

NAG Library Routine Document

G13MGF

1 Purpose

G13MGF provides a moving average, moving norm, moving variance and moving standard deviation operator for an inhomogeneous time series.

2 Specification

```

SUBROUTINE G13MGF (NB, MA, T, TAU, M1, M2, SINIT, INTER, FTYPE, P, PN,      &
                  WMA, RCOMM, LRCOMM, IFAIL)
INTEGER          NB, M1, M2, INTER(2), FTYPE, PN, LRCOMM, IFAIL
REAL (KIND=nag_wp) MA(NB), T(NB), TAU, SINIT(*), P, WMA(NB),      &
                  RCOMM(LRCOMM)

```

3 Description

G13MGF provides a number of operators for an inhomogeneous time series. The time series is represented by two vectors of length n ; a vector of times, t ; and a vector of values, z . Each element of the time series is therefore composed of the pair of scalar values (t_i, z_i) , for $i = 1, 2, \dots, n$. Time t can be measured in any arbitrary units, as long as all elements of t use the same units.

The main operator available, the moving average (MA), with parameter τ is defined as

$$\text{MA}[\tau, m_1, m_2; y](t_i) = \frac{1}{m_2 - m_1 + 1} \sum_{j=m_1}^{m_2} \text{EMA}[\tilde{\tau}, j; y](t_i) \quad (1)$$

where $\tilde{\tau} = \frac{2\tau}{m_2 + m_1}$, m_1 and m_2 are user-supplied integers controlling the amount of lag and smoothing respectively, with $m_2 \geq m_1$ and $\text{EMA}[\cdot]$ is the iterated exponential moving average operator.

The iterated exponential moving average, $\text{EMA}[\tilde{\tau}, m; y](t_i)$, is defined using the recursive formula:

$$\text{EMA}[\tilde{\tau}, m; y](t_i) = \text{EMA}[\tilde{\tau}; \text{EMA}[\tilde{\tau}, m - 1; y](t_i)](t_i)$$

with

$$\text{EMA}[\tilde{\tau}, 1; y](t_i) = \text{EMA}[\tilde{\tau}; y](t_i)$$

and

$$\text{EMA}[\tilde{\tau}; y](t_i) = \mu \text{EMA}[\tilde{\tau}; y](t_{i-1}) + (\nu - \mu)y_{i-1} + (1 - \nu)y_i$$

where

$$\mu = e^{-\alpha} \quad \text{and} \quad \alpha = \frac{t_i - t_{i-1}}{\tilde{\tau}}.$$

The value of ν depends on the method of interpolation chosen and the relationship between y and the input series z depends on the transformation function chosen. G13MGF gives the option of three interpolation methods:

1. Previous point: $\nu = 1$.
2. Linear: $\nu = (1 - \mu)/\alpha$.
3. Next point: $\nu = \mu$.

and three transformation functions:

1. Identity: $y_i = z_i^{[p]}$.
2. Absolute value: $y_i = |z_i|^p$.
3. Absolute difference: $y_i = |z_i - \text{MA}[\tau, m_1, m_2; z](t_i)|^p$.

where the notation $[p]$ is used to denote the integer nearest to p . In addition, if either the absolute value or absolute difference transformation are used then the resulting moving average can be scaled by p^{-1} .

The various parameter options allow a number of different operators to be applied by G13MGF, a few of which are:

(i) **Moving Average (MA)**, as defined in (1) (obtained by setting $\text{FTYPE} = 1$ and $P = 1$).

(ii) **Moving Norm (MNorm)**, defined as

$$\text{MNorm}(\tau, m, p; z) = \text{MA}[\tau, 1, m; |z|^p]^{1/p}$$

(obtained by setting $\text{FTYPE} = 4$, $M1 = 1$ and $M2 = m$).

(iii) **Moving Variance (MVar)**, defined as

$$\text{MVar}(\tau, m, p; z) = \text{MA}[\tau, 1, m; |z - \text{MA}[\tau, 1, m; z]|^p]$$

(obtained by setting $\text{FTYPE} = 3$, $M1 = 1$ and $M2 = m$).

(iv) **Moving Standard Deviation (MSD)**, defined as

$$\text{MSD}(\tau, m, p; z) = \text{MA}[\tau, 1, m; |z - \text{MA}[\tau, 1, m; z]|^p]^{1/p}$$

(obtained by setting $\text{FTYPE} = 5$, $M1 = 1$ and $M2 = m$).

For large datasets or where all the data is not available at the same time, z and t can be split into arbitrary sized blocks and G13MGF called multiple times.

4 References

Dacorogna M M, Gencay R, Müller U, Olsen R B and Pictet O V (2001) *An Introduction to High-frequency Finance* Academic Press

Zumbach G O and Müller U A (2001) Operators on inhomogeneous time series *International Journal of Theoretical and Applied Finance* **4(1)** 147–178

5 Parameters

1: NB – INTEGER *Input*

On entry: b , the number of observations in the current block of data. At each call the size of the block of data supplied in MA and T can vary; therefore NB can change between calls to G13MGF.

Constraint: $\text{NB} \geq 0$.

2: MA(NB) – REAL (KIND=nag_wp) array *Input/Output*

On entry: z_i , the current block of observations, for $i = k + 1, \dots, k + b$, where k is the number of observations processed so far, i.e., the value supplied in PN on entry.

On exit: the moving average:

if $\text{FTYPE} = 4$ or 5

$$\text{MA}(i) = \{\text{MA}[\tau, m_1, m_2; y](t_i)\}^{1/p},$$

otherwise

$$\text{MA}(i) = \text{MA}[\tau, m_1, m_2; y](t_i).$$

- 3: T(NB) – REAL (KIND=nag_wp) array *Input*
On entry: t_i , the times for the current block of observations, for $i = k + 1, \dots, k + b$, where k is the number of observations processed so far, i.e., the value supplied in PN on entry.
 If $t_i \leq t_{i-1}$, IFAIL = 31 will be returned, but G13MGF will continue as if t was strictly increasing by using the absolute value. The lagged difference, $t_i - t_{i-1}$ must be sufficiently small that $e^{-\alpha}$, $\alpha = (t_i - t_{i-1})/\tilde{\tau}$ can be calculated without overflowing, for all i .
- 4: TAU – REAL (KIND=nag_wp) *Input*
On entry: τ , the parameter controlling the rate of decay. τ must be sufficiently large that $e^{-\alpha}$, $\alpha = (t_i - t_{i-1})/\tilde{\tau}$ can be calculated without overflowing, for all i , where $\tilde{\tau} = \frac{2\tau}{m_2 + m_1}$.
Constraint: TAU > 0.0.
- 5: M1 – INTEGER *Input*
On entry: m_1 , the iteration of the EMA operator at which the sum is started.
Constraint: M1 \geq 1.
- 6: M2 – INTEGER *Input*
On entry: m_2 , the iteration of the EMA operator at which the sum is ended.
Constraint: M2 \geq M1.
- 7: SINIT(*) – REAL (KIND=nag_wp) array *Input*
Note: the dimension of the array SINIT must be at least $2 \times M2 + 3$ if FTYPE = 3 or 5, and at least $M2 + 2$ otherwise.
On entry: if PN = 0, the values used to start the iterative process, with

$$\begin{aligned} \text{SINIT}(1) &= t_0, \\ \text{SINIT}(2) &= y_0, \\ \text{SINIT}(j+2) &= \text{EMA}[\tau, j; y](t_0), \text{ for } i = 1, 2, \dots, M2. \end{aligned}$$
 In addition, if FTYPE = 3 or 5 then

$$\begin{aligned} \text{SINIT}(M2+3) &= z_0, \\ \text{SINIT}(M2+j+2) &= \text{EMA}[\tau, j; z](t_0), \text{ for } j = 1, 2, \dots, M2. \end{aligned}$$
 i.e., initial values based on the original data z as opposed to the transformed data y .
 If PN \neq 0, SINIT is not referenced.
Constraint: if FTYPE \neq 1, SINIT(j) \geq 0, for $j = 2, 3, \dots, M2 + 2$.
- 8: INTER(2) – INTEGER array *Input*
On entry: the type of interpolation used with INTER(1) indicating the interpolation method to use when calculating $\text{EMA}[\tau, 1; z]$ and INTER(2) the interpolation method to use when calculating $\text{EMA}[\tau, j; z]$, $j > 1$.
 Three types of interpolation are possible:

$$\begin{aligned} \text{INTER}(i) = 1 & \\ & \text{Previous point, with } \nu = 1. \\ \text{INTER}(i) = 2 & \\ & \text{Linear, with } \nu = (1 - \mu)/\alpha. \\ \text{INTER}(i) = 3 & \\ & \text{Next point, } \nu = \mu. \end{aligned}$$

Zumbach and Müller (2001) recommend that linear interpolation is used in second and subsequent iterations, i.e., $\text{INTER}(2) = 2$, irrespective of the interpolation method used at the first iteration, i.e., the value of $\text{INTER}(1)$.

Constraint: $\text{INTER}(i) = 1, 2$ or 3 , for $i = 1, 2$.

9: FTYPE – INTEGER *Input*

On entry: the function type used to define the relationship between y and z when calculating $\text{EMA}[\tau, 1; y]$. Three functions are provided:

FTYPE = 1

The identity function, with $y_i = z_i^{[p]}$.

FTYPE = 2 or 4

The absolute value, with $y_i = |z_i|^p$.

FTYPE = 3 or 5

The absolute difference, with $y_i = |z_i - \text{MA}[\tau, m; y](t_i)|^p$.

If FTYPE = 4 or 5 then the resulting vector of averages is scaled by p^{-1} as described in MA.

Constraint: FTYPE = 1, 2, 3, 4 or 5.

10: P – REAL (KIND=nag_wp) *Input/Output*

On entry: p , the power used in the transformation function.

On exit: if FTYPE = 1, then $[p]$, the actual power used in the transformation function is returned, otherwise P is unchanged.

Constraint: $P \neq 0$.

11: PN – INTEGER *Input/Output*

On entry: k , the number of observations processed so far. On the first call to G13MGF, or when starting to summarise a new dataset, PN must be set to 0. On subsequent calls it must be the same value as returned by the last call to G13MGF.

On exit: $k + b$, the updated number of observations processed so far.

Constraint: $\text{PN} \geq 0$.

12: WMA(NB) – REAL (KIND=nag_wp) array *Output*

On exit: either the moving average or exponential moving average, depending on the value of FTYPE.

if FTYPE = 3 or 5

$$\text{WMA}(i) = \text{MA}[\tau; y](t_i)$$

otherwise

$$\text{WMA}(i) = \text{EMA}[\tilde{\tau}; y](t_i).$$

13: RCOMM(LRCOMM) – REAL (KIND=nag_wp) array *Communication Array*

On entry: communication array, used to store information between calls to G13MGF. If LRCOMM = 0, RCOMM is not referenced, PN must be set to 0 and all the data must be supplied in one go.

14: LRCOMM – INTEGER *Input*

On entry: the dimension of the array RCOMM as declared in the (sub)program from which G13MGF is called.

Constraint: $\text{LRCOMM} = 0$ or $\text{LRCOMM} \geq 2 \times \text{M2} + 20$.

15: IFAIL – INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 11

On entry, NB = $\langle value \rangle$.
Constraint: NB \geq 0.

IFAIL = 31

On entry, $i = \langle value \rangle$, $T(i - 1) = \langle value \rangle$ and $T(i) = \langle value \rangle$.
Constraint: T should be strictly increasing.

IFAIL = 32

On entry, $i = \langle value \rangle$, $T(i - 1) = \langle value \rangle$ and $T(i) = \langle value \rangle$.
Constraint: $T(i) \neq T(i - 1)$ if linear interpolation is being used.

IFAIL = 41

On entry, TAU = $\langle value \rangle$.
Constraint: TAU $>$ 0.0.

IFAIL = 42

On entry, TAU = $\langle value \rangle$.
On entry at previous call, TAU = $\langle value \rangle$.
Constraint: if PN $>$ 0 then TAU must be unchanged since previous call.

IFAIL = 51

On entry, M1 = $\langle value \rangle$.
Constraint: M1 \geq 1.

IFAIL = 52

On entry, M1 = $\langle value \rangle$.
On entry at previous call, M1 = $\langle value \rangle$.
Constraint: if PN $>$ 0 then M1 must be unchanged since previous call.

IFAIL = 61

On entry, M1 = $\langle value \rangle$ and M2 = $\langle value \rangle$.
Constraint: M2 \geq M1.

IFAIL = 62

On entry, $M2 = \langle value \rangle$.

On entry at previous call, $M2 = \langle value \rangle$.

Constraint: if $PN > 0$ then $M2$ must be unchanged since previous call.

IFAIL = 71

On entry, $FTYPE \neq 1$, $j = \langle value \rangle$ and $SINIT(j) = \langle value \rangle$.

Constraint: if $FTYPE \neq 1$, $SINIT(j) \geq 0.0$, for $j = 2, 3, \dots, M2 + 2$.

IFAIL = 81

On entry, $INTER(1) = \langle value \rangle$.

Constraint: $INTER(1) = 1, 2$ or 3 .

IFAIL = 82

On entry, $INTER(2) = \langle value \rangle$.

Constraint: $INTER(2) = 1, 2$ or 3 .

IFAIL = 83

On entry, $INTER(1) = \langle value \rangle$ and $INTER(2) = \langle value \rangle$.

On entry at previous call, $INTER(1) = \langle value \rangle$, $INTER(2) = \langle value \rangle$.

Constraint: if $PN \neq 0$, $INTER$ must be unchanged since the last call.

IFAIL = 91

On entry, $FTYPE = \langle value \rangle$.

Constraint: $FTYPE = 1, 2, 3, 4$ or 5 .

IFAIL = 92

On entry, $FTYPE = \langle value \rangle$, On entry at previous call, $FTYPE = \langle value \rangle$.

Constraint: if $PN \neq 0$, $FTYPE$ must be unchanged since the previous call.

IFAIL = 101

On entry, $P = \langle value \rangle$.

Constraint: absolute value of P must be representable as an integer.

IFAIL = 102

On entry, $P = \langle value \rangle$.

Constraint: if $FTYPE \neq 1$, $P \neq 0.0$. If $FTYPE = 1$, the nearest integer to P must not be 0.

IFAIL = 103

On entry, $i = \langle value \rangle$, $MA(i) = \langle value \rangle$ and $P = \langle value \rangle$.

Constraint: if $FTYPE = 1, 2$ or 4 and $MA(i) = 0$ for any i then $P > 0.0$.

IFAIL = 104

On entry, $i = \langle value \rangle$, $MA(i) = \langle value \rangle$, $WMA(i) = \langle value \rangle$ and $P = \langle value \rangle$.

Constraint: if $P < 0.0$, $MA(i) - WMA(i) \neq 0.0$, for any i .

IFAIL = 105

On entry, $P = \langle value \rangle$.

On exit from previous call, $P = \langle value \rangle$.

Constraint: if $PN > 0$ then P must be unchanged since previous call.

IFAIL = 111

On entry, PN = $\langle value \rangle$.
Constraint: $PN \geq 0$.

IFAIL = 112

On entry, PN = $\langle value \rangle$.
On exit from previous call, PN = $\langle value \rangle$.
Constraint: if $PN > 0$ then PN must be unchanged since previous call.

IFAIL = 131

RCOMM has been corrupted between calls.

IFAIL = 141

On entry, PN = 0, LRCOMM = $\langle value \rangle$ and M2 = $\langle value \rangle$.
Constraint: if $PN = 0$, $LRCOMM = 0$ or $LRCOMM \geq 2M2 + 20$.

IFAIL = 142

On entry, $PN \neq 0$, LRCOMM = $\langle value \rangle$ and M2 = $\langle value \rangle$.
Constraint: if $PN \neq 0$, $LRCOMM \geq 2M2 + 20$.

IFAIL = 301

Truncation occurred to avoid overflow, check for extreme values in T, MA or for TAU. Results are returned using the truncated values.

IFAIL = -99

An unexpected error has been triggered by this routine. Please contact NAG.
See Section 3.8 in the Essential Introduction for further information.

IFAIL = -399

Your licence key may have expired or may not have been installed correctly.
See Section 3.7 in the Essential Introduction for further information.

IFAIL = -999

Dynamic memory allocation failed.
See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

Not applicable.

8 Parallelism and Performance

G13MGF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

G13MGF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Approximately $4m_2$ real elements are internally allocated by G13MGF. If `FTYPE = 3` or `5` then a further `NB` real elements are also allocated.

The more data you supply to G13MGF in one call, i.e., the larger `NB` is, the more efficient the routine will be, particularly if the routine is being run using more than one thread.

Checks are made during the calculation of α and y_i to avoid overflow. If a potential overflow is detected the offending value is replaced with a large positive or negative value, as appropriate, and the calculations performed based on the replacement values. In such cases `IFAIL = 301` is returned. This should not occur in standard usage and will only occur if extreme values of `MA`, `T` or `TAU` are supplied.

10 Example

The example reads in a simulated time series, (t, z) and calculates the moving average. The data is supplied in three blocks of differing sizes.

10.1 Program Text

```

Program g13mgfe
!   G13MGF Example Program Text

!   Mark 25 Release. NAG Copyright 2014.

!   .. Use Statements ..
Use nag_library, Only: g13mgf, nag_wp
!   .. Implicit None Statement ..
Implicit None
!   .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!   .. Local Scalars ..
Real (Kind=nag_wp)         :: p, tau
Integer                    :: ftype, i, ierr, ifail, lrcomm,      &
                             lsinit, m1, m2, nb, pn
!   .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: ma(:), rcomm(:), sinit(:), t(:),      &
                             wma(:)
Integer                    :: inter(2)
!   .. Intrinsic Procedures ..
Intrinsic                  :: repeat
!   .. Executable Statements ..
Write (nout,*) 'G13MGF Example Program Results'
Write (nout,*)

!   Skip heading in data file
Read (nin,*)

!   Read in the number of iterations required
Read (nin,*) m1, m2

!   Read in the transformation function, its parameter, the interpolation
!   method to use and the decay parameter tau
Read (nin,*) ftype, p, inter(1:2), tau

!   Read in the initial values
If (ftype==3 .Or. ftype==5) Then
  lsinit = 2*m2 + 3
Else
  lsinit = m2 + 2
End If
Allocate (sinit(lsinit))
Read (nin,*) sinit(1:lsinit)

!   Print some titles
Write (nout,99997) 'Time', 'MA'
Write (nout,99998) repeat('-',32)

```



```

      lrcomm = 20 + 2*m2
      Allocate (rcomm(lrcomm))

!      Loop over each block of data
      pn = 0
      Do
!      Read in the number of observations in this block
      Read (nin,*,Iostat=ierr) nb
      If (ierr/=0) Exit

!      Allocate MA, T and WMA to the required size
      Allocate (ma(nb),t(nb),wma(nb))

!      Read in the data for this block
      Do i = 1, nb
        Read (nin,*) t(i), ma(i)
      End Do

!      Update the moving average operator for this block of data
!      G13MGF overwrites the input data
      ifail = 0
      Call g13mgf(nb,ma,t,tau,m1,m2,sinit,inter,ftype,p,pn,wma,rcomm,lrcomm, &
        ifail)

!      Display the results for this block of data
      Write (nout,99999)(pn-nb+i,t(i),ma(i),i=1,nb)
      Write (nout,*)

      Deallocate (t,ma,wma)
      End Do

99999 Format (1X,I3,4X,F10.1,4X,F10.3)
99998 Format (1X,A)
99997 Format (14X,A,10X,A)
      End Program g13mgfe

```

10.2 Program Data

G13MGF Example Program Data

```

1 2          :: M1,M2
1 1.0 3 2 2.0 :: FTYPE,P,INTER(1:2),TAU
0.0 0.0 0.0 0.0 :: SINIT

5          :: NB
7.5 0.6
8.2 0.6
18.1 0.8
22.8 0.1
25.8 0.2    :: End of T and Z for first block

10         :: NB
26.8 0.2
31.1 0.5
38.4 0.7
45.9 0.1
48.2 0.4
48.9 0.7
57.9 0.8
58.5 0.3
63.9 0.2
65.2 0.5    :: End of T and Z for second block

15         :: NB
66.6 0.2
67.4 0.3
69.3 0.8
69.9 0.6
73.0 0.1
75.6 0.7
77.0 0.9

```

84.7 0.6
 86.8 0.3
 88.0 0.1
 88.5 0.1
 91.0 0.4
 93.0 1.0
 93.7 1.0
 94.0 0.1

:: End of T and Z for third block

10.3 Program Results

G13MGF Example Program Results

	Time	MA
1	7.5	0.545
2	8.2	0.567
3	18.1	0.786
4	22.8	0.214
5	25.8	0.187
6	26.8	0.192
7	31.1	0.444
8	38.4	0.680
9	45.9	0.155
10	48.2	0.298
11	48.9	0.406
12	57.9	0.777
13	58.5	0.677
14	63.9	0.258
15	65.2	0.351
16	66.6	0.291
17	67.4	0.289
18	69.3	0.572
19	69.9	0.593
20	73.0	0.244
21	75.6	0.532
22	77.0	0.715
23	84.7	0.618
24	86.8	0.426
25	88.0	0.284
26	88.5	0.240
27	91.0	0.332
28	93.0	0.723
29	93.7	0.814
30	94.0	0.744
