NAG Library Routine Document

F07CEF (DGTTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07CEF (DGTTRS) computes the solution to a real system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, where A is an n by n tridiagonal matrix and X and B are n by r matrices, using the LU factorization returned by F07CDF (DGTTRF).

2 Specification

```
SUBROUTINE F07CEF (TRANS, N, NRHS, DL, D, DU, DU2, IPIV, B, LDB, INFO)

INTEGER             N, NRHS, IPIV(*), LDB, INFO
REAL (KIND=nag_wp) DL(*), D(*), DU(*), DU2(*), B(LDB,*)
CHARACTER(1)        TRANS
```

The routine may be called by its LAPACK name ***dgttrs***.

3 Description

F07CEF (DGTTRS) should be preceded by a call to F07CDF (DGTTRF), which uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals. F07CEF (DGTTRS) then utilizes the factorization to solve the required equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

5 Arguments

1: TRANS – CHARACTER(1) *Input*

On entry: specifies the equations to be solved as follows:

TRANS = 'N'
  Solve $AX = B$ for X.

TRANS = 'T' or 'C'
  Solve $A^{\mathrm{T}}X = B$ for X.

Constraint: TRANS = 'N', 'T' or 'C'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\text{NRHS} \geq 0$.

4: DL($*$) – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array DL must be at least $\max(1, \text{N} - 1)$.

On entry: must contain the $(n-1)$ multipliers that define the matrix L of the LU factorization of A.

5: D($*$) – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array D must be at least $\max(1, \text{N})$.

On entry: must contain the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

6: DU($*$) – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array DU must be at least $\max(1, \text{N} - 1)$.

On entry: must contain the $(n-1)$ elements of the first superdiagonal of U.

7: DU2($*$) – REAL (KIND=nag_wp) array *Input*

Note: the dimension of the array DU2 must be at least $\max(1, \text{N} - 2)$.

On entry: must contain the $(n-2)$ elements of the second superdiagonal of U.

8: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \text{N})$.

On entry: must contain the n pivot indices that define the permutation matrix P. At the ith step, row i of the matrix was interchanged with row IPIV(i), and IPIV(i) must always be either i or $(i+1)$, $\text{IPIV}(i) = i$ indicating that a row interchange was not performed.

9: B(LDB, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.

On entry: the n by r matrix of right-hand sides B.

On exit: the n by r solution matrix X.

10: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07CEF (DGTTRS) is called.

Constraint: $\text{LDB} \geq \max(1, \text{N})$.

11: INFO – INTEGER *Output*

On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If $\text{INFO} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of this routine F07CGF (DGTCON) can be used to estimate the condition number of A and F07CHF (DGTRFS) can be used to obtain approximate error bounds.

8 Parallelism and Performance

F07CEF (DGTTRS) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ or $A^{\mathrm{T}}X = B$ is proportional to nr.

The complex analogue of this routine is F07CSF (ZGTTRS).

10 Example

This example solves the equations

$$AX = B,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2.7 & 6.6 \\ -0.5 & 10.8 \\ 2.6 & -3.2 \\ 0.6 & -11.2 \\ 2.7 & 19.1 \end{pmatrix}.$$

10.1 Program Text

```
    Program f07cefe

!     F07CEF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: dgttrf, dgttrs, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                           :: i, ifail, info, ldb, n, nrhs
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable   :: b(:,:), d(:), dl(:), du(:), du2(:)
      Integer, Allocatable              :: ipiv(:)
!     .. Executable Statements ..
```

```
      Write (nout,*) 'F07CEF Example Program Results'
      Write (nout,*)
      Flush (nout)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs
      ldb = n
      Allocate (b(ldb,nrhs),d(n),dl(n-1),du(n-1),du2(n-2),ipiv(n))

!     Read the tridiagonal matrix A from data file

      Read (nin,*) du(1:n-1)
      Read (nin,*) d(1:n)
      Read (nin,*) dl(1:n-1)

!     Read the right hand matrix B
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Factorize the tridiagonal matrix A
!     The NAG name equivalent of dgttrf is f07cdf
      Call dgttrf(n,dl,d,du,du2,ipiv,info)

      If (info==0) Then

!       Solve the equations AX = B
!       The NAG name equivalent of dgttrs is f07cef
        Call dgttrs('No transpose',n,nrhs,dl,d,du,du2,ipiv,b,ldb,info)

!       Print the solution

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)

      Else
        Write (nout,99999) 'The (', info, ',', info, ')',                      &
          ' element of the factor U is zero'
      End If

99999 Format (1X,A,I3,A,I3,A,A)
    End Program f07cefe
```

10.2 Program Data

```
F07CEF Example Program Data
  5     2                       :Values of N and NRHS
        2.1  -1.0   1.9   8.0
  3.0   2.3  -5.0  -0.9   7.1
  3.4   3.6   7.0  -6.0         :End of matrix A
  2.7   6.6
 -0.5  10.8
  2.6  -3.2
  0.6 -11.2
  2.7  19.1                     :End of matrix B
```

10.3 Program Results

```
 F07CEF Example Program Results

 Solution(s)
             1          2
 1     -4.0000     5.0000
 2      7.0000    -4.0000
 3      3.0000    -3.0000
 4     -4.0000    -2.0000
 5     -3.0000     1.0000
```