NAG Library Routine Document

F07CRF (ZGTTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07CRF (ZGTTRF) computes the LU factorization of a complex n by n tridiagonal matrix A.

2 Specification

```
SUBROUTINE F07CRF (N, DL, D, DU, DU2, IPIV, INFO)
INTEGER                 N, IPIV(N), INFO
COMPLEX (KIND=nag_wp) DL(*), D(*), DU(*), DU2(N-2)
```

The routine may be called by its LAPACK name ***zgttrf***.

3 Description

F07CRF (ZGTTRF) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

5 Arguments

1: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: DL($*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array DL must be at least $\max(1, \mathrm{N} - 1)$.

On entry: must contain the $(n-1)$ subdiagonal elements of the matrix A.

On exit: is overwritten by the $(n-1)$ multipliers that define the matrix L of the LU factorization of A.

3: D($*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the matrix A.

On exit: is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

4: DU($*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array DU must be at least $\max(1, \mathrm{N}-1)$.

On entry: must contain the $(n-1)$ superdiagonal elements of the matrix A.

On exit: is overwritten by the $(n-1)$ elements of the first superdiagonal of U.

5: DU2(N $-$ 2) – COMPLEX (KIND=nag_wp) array *Output*

On exit: contains the $(n-2)$ elements of the second superdiagonal of U.

6: IPIV(N) – INTEGER array *Output*

On exit: contains the n pivot indices that define the permutation matrix P. At the ith step, row i of the matrix was interchanged with row IPIV(i). IPIV(i) will always be either i or $(i+1)$, IPIV(i) $= i$ indicating that a row interchange was not performed.

7: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

Element $\langle$*value*$\rangle$ of the diagonal is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = PLU,$$

where

$$\|E\|_\infty = O(\epsilon)\|A\|_\infty$$

and ϵ is the ***machine precision***.

Following the use of this routine, F07CSF (ZGTTRS) can be used to solve systems of equations $AX = B$ or $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$, and F07CUF (ZGTCON) can be used to estimate the condition number of A.

8 Parallelism and Performance

F07CRF (ZGTTRF) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to factorize the matrix A is proportional to n.

The real analogue of this routine is F07CDF (DGTTRF).

10 Example

This example factorizes the tridiagonal matrix A given by

$$A = \begin{pmatrix} -1.3+1.3i & 2.0-1.0i & 0 & 0 & 0 \\ 1.0-2.0i & -1.3+1.3i & 2.0+1.0i & 0 & 0 \\ 0 & 1.0+1.0i & -1.3+3.3i & -1.0+1.0i & 0 \\ 0 & 0 & 2.0-3.0i & -0.3+4.3i & 1.0-1.0i \\ 0 & 0 & 0 & 1.0+1.0i & -3.3+1.3i \end{pmatrix}.$$

10.1 Program Text

```
    Program f07crfe

!     F07CRF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, zgttrf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: info, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: d(:), dl(:), du(:), du2(:)
      Integer, Allocatable             :: ipiv(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07CRF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (d(n),dl(n-1),du(n-1),du2(n-2),ipiv(n))

!     Read the tridiagonal matrix A from data file

      Read (nin,*) du(1:n-1)
      Read (nin,*) d(1:n)
      Read (nin,*) dl(1:n-1)

!     Factorize the tridiagonal matrix A
!     The NAG name equivalent of zgttrf is f07crf
      Call zgttrf(n,dl,d,du,du2,ipiv,info)

      If (info>0) Then
        Write (nout,99999) 'The (', info, ',', info, ')',                    &
          ' element of the factor U is zero'
      End If

!     Print details of the factorization

      Write (nout,*) 'Details of factorization'
      Write (nout,*)
      Write (nout,*) ' Second superdiagonal of U'
      Write (nout,99998) du2(1:n-2)
      Write (nout,*)
      Write (nout,*) ' First superdiagonal of U'
      Write (nout,99998) du(1:n-1)
      Write (nout,*)
      Write (nout,*) ' Main diagonal of U'
      Write (nout,99998) d(1:n)
      Write (nout,*)
      Write (nout,*) ' Multipliers'
      Write (nout,99998) dl(1:n-1)
      Write (nout,*)
```

```
      Write (nout,*) ' Vector of interchanges'
      Write (nout,99997) ipiv(1:n)

99999 Format (1X,A,I3,A,I3,A,A)
99998 Format (4(' (',F8.4,',',F8.4,')',:))
99997 Format (1X,5I7)
    End Program f07crfe
```

10.2 Program Data

```
F07CRF Example Program Data
   5                                                :Value of N
 ( 2.0,-1.0) ( 2.0, 1.0) (-1.0, 1.0) ( 1.0,-1.0) :End of DU
 (-1.3, 1.3) (-1.3, 1.3) (-1.3, 3.3) (-0.3, 4.3)
 (-3.3, 1.3)                                      :End of D
 ( 1.0,-2.0) ( 1.0, 1.0) ( 2.0,-3.0) ( 1.0, 1.0) :End of DL
```

10.3 Program Results

```
 F07CRF Example Program Results

 Details of factorization

  Second superdiagonal of U
 (  2.0000,  1.0000) ( -1.0000,  1.0000) (  1.0000, -1.0000)

  First superdiagonal of U
 ( -1.3000,  1.3000) ( -1.3000,  3.3000) ( -0.3000,  4.3000) ( -3.3000,  1.3000)

  Main diagonal of U
 (  1.0000, -2.0000) (  1.0000,  1.0000) (  2.0000, -3.0000) (  1.0000,  1.0000)
 ( -1.3399,  0.2875)

  Multipliers
 ( -0.7800, -0.2600) (  0.1620, -0.4860) ( -0.0452, -0.0010) ( -0.3979, -0.0562)

  Vector of interchanges
       2      3      4      5      5
```