NAG Library Routine Document

F07BSF (ZGBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07BSF (ZGBTRS) solves a complex band system of linear equations with multiple right-hand sides,

$$AX = B, \quad A^{\mathrm{T}}X = B \quad \text{or} \quad A^{\mathrm{H}}X = B,$$

where A has been factorized by F07BRF (ZGBTRF).

2 Specification

```
SUBROUTINE F07BSF (TRANS, N, KL, KU, NRHS, AB, LDAB, IPIV, B, LDB, INFO)
INTEGER                N, KL, KU, NRHS, LDAB, IPIV(*), LDB, INFO
COMPLEX (KIND=nag_wp) AB(LDAB,*), B(LDB,*)
CHARACTER(1)           TRANS
```

The routine may be called by its LAPACK name ***zgbtrs***.

3 Description

F07BSF (ZGBTRS) is used to solve a complex band system of linear equations $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$, the routine must be preceded by a call to F07BRF (ZGBTRF) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If TRANS = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If TRANS = 'T', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If TRANS = 'C', the solution is computed by solving $U^{\mathrm{H}}Y = B$ and then $L^{\mathrm{H}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: TRANS – CHARACTER(1) *Input*

On entry: indicates the form of the equations.

TRANS = 'N'
: $AX = B$ is solved for X.

TRANS = 'T'
: $A^{\mathrm{T}}X = B$ is solved for X.

TRANS = 'C'
: $A^{\mathrm{H}}X = B$ is solved for X.

Constraint: TRANS = 'N', 'T' or 'C'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: KL – INTEGER *Input*

On entry: k_l, the number of subdiagonals within the band of the matrix A.

Constraint: $\mathrm{KL} \geq 0$.

4: KU – INTEGER *Input*

On entry: k_u, the number of superdiagonals within the band of the matrix A.

Constraint: $\mathrm{KU} \geq 0$.

5: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: $\mathrm{NRHS} \geq 0$.

6: AB(LDAB, $*$) – COMPLEX (KIND=nag_wp) array *Input*

Note: the second dimension of the array AB must be at least $\max(1, \mathrm{N})$.

On entry: the LU factorization of A, as returned by F07BRF (ZGBTRF).

7: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07BSF (ZGBTRS) is called.

Constraint: $\mathrm{LDAB} \geq 2 \times \mathrm{KL} + \mathrm{KU} + 1$.

8: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On entry: the pivot indices, as returned by F07BRF (ZGBTRF).

9: B(LDB, $*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

10: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07BSF (ZGBTRS) is called.

Constraint: $\mathrm{LDB} \geq \max(1, \mathrm{N})$.

11: INFO – INTEGER *Output*

On exit: $\mathrm{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathrm{INFO} < 0$

If $\mathrm{INFO} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(k)\epsilon|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll n$.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \big\||A^{-1}||A||x|\big\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \big\||A^{-1}||A|\big\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}(A^{\mathrm{H}})$ (which is the same as $\mathrm{cond}(A^{\mathrm{T}})$) can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07BVF (ZGBRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07BUF (ZGBCON) with NORM = 'I'.

8 Parallelism and Performance

F07BSF (ZGBTRS) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

F07BSF (ZGBTRS) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8n(2k_l + k_u)r$, assuming $n \gg k_l$ and $n \gg k_u$.

This routine may be followed by a call to F07BVF (ZGBRFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07BEF (DGBTRS).

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.65+2.26i & -2.05-0.85i & 0.97-2.84i & 0.00+0.00i \\ 0.00+6.30i & -1.48-1.75i & -3.99+4.01i & 0.59-0.48i \\ 0.00+0.00i & -0.77+2.83i & -1.06+1.94i & 3.33-1.04i \\ 0.00+0.00i & 0.00+0.00i & 4.48-1.09i & -0.46-1.72i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -1.06+21.50i & 12.85+\ \ 2.84i \\ -22.72-53.90i & -70.22+21.57i \\ 28.24-38.60i & -20.7\ \ -31.23i \\ -34.56+16.73i & 26.01+31.97i \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by F07BRF (ZGBTRF).

10.1 Program Text

```
    Program f07bsfe

!     F07BSF Example Program Text

!     Mark 26 Release. NAG Copyright 2016.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zgbtrf, zgbtrs
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
      Character (1), Parameter         :: trans = 'N'
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, j, k, kl, ku, ldab,  &
                                          ldb, n, nrhs
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: ab(:,:), b(:,:)
      Integer, Allocatable             :: ipiv(:)
      Character (1)                    :: clabs(1), rlabs(1)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: max, min
!     .. Executable Statements ..
      Write (nout,*) 'F07BSF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs, kl, ku
      ldab = 2*kl + ku + 1
      ldb = n
      Allocate (ab(ldab,n),b(ldb,nrhs),ipiv(n))

!     Read A and B from data file

      k = kl + ku + 1
      Read (nin,*)((ab(k+i-j,j),j=max(i-kl,1),min(i+ku,n)),i=1,n)
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Factorize A

!     The NAG name equivalent of zgbtrf is f07brf
      Call zgbtrf(n,n,kl,ku,ab,ldab,ipiv,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute solution
!       The NAG name equivalent of zgbtrs is f07bsf
        Call zgbtrs(trans,n,kl,ku,nrhs,ab,ldab,ipiv,b,ldb,info)

!       Print solution

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04dbf('General',' ',n,nrhs,b,ldb,'Bracketed','F7.4',            &
          'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

      Else
        Write (nout,*) 'The factor U is singular'
      End If

    End Program f07bsfe
```

10.2 Program Data

```
F07BSF Example Program Data
  4  2  1  2                                       :Values of N, NRHS, KL and KU
 (-1.65, 2.26) (-2.05,-0.85) ( 0.97,-2.84)
 ( 0.00, 6.30) (-1.48,-1.75) (-3.99, 4.01) ( 0.59,-0.48)
               (-0.77, 2.83) (-1.06, 1.94) ( 3.33,-1.04)
                             ( 4.48,-1.09) (-0.46,-1.72)  :End of matrix A
 ( -1.06, 21.50) ( 12.85,  2.84)
 (-22.72,-53.90) (-70.22, 21.57)
 ( 28.24,-38.60) (-20.73, -1.23)
 (-34.56, 16.73) ( 26.01, 31.97)                          :End of matrix B
```

10.3 Program Results

```
 F07BSF Example Program Results

 Solution(s)
                    1                  2
 1  (-3.0000, 2.0000) ( 1.0000, 6.0000)
 2  ( 1.0000,-7.0000) (-7.0000,-4.0000)
 3  (-5.0000, 4.0000) ( 3.0000, 5.0000)
 4  ( 6.0000,-8.0000) (-8.0000, 2.0000)
```