

NAG Library Routine Document

E04RJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

E04RJF is a part of the NAG optimization modelling suite and defines the block of linear constraints of the problem.

2 Specification

```
SUBROUTINE E04RJF (HANDLE, NCLIN, BL, BU, NNZB, IROWB, ICOLB, B, IDLC,      &
                   IFAIL)

INTEGER           NCLIN, NNZB, IROWB(NNZB), ICOLB(NNZB), IDLC, IFAIL
REAL   (KIND=nag_wp) BL(NCLIN), BU(NCLIN), B(NNZB)
TYPE   (C_PTR)      HANDLE
```

3 Description

After the initialization routine E04RAF has been called, E04RJF may be used to define the linear constraints $l_B \leq Bx \leq u_B$ of the problem unless the linear constraints have already been defined. This will typically be used for problems, such as quadratic programming (QP)

$$\begin{array}{ll} \text{minimize}_{x \in \mathbb{R}^n} & \frac{1}{2}x^T Hx + c^T x & (\text{a}) \\ \text{subject to} & l_B \leq Bx \leq u_B & (\text{b}) \\ & l_x \leq x \leq u_x, & (\text{c}) \end{array} \quad (1)$$

nonlinear programming (NLP)

$$\begin{array}{ll} \text{minimize}_{x \in \mathbb{R}^n} & f(x) & (\text{a}) \\ \text{subject to} & l_g \leq g(x) \leq u_g & (\text{b}) \\ & l_B \leq Bx \leq u_B & (\text{c}) \\ & l_x \leq x \leq u_x & (\text{d}) \end{array} \quad (2)$$

linear semidefinite programming (SDP)

$$\begin{array}{ll} \text{minimize}_{x \in \mathbb{R}^n} & c^T x & (\text{a}) \\ \text{subject to} & \sum_{i=1}^n x_i A_i^k - A_0^k \succeq 0, \quad k = 1, \dots, m_A & (\text{b}) \\ & l_B \leq Bx \leq u_B & (\text{c}) \\ & l_x \leq x \leq u_x & (\text{d}) \end{array} \quad (3)$$

or semidefinite programming with bilinear matrix inequalities (BMI-SDP)

$$\begin{array}{ll} \text{minimize}_{x \in \mathbb{R}^n} & \frac{1}{2}x^T Hx + c^T x & (\text{a}) \\ \text{subject to} & \sum_{i,j=1}^n x_i x_j Q_{ij}^k + \sum_{i=1}^n x_i A_i^k - A_0^k \succeq 0, \quad k = 1, \dots, m_A & (\text{b}) \\ & l_B \leq Bx \leq u_B & (\text{c}) \\ & l_x \leq x \leq u_x & (\text{d}) \end{array} \quad (4)$$

where n is the number of decision variables, B is a general $m_B \times n$ rectangular matrix and l_B and u_B are m_B -dimensional vectors. Note that upper and lower bounds are specified for all the constraints. This form allows full generality in specifying various types of constraint. In particular, the j th constraint may be defined as an equality by setting $l_j = u_j$. If certain bounds are not present, the associated elements of

l_B or u_B may be set to special values that are treated as $-\infty$ or $+\infty$. See the description of the optional parameter **Infinite Bound Size** of the solvers in the suite, E04STF and E04SVF. Its value is denoted as *bigbnd* further in this text. Note that the bounds are interpreted based on its value at the time of calling this routine and any later alterations to **Infinite Bound Size** will not affect these constraints.

See E04RAF for more details.

4 References

None.

5 Arguments

1: HANDLE – TYPE (C_PTR) *Input*

On entry: the handle to the problem. It needs to be initialized by E04RAF and **must not** be changed.

2: NCLIN – INTEGER *Input*

On entry: m_B , the number of linear constraints (number of rows of the matrix B).

If NCLIN = 0, no linear constraints will be defined and BL, BU, NNZB, IROWB, ICOLB and B will not be referenced.

Constraint: $NCLIN \geq 0$.

3: BL(NCLIN) – REAL (KIND=nag_wp) array *Input*

4: BU(NCLIN) – REAL (KIND=nag_wp) array *Input*

On entry: BL and BU define lower and upper bounds of the linear constraints, l_B and u_B , respectively. To define the j th constraint as equality, set $BL(j) = BU(j) = \beta$, where $|\beta| < bigbnd$. To specify a nonexistent lower bound (i.e., $l_j = -\infty$), set $BL(j) \leq -bigbnd$; to specify a nonexistent upper bound, set $BU(j) \geq bigbnd$.

Constraints:

$BL(j) \leq BU(j)$, for $j = 1, 2, \dots, NCLIN$;

$BL(j) < bigbnd$, for $j = 1, 2, \dots, NCLIN$;

$BU(j) > -bigbnd$, for $j = 1, 2, \dots, NCLIN$;

if $BL(j) = BU(j)$, $|BL(j)| < bigbnd$, for $j = 1, 2, \dots, NCLIN$.

5: NNZB – INTEGER *Input*

On entry: NNZB gives the number of nonzeros in matrix B .

Constraint: if $NCLIN > 0$, $NNZB > 0$.

6: IROWB(NNZB) – INTEGER array *Input*

7: ICOLB(NNZB) – INTEGER array *Input*

8: B(NNZB) – REAL (KIND=nag_wp) array *Input*

On entry: arrays IROWB, ICOLB and B store NNZB nonzeros of the sparse matrix B in coordinate storage (CS) format (see Section 2.1.1 in the F11 Chapter Introduction). The matrix B has dimensions $m_B \times n$, where n , the number of variables in the problem, was set in NVAR during the initialization of the handle by E04RAF. IROWB specifies one-based row indices, ICOLB specifies one-based column indices and B specifies the values of the nonzero elements in such a way that $b_{ij} = B(l)$ where $i = IROWB(l)$ and $j = ICOLB(l)$, for $l = 1, 2, \dots, NNZB$. No particular order of elements is expected, but elements should not repeat.

Constraint: $1 \leq IROWB(l) \leq NCLIN$, $1 \leq ICOLB(l) \leq n$, for $l = 1, 2, \dots, NNZB$.

9:	IDLC – INTEGER	<i>Input/Output</i>
Note: IDLC is reserved for future releases of the NAG Library.		
<i>On entry:</i> if IDLC = 0, new linear constraints are added to the problem definition. This is the only value allowed at the moment.		
<i>Constraint:</i> IDLC = 0.		
<i>On exit:</i> the number of the last linear constraint added, thus NCLIN.		
10:	IFAIL – INTEGER	<i>Input/Output</i>
<i>On entry:</i> IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.		
For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, the recommended value is -1. When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.		
<i>On exit:</i> IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).		

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

The supplied HANDLE does not define a valid handle to the data structure for the NAG optimization modelling suite. It has not been initialized by E04RAF or it has been corrupted.

IFAIL = 2

The problem cannot be modified in this phase any more, the solver has already been called.

IFAIL = 3

A set of linear constraints has already been defined.

IFAIL = 4

On entry, IDLC = $\langle \text{value} \rangle$.

Constraint: IDLC = 0.

IFAIL = 6

On entry, NCLIN = $\langle \text{value} \rangle$.

Constraint: NCLIN ≥ 0 .

On entry, NNZB = $\langle \text{value} \rangle$.

Constraint: NNZB > 0 .

IFAIL = 8

On entry, $i = \langle \text{value} \rangle$, ICOLB(i) = $\langle \text{value} \rangle$ and $n = \langle \text{value} \rangle$.

Constraint: $1 \leq \text{ICOLB}(i) \leq n$.

On entry, $i = \langle \text{value} \rangle$, IROWB(i) = $\langle \text{value} \rangle$ and NCLIN = $\langle \text{value} \rangle$.

Constraint: $1 \leq \text{IROWB}(i) \leq \text{NCLIN}$.

On entry, more than one element of B has row index $\langle \text{value} \rangle$ and column index $\langle \text{value} \rangle$.

Constraint: each element of B must have a unique row and column index.

IFAIL = 10

On entry, $j = \langle value \rangle$, $\text{BL}(j) = \langle value \rangle$, $\text{bigbnd} = \langle value \rangle$.
 Constraint: $\text{BL}(j) < \text{bigbnd}$.

On entry, $j = \langle value \rangle$, $\text{BL}(j) = \langle value \rangle$ and $\text{BU}(j) = \langle value \rangle$.
 Constraint: $\text{BL}(j) \leq \text{BU}(j)$.

On entry, $j = \langle value \rangle$, $\text{BU}(j) = \langle value \rangle$, $\text{bigbnd} = \langle value \rangle$.
 Constraint: $\text{BU}(j) > -\text{bigbnd}$.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

IFAIL = -999

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

Not applicable.

8 Parallelism and Performance

E04RJF is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example demonstrates how to use the MPS file reader E04MXF and this suite of routines to define and solve a QP problem. E04MXF uses a different output format to the one required by E04RJF, in particular, it uses the compressed column storage (CCS) (see Section 2.1.3 in the F11 Chapter Introduction) instead of the coordinate storage and the linear objective vector is included in the system matrix. Therefore a simple transformation is needed before calling E04RJF as demonstrated in the example program.

The data file stores the following problem:

$$\text{minimize } c^T x + \frac{1}{2} x^T H x \quad \text{subject to} \quad \begin{array}{lcl} l_B & \leq & Bx \\ -2 & \leq & x \leq 2, \end{array}$$

where

$$c = \begin{pmatrix} -4.0 \\ -1.0 \\ -1.0 \\ -1.0 \\ -1.0 \\ -1.0 \\ -1.0 \\ -1.0 \\ -0.1 \\ -0.3 \end{pmatrix}, \quad H = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 4.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & -2.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & -1.0 & 1.0 & -1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{pmatrix},$$

$$l_B = \begin{pmatrix} -2.0 \\ -2.0 \\ -2.0 \end{pmatrix} \quad \text{and} \quad u_B = \begin{pmatrix} 1.5 \\ 1.5 \\ 4.0 \end{pmatrix}.$$

The optimal solution (to five figures) is

$$x^* = (2.0, -0.23333, -0.26667, -0.3, -0.1, 2.0, 2.0, -1.7777, -0.45555)^T.$$

See also Section 10 in E04RAF for links to further examples in this suite.

10.1 Program Text

```
Program e04rjfe
!
! E04RJF Example Program Text
!
! Read in LP/QP problem stored in a MPS file, formulated it
! as a handle and pass it to the solver.
!
! Mark 26 Release. NAG Copyright 2016.
!
! .. Use Statements ..
Use nag_library, Only: e04mxf, e04raf, e04rff, e04rhf, e04rjf, e04rzf,   &
                      e04svf, e04zmf, nag_wp, x04acf, x04adf
Use, Intrinsic          :: iso_c_binding, Only: c_null_ptr,      &
                          c_ptr
!
! .. Implicit None Statement ..
Implicit None
!
! .. Parameters ..
Integer, Parameter       :: mpslst = 1, nin = 7, nout = 6
Character (*), Parameter :: fname_default = 'e04rjfe.opt'
!
! .. Local Scalars ..
Type (c_ptr)
Integer                  :: handle
!
! .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: a(:), bl(:), bu(:), c(:), h(:),      &
                                    u(:), ua(:), uc(:), x(:)
Real (Kind=nag_wp)
Integer, Allocatable      :: rinfo(32), stats(32)
!
! .. Intrinsic Procedures ..
Intrinsic                 :: command_argument_count, count,           &
                           get_command_argument, trim
```

```

!     .. Executable Statements ..
Continue

Write (nout,*) 'E04RJF Example Program Results'
Write (nout,*)

! Use the first command line argument as the filename or
! choose default hard-coded filename in 'fname_default'.
nargs = command_argument_count()
If (nargs>=1) Then
    Call get_command_argument(1,fname)
Else
    fname = fname_default
End If

Write (nout,*) 'Reading MPS file: ', trim(fname)
Flush (nout)

! Read the input MPS file.
pnames(1:5) = ','
maxm = 0
maxn = 0
maxnnz = 0
maxnnzh = 0
maxncolh = 0
maxlintvar = -1

! Open the data file for reading.
mode = 0
ifail = 0
Call x04acf(nin,fname,mode,ifail)

! Call E04MXF in query mode to obtain an approximate problem size.
Allocate (a(maxnnz),irowa(maxnnz),iccola(maxn+1),bl(maxn+maxm),
         & bu(maxn+maxm),crname(maxn+maxm),h(maxnnzh),irowh(maxnnzh),
         & iccolh(maxncolh+1),intvar(maxlintvar))
ifail = 0
Call e04mxf(nin,maxn,maxm,maxnnz,maxncolh,maxnnzh,maxlintvar,mpslist,n,m,
            & nnz,ncolh,nnzh,lintvar,iobj,a,irowa,iccola,bl,bu,pnames,nnname,crname,
            & h,irowh,iccolh,minmax,intvar,ifail)
Deallocate (a,irowa,iccola,bl,bu,crname,h,irowh,iccolh)

! Close the data file.
ifail = 0
Call x04adf(nin,ifail)

! Set maximal problem size.
maxm = m
maxn = n
maxnnz = nnz
maxnnzh = nnzh
maxncolh = ncolh

Allocate (irowa(maxnnz),iccola(maxn+1),a(maxnnz),bl(maxn+maxm),
         & bu(maxn+maxm),crname(maxn+maxm),irowh(maxnnzh),iccolh(maxncolh+1),
         & h(maxnnzh),x(maxn),icolh(maxnnzh),icola(maxnnz))

! Open the data file for reading.
mode = 0
ifail = 0
Call x04acf(nin,fname,mode,ifail)

! Call E04MXF to read the problem.
ifail = 0
Call e04mxf(nin,maxn,maxm,maxnnz,maxncolh,maxnnzh,maxlintvar,mpslist,n,m,
            & nnz,ncolh,nnzh,lintvar,iobj,a,irowa,iccola,bl,bu,pnames,nnname,crname,
            & h,irowh,iccolh,minmax,intvar,ifail)

Write (nout,*) 'MPS/QPS file read'
Flush (nout)

```

```

!      Close the data file.
ifail = 0
Call x04adf(nin,ifail)

!      Data has been read. Set up the problem to the solver.

!      Initialize handle.
handle = c_null_ptr
ifail = 0
Call e04raf(handle,n,ifail)

!      Move linear objective from A to C.
If (iobj>0) Then
    Shift bounds.
    Do j = iobj, m - 1
        bl(n+j) = bl(n+j+1)
        bu(n+j) = bu(n+j+1)
    End Do
    m = m - 1
Extract row IOBJ.
Count how many nonzeros will be needed in C.
nnzc = count(irowa(1:nnz)==iobj)
Allocate (idxc(nnzc),c(nnzc))
idx = 1
idx_c = 1
idx_dest = 1
Do j = 1, n
    Do idx = idx, iccola(j+1) - 1
        If (irowa(idx)<iobj) Then
            a(idx_dest) = a(idx)
            irowa(idx_dest) = irowa(idx)
            idx_dest = idx_dest + 1
        Else If (irowa(idx)==iobj) Then
            idxc(idx_c) = j
            c(idx_c) = a(idx)
            idx_c = idx_c + 1
        Else
            a(idx_dest) = a(idx)
            irowa(idx_dest) = irowa(idx) - 1
            idx_dest = idx_dest + 1
        End If
    End Do
    iccola(j+1) = idx_dest
End Do
nnz = idx_dest - 1
Else
    There is no linear part of the objective function.
    nnzc = 0
    Allocate (idxc(nnzc),c(nnzc))
End If
Convert (decompress) ICCOLA() to ICOLA().
Do j = 1, n
    icola(iccola(j):iccola(j+1)-1) = j
End Do

!      Add objective function to the problem formulation.
If (nnzh==0) Then
    The objective is a (sparse) linear function.
    ifail = 0
    Call e04rff(handle,nnzc,idxc,c,nnzh,irowh,icolh,h,ifail)
Else
    The objective is a quadratic function.
    Transform (decompress) ICCOLH() -> ICOLH().
    Do j = 1, ncolh
        icolh(iccolh(j):iccolh(j+1)-1) = j
    End Do
EO4MX returned L triangle, E04RFF needs U triangle -> swap.
    ifail = 0
    Call e04rff(handle,nnzc,idxc,c,nnzh,icolh,irowh,h,ifail)
End If

```

```

!      Add box constraints to the formulation.
ifail = 0
Call e04rhf(handle,n,bl,bu,ifail)

!      Add linear constraints.
idlc = 0
ifail = 0
Call e04rjf(handle,m,bl(n+1:n+m),bu(n+1:n+m),nnz,irowa,icola,a,idlc,      &
            ifail)

Write (nout,*) 'The problem was set-up'
Flush (nout)

!      Call the solver.

!      Set optional arguments.
ifail = 0
Call e04zmf(handle,'Print Options = No',ifail)

!      Set up a starting point and call the solver.
!      Let's ignore Lagrangian multipliers U/UA.
x(:) = 0.0_nag_wp
nnzu = 0
nnzuc = 0
nnzua = 0
Allocate (u(nnzu),uc(nnzuc),ua(nnzua))

ifail = 0
Call e04svf(handle,n,x,nnzu,u,nnzuc,uc,nnzua,ua,rinfo,stats,inform,      &
            ifail)

Write (nout,*)
Write (nout,*) 'Optimal solution:'
Write (nout,99999) x(1:n)
99999 Format (1X,'X = ',3F9.2)
Flush (nout)

!      Destroy the handle.
ifail = 0
Call e04rzf(handle,ifail)

End Program e04rjfe

```

10.2 Program Data

```

NAME          E04RJ.EX
ROWS
L ..ROW1..
L ..ROW2..
L ..ROW3..
N ..COST..
COLUMNS
...X1...  ..ROW1..    1.0    ..ROW2..    1.0
...X1...  ..ROW3..    1.0    ..COST..   -4.0
...X2...  ..ROW1..    1.0    ..ROW2..    2.0
...X2...  ..ROW3..   -1.0    ..COST..   -1.0
...X3...  ..ROW1..    1.0    ..ROW2..    3.0
...X3...  ..ROW3..    1.0    ..COST..   -1.0
...X4...  ..ROW1..    1.0    ..ROW2..    4.0
...X4...  ..ROW3..   -1.0    ..COST..   -1.0
...X5...  ..ROW1..    1.0    ..ROW2..   -2.0
...X5...  ..ROW3..    1.0    ..COST..   -1.0
...X6...  ..ROW1..    1.0    ..ROW2..    1.0
...X6...  ..ROW3..    1.0    ..COST..   -1.0
...X7...  ..ROW1..    1.0    ..ROW2..    1.0
...X7...  ..ROW3..    1.0    ..COST..   -1.0
...X8...  ..ROW1..    1.0    ..ROW2..    1.0
...X8...  ..ROW3..    1.0    ..COST..   -0.1
...X9...  ..ROW1..    4.0    ..ROW2..    1.0
...X9...  ..ROW3..    1.0    ..COST..   -0.3

```

```

RHS
  RHS1    ..ROW1..      1.5
  RHS1    ..ROW2..      1.5
  RHS1    ..ROW3..      4.0
  RHS1    ..COST..   1000.0
RANGES
  RANGE1   ..ROW1..      3.5
  RANGE1   ..ROW2..      3.5
  RANGE1   ..ROW3..      6.0
BOUNDS
  LO BOUND   ...X1...     -2.0
  LO BOUND   ...X2...     -2.0
  LO BOUND   ...X3...     -2.0
  LO BOUND   ...X4...     -2.0
  LO BOUND   ...X5...     -2.0
  LO BOUND   ...X6...     -2.0
  LO BOUND   ...X7...     -2.0
  LO BOUND   ...X8...     -2.0
  LO BOUND   ...X9...     -2.0
  UP BOUND   ...X1...      2.0
  UP BOUND   ...X2...      2.0
  UP BOUND   ...X3...      2.0
  UP BOUND   ...X4...      2.0
  UP BOUND   ...X5...      2.0
  UP BOUND   ...X6...      2.0
  UP BOUND   ...X7...      2.0
  UP BOUND   ...X8...      2.0
  UP BOUND   ...X9...      2.0
QUADOBJ
  ...X1...   ...X1...  2.000000000E0  ...X2...  1.000000000E0
  ...X1...   ...X3...  1.000000000E0  ...X4...  1.000000000E0
  ...X1...   ...X5...  1.000000000E0
  ...X2...   ...X2...  2.000000000E0  ...X3...  1.000000000E0
  ...X2...   ...X4...  1.000000000E0  ...X5...  1.000000000E0
  ...X3...   ...X3...  2.000000000E0  ...X4...  1.000000000E0
  ...X3...   ...X5...  1.000000000E0
  ...X4...   ...X4...  2.000000000E0  ...X5...  1.000000000E0
  ...X5...   ...X5...  2.000000000E0
ENDATA

```

10.3 Program Results

E04RJF Example Program Results

Reading MPS file: e04rjfe.opt

```

MPSX INPUT LISTING
-----
Searching for indicator line
Line      1: Found NAME indicator line
          Query mode - Ignoring NAME data.
Line      2: Found ROWS indicator line
          Query mode - Counting ROWS data.
Line      7: Found COLUMNS indicator line
          Query mode - Counting COLUMNS data.
Line     26: Found RHS indicator line
          Query mode - Ignoring RHS data.
Line    31: Found RANGES indicator line
          Query mode - Ignoring RANGES data.
Line    35: Found BOUNDS indicator line
          Query mode - Ignoring BOUNDS data.
Line   54: Found QUADOBJ indicator line
          Query mode - Counting QUADOBJ data.
          Query mode - End of QUADOBJ data. Exit

```

```

MPSX INPUT LISTING
-----
Searching for indicator line
Line      1: Found NAME indicator line
Line      2: Found ROWS indicator line

```

```

Line      7: Found COLUMNS indicator line
Line     26: Found RHS indicator line
Line    31: Found RANGES indicator line
Line    35: Found BOUNDS indicator line
Line    54: Found QUADOBJ indicator line
Line    64: Found ENDATA indicator line
MPS/QPS file read
The problem was set-up
E04SV, NLP-SDP Solver (Pennon)
-----
Number of variables          9           [eliminated]          0]
                           simple   linear   nonlin
(Standard) inequalities      18         6         0
(Standard) equalities        0         0
Matrix inequalities           0         0 [dense      0, sparse   0]
                                         [max dimension]  0]

-----
it| objective | optim | feas | compl | pen min | inner
-----
  0  0.00000E+00  4.70E+00  0.00E+00  1.00E+01  1.00E+00  0
  1 -6.76762E+00  9.46E-04  0.00E+00  6.25E-01  1.00E+00  4
  2 -7.82467E+00  1.79E-03  3.69E-02  1.45E-01  4.65E-01  4
  3 -8.02059E+00  7.27E-03  2.27E-02  3.38E-02  2.16E-01  4
  4 -8.06187E+00  3.18E-03  5.75E-03  7.86E-03  1.01E-01  4
  5 -8.06653E+00  1.30E-03  1.13E-03  1.83E-03  4.68E-02  5
  6 -8.06739E+00  6.98E-03  1.42E-04  4.25E-04  2.18E-02  3
  7 -8.06775E+00  2.16E-04  2.80E-05  9.89E-05  1.01E-02  2
  8 -8.06778E+00  4.44E-05  6.94E-05  2.30E-05  4.71E-03  1
  9 -8.06778E+00  1.88E-06  1.15E-05  5.35E-06  2.19E-03  1
 10 -8.06778E+00  4.38E-08  1.52E-06  1.24E-06  1.02E-03  1
 11 -8.06778E+00  6.52E-10  1.74E-07  2.90E-07  4.74E-04  1
 12 -8.06778E+00  8.12E-12  1.90E-08  6.73E-08  2.21E-04  1

-----
Status: converged, an optimal solution found
-----
Final objective value          -8.067778E+00
Relative precision            1.518278E-09
Optimality                   8.116995E-12
Feasibility                  1.900689E-08
Complementarity              6.734260E-08
Iteration counts
  Outer iterations             12
  Inner iterations             31
  Linesearch steps             43
Evaluation counts
  Augm. Lagr. values          56
  Augm. Lagr. gradient         44
  Augm. Lagr. hessian          31
-----
Optimal solution:
X =      2.00     -0.23     -0.27
X =     -0.30     -0.10      2.00
X =      2.00     -1.78     -0.46

```