $b$  and solution  $x$ .

Given a computed solution  $x$ , the routine computes the *component-wise backward error*  $\beta$ . This is the size of the smallest relative perturbation in each element of  $A$  and  $b$  such that  $x$  is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$

$$|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.$$

Then the routine estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where  $\hat{x}$  is the true solution.

For details of the method, see the F07 Chapter Introduction.

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Arguments

- 1: UPLO – CHARACTER(1) *Input*  
*On entry:* specifies whether the upper or lower triangular part of  $A$  is stored and how  $A$  is to be factorized.  
 UPLO = 'U'  
 The upper triangular part of  $A$  is stored and  $A$  is factorized as  $PUDU^H P^T$ , where  $U$  is upper triangular.  
 UPLO = 'L'  
 The lower triangular part of  $A$  is stored and  $A$  is factorized as  $PLDL^H P^T$ , where  $L$  is lower triangular.  
*Constraint:* UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .
- 3: NRHS – INTEGER *Input*  
*On entry:*  $r$ , the number of right-hand sides.  
*Constraint:* NRHS  $\geq 0$ .
- 4: AP(\*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array AP must be at least  $\max(1, N \times (N + 1)/2)$ .  
*On entry:* the  $n$  by  $n$  original Hermitian matrix  $A$  as supplied to F07PRF (ZHPTRF).
- 5: AFP(\*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array AFP must be at least  $\max(1, N \times (N + 1)/2)$ .  
*On entry:* the factorization of  $A$  stored in packed form, as returned by F07PRF (ZHPTRF).
- 6: IPIV(\*) – INTEGER array *Input*  
**Note:** the dimension of the array IPIV must be at least  $\max(1, N)$ .  
*On entry:* details of the interchanges and the block structure of  $D$ , as returned by F07PRF (ZHPTRF).
- 7: B(LDB, \*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the second dimension of the array B must be at least  $\max(1, NRHS)$ .  
*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .
- 8: LDB – INTEGER *Input*  
*On entry:* the first dimension of the array B as declared in the (sub)program from which F07PVF (ZHPTRF) is called.  
*Constraint:* LDB  $\geq \max(1, N)$ .
- 9: X(LDX, \*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array X must be at least  $\max(1, NRHS)$ .  
*On entry:* the  $n$  by  $r$  solution matrix  $X$ , as returned by F07PSF (ZHPTRS).  
*On exit:* the improved solution matrix  $X$ .

- 10: LDX – INTEGER *Input*  
*On entry:* the first dimension of the array X as declared in the (sub)program from which F07PVF (ZHPRFS) is called.  
*Constraint:*  $LDX \geq \max(1, N)$ .
- 11: FERR(NRHS) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* FERR( $j$ ) contains an estimated error bound for the  $j$ th solution vector, that is, the  $j$ th column of  $X$ , for  $j = 1, 2, \dots, r$ .
- 12: BERR(NRHS) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* BERR( $j$ ) contains the component-wise backward error bound  $\beta$  for the  $j$ th solution vector, that is, the  $j$ th column of  $X$ , for  $j = 1, 2, \dots, r$ .
- 13: WORK( $2 \times N$ ) – COMPLEX (KIND=nag\_wp) array *Workspace*
- 14: RWORK( $N$ ) – REAL (KIND=nag\_wp) array *Workspace*
- 15: INFO – INTEGER *Output*  
*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO < 0

If INFO =  $-i$ , argument  $i$  had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The bounds returned in FERR are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

## 8 Parallelism and Performance

F07PVF (ZHPRFS) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

F07PVF (ZHPRFS) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

For each right-hand side, computation of the backward error involves a minimum of  $16n^2$  real floating-point operations. Each step of iterative refinement involves an additional  $24n^2$  real operations. At most five steps of iterative refinement are performed, but usually only 1 or 2 steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form  $Ax = b$ ; the number is usually 5 and never more than 11. Each solution involves approximately  $8n^2$  real operations.

The real analogue of this routine is F07PHF (DSPRFS).

## 10 Example

This example solves the system of equations  $AX = B$  using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 7.79 + 5.48i & -35.39 + 18.01i \\ -0.77 - 16.05i & 4.23 - 70.02i \\ -9.58 + 3.88i & -24.79 - 8.40i \\ 2.98 - 10.18i & 28.68 - 39.89i \end{pmatrix}.$$

Here  $A$  is Hermitian indefinite, stored in packed form, and must first be factorized by F07PRF (ZHPTRF).

### 10.1 Program Text

Program f07pvfe

```
!      F07PVF Example Program Text
!
!      Mark 26 Release. NAG Copyright 2016.
!
!      .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zhprfs, zhptraf, zhptra
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Integer                     :: aplen, i, ifail, info, j, ldb, ldx, &
                                   n, nrhs
      Character (1)               :: uplo
!      .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: afp(:), ap(:), b(:,,:), work(:), &
                                   x(:,:)
      Real (Kind=nag_wp), Allocatable  :: berr(:), ferr(:), rwork(:)
      Integer, Allocatable             :: ipiv(:)
      Character (1)                   :: clabs(1), rlabs(1)
!      .. Executable Statements ..
      Write (nout,*) 'F07PVF Example Program Results'
!      Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs
      ldb = n
      ldx = n
      aplen = n*(n+1)/2
      Allocate (afp(aplen),ap(aplen),b(ldb,nrhs),work(2*n),x(ldx,n), &
               berr(nrhs),ferr(nrhs),rwork(n),ipiv(n))
!
!      Read A and B from data file, and copy A to AFP and B to X
!
      Read (nin,*) uplo
      If (uplo=='U') Then
         Read (nin,*)((ap(i+j*(j-1)/2),j=i,n),i=1,n)
      Else If (uplo=='L') Then
         Read (nin,*)((ap(i+(2*n-j)*(j-1)/2),j=1,i),i=1,n)
      End If
      Read (nin,*)(b(i,1:nrhs),i=1,n)
!
      afp(1:aplen) = ap(1:aplen)
      x(1:n,1:nrhs) = b(1:n,1:nrhs)
```

```

!      Factorize A in the array AFP
!      The NAG name equivalent of zhpvtrf is f07prf
!      Call zhpvtrf(uplo,n,afp,ipiv,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!      Compute solution in the array X
!      The NAG name equivalent of zhpvtrs is f07psf
!      Call zhpvtrs(uplo,n,nrhs,afp,ipiv,x,ldx,info)

!      Improve solution, and compute backward errors and
!      estimated bounds on the forward errors

!      The NAG name equivalent of zhprfs is f07puf
!      Call zhprfs(uplo,n,nrhs,ap,afp,ipiv,b,ldb,x,ldx,ferr,berr,work,rwork, &
!      info)

!      Print solution

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
!      Call x04dbf('General',' ',n,nrhs,x,ldx,'Bracketed','F7.4', &
!      'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

      Write (nout,*)
      Write (nout,*) 'Backward errors (machine-dependent)'
      Write (nout,99999) berr(1:nrhs)
      Write (nout,*) 'Estimated forward error bounds (machine-dependent)'
      Write (nout,99999) ferr(1:nrhs)
      Else
        Write (nout,*) 'The factor D is singular'
      End If

99999 Format ((5X,1P,4(E11.1,7X)))
      End Program f07pvfe

```

## 10.2 Program Data

F07PVF Example Program Data

```

4 2                                     :Values of N and NRHS
'L'                                     :Value of UPLO
(-1.36, 0.00)
( 1.58,-0.90) (-8.87, 0.00)
( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A
( 7.79, 5.48) (-35.39, 18.01)
(-0.77,-16.05) ( 4.23,-70.02)
(-9.58, 3.88) (-24.79, -8.40)
( 2.98,-10.18) ( 28.68,-39.89)         :End of matrix B

```

## 10.3 Program Results

F07PVF Example Program Results

Solution(s)

```

           1           2
1 ( 1.0000,-1.0000) ( 3.0000,-4.0000)
2 (-1.0000, 2.0000) (-1.0000, 5.0000)
3 ( 3.0000,-2.0000) ( 7.0000,-2.0000)
4 ( 2.0000, 1.0000) (-8.0000, 6.0000)

```

Backward errors (machine-dependent)

```

5.1E-17           3.5E-17

```

Estimated forward error bounds (machine-dependent)

```

2.5E-15           3.0E-15

```