NAG Library Routine Document

F07CNF (ZGTSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07CNF (ZGTSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n tridiagonal matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07CNF (N, NRHS, DL, D, DU, B, LDB, INFO)
INTEGER                N, NRHS, LDB, INFO
COMPLEX (KIND=nag_wp)  DL(*), D(*), DU(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***zgtsv***.

3 Description

F07CNF (ZGTSV) uses Gaussian elimination with partial pivoting and row interchanges to solve the equations $AX = B$. The matrix A is factorized as $A = PLU$, where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element per column, and U is an upper triangular band matrix, with two superdiagonals.

Note that the equations $A^{\mathrm{T}}X = B$ may be solved by interchanging the order of the arguments DU and DL.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

5 Arguments

1: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

3: DL($*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array DL must be at least $\max(1, \mathrm{N}-1)$.

On entry: must contain the $(n-1)$ subdiagonal elements of the matrix A.

On exit: if no constraints are violated, DL is overwritten by the $(n-2)$ elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A, in $\mathrm{DL}(1), \mathrm{DL}(2), \ldots, \mathrm{DL}(n-2)$.

4: $\mathrm{D}(*)$ – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: must contain the n diagonal elements of the matrix A.

On exit: if no constraints are violated, D is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

5: $\mathrm{DU}(*)$ – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array DU must be at least $\max(1, \mathrm{N}-1)$.

On entry: must contain the $(n-1)$ superdiagonal elements of the matrix A.

On exit: if no constraints are violated, DU is overwritten by the $(n-1)$ elements of the first superdiagonal of U.

6: $\mathrm{B}(\mathrm{LDB}, *)$ – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

Note: to solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length $\mathrm{LDB} = \max(1, \mathrm{N})$.

On entry: the n by r right-hand side matrix B.

On exit: if $\mathrm{INFO} = 0$, the n by r solution matrix X.

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07CNF (ZGTSV) is called.

Constraint: $\mathrm{LDB} \geq \max(1, \mathrm{N})$.

8: INFO – INTEGER *Output*

On exit: $\mathrm{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathrm{INFO} < 0$

If $\mathrm{INFO} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

$\mathrm{INFO} > 0$

Element $\langle value \rangle$ of the diagonal is exactly zero, and the solution has not been computed. The factorization has not been completed unless $\mathrm{N} = \langle value \rangle$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x}-x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Alternatives to F07CNF (ZGTSV), which return condition and error estimates are F04CCF and F07CPF (ZGTSVX).

8 Parallelism and Performance

F07CNF (ZGTSV) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr.

The real analogue of this routine is F07CAF (DGTSV).

10 Example

This example solves the equations

$$Ax = b,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} -1.3+1.3i & 2.0-1.0i & 0 & 0 & 0 \\ 1.0-2.0i & -1.3+1.3i & 2.0+1.0i & 0 & 0 \\ 0 & 1.0+1.0i & -1.3+3.3i & -1.0+1.0i & 0 \\ 0 & 0 & 2.0-3.0i & -0.3+4.3i & 1.0-1.0i \\ 0 & 0 & 0 & 1.0+1.0i & -3.3+1.3i \end{pmatrix}$$

and

$$b = \begin{pmatrix} 2.4-5.0i \\ 3.4+18.2i \\ -14.7+9.7i \\ 31.9-7.7i \\ -1.0+1.6i \end{pmatrix}.$$

10.1 Program Text

```
    Program f07cnfe

!     F07CNF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, zgtsv
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                 :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                            :: info, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: b(:), d(:), dl(:), du(:)
!     .. Executable Statements ..
      Write (nout,*) 'F07CNF Example Program Results'
      Write (nout,*)
```

```
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n

      Allocate (b(n),d(n),dl(n-1),du(n-1))

!     Read the tridiagonal matrix A and the right hand side B from
!     data file

      Read (nin,*) du(1:n-1)
      Read (nin,*) d(1:n)
      Read (nin,*) dl(1:n-1)
      Read (nin,*) b(1:n)

!     Solve the equations Ax = b for x
!     The NAG name equivalent of zgtsv is f07cnf
      Call zgtsv(n,1,dl,d,du,b,n,info)

      If (info==0) Then

!       Print solution

        Write (nout,*) 'Solution'
        Write (nout,99999) b(1:n)

      Else
        Write (nout,99998) 'The (', info, ',', info, ')',                      &
          ' element of the factor U is zero'
      End If

99999 Format (4(' (',F8.4,',',F8.4,')',:))
99998 Format (1X,A,I3,A,I3,A,A)
    End Program f07cnfe
```

10.2 Program Data

```
F07CNF Example Program Data
   5                                                       :Value of N
 (  2.0, -1.0) (  2.0,  1.0) ( -1.0,  1.0) (  1.0, -1.0) :End of DU
 ( -1.3,  1.3) ( -1.3,  1.3) ( -1.3,  3.3) ( -0.3,  4.3)
 ( -3.3,  1.3)                                             :End of D
 (  1.0, -2.0) (  1.0 , 1.0) (  2.0, -3.0) (  1.0,  1.0) :End of  DL
 (  2.4, -5.0) (  3.4, 18.2) (-14.7,  9.7) ( 31.9, -7.7)
 ( -1.0,  1.6)                                             :End of B
```

10.3 Program Results

```
 F07CNF Example Program Results

 Solution
 (  1.0000,  1.0000) (  3.0000, -1.0000) (  4.0000,  5.0000) ( -1.0000, -2.0000)
 (  1.0000, -1.0000)
```