

# NAG Library Routine Document

## G13CCF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

G13CCF calculates the smoothed sample cross spectrum of a bivariate time series using one of four lag windows: rectangular, Bartlett, Tukey or Parzen.

### 2 Specification

SUBROUTINE G13CCF (NXY, MTXY, PXY, IW, MW, ISH, IC, NC, CXY, CYX, KC, L, &  
 NXYG, XG, YG, NG, IFAIL)  
 INTEGER NXY, MTXY, IW, MW, ISH, IC, NC, KC, L, NXYG, NG, &  
 IFAIL  
 REAL (KIND=nag\_wp) PXY, CXY(NC), CYX(NC), XG(NXYG), YG(NXYG)

### 3 Description

The smoothed sample cross spectrum is a complex valued function of frequency  $\omega$ ,  $f_{xy}(\omega) = cf(\omega) + iqf(\omega)$ , defined by its real part or co-spectrum

$$cf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S) \cos(\omega k)$$

and imaginary part or quadrature spectrum

$$qf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S) \sin(\omega k)$$

where  $w_k = w_{-k}$ , for  $k = 0, 1, \dots, M-1$ , is the smoothing lag window as defined in the description of G13CAF. The alignment shift  $S$  is recommended to be chosen as the lag  $k$  at which the cross-covariances  $c_{xy}(k)$  peak, so as to minimize bias.

The results are calculated for frequency values

$$\omega_j = \frac{2\pi j}{L}, \quad j = 0, 1, \dots, [L/2],$$

where  $[\ ]$  denotes the integer part.

The cross-covariances  $c_{xy}(k)$  may be supplied by you, or constructed from supplied series  $x_1, x_2, \dots, x_n$ ;  $y_1, y_2, \dots, y_n$  as

$$c_{xy}(k) = \frac{\sum_{t=1}^{n-k} x_t y_{t+k}}{n}, \quad k \geq 0$$

$$c_{xy}(k) = \frac{\sum_{t=1-k}^n x_t y_{t+k}}{n} = c_{yx}(-k), \quad k < 0$$

this convolution being carried out using the finite Fourier transform.

The supplied series may be mean and trend corrected and tapered before calculation of the cross-covariances, in exactly the manner described in G13CAF for univariate spectrum estimation. The results are corrected for any bias due to tapering.

The bandwidth associated with the estimates is not returned. It will normally already have been calculated in previous calls of G13CAF for estimating the univariate spectra of  $y_t$  and  $x_t$ .

## 4 References

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

## 5 Parameters

- 1: NXY – INTEGER *Input*  
*On entry:*  $n$ , the length of the time series  $x$  and  $y$ .  
*Constraint:*  $NXY \geq 1$ .
- 2: MTXY – INTEGER *Input*  
*On entry:* if cross-covariances are to be calculated by the routine ( $IC = 0$ ), MTXY must specify whether the data is to be initially mean or trend corrected.  
 MTXY = 0  
     For no correction.  
 MTXY = 1  
     For mean correction.  
 MTXY = 2  
     For trend correction.  
 If cross-covariances are supplied ( $IC \neq 0$ ), MTXY is not used.  
*Constraint:* if  $IC = 0$ , MTXY = 0, 1 or 2.
- 3: PXY – REAL (KIND=nag\_wp) *Input*  
*On entry:* if cross-covariances are to be calculated by the routine ( $IC = 0$ ), PXY must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. A value of 0.0 implies no tapering.  
 If cross-covariances are supplied ( $IC \neq 0$ ), PXY is not used.  
*Constraint:* if  $IC = 0$ ,  $0.0 \leq PXY \leq 1.0$ .
- 4: IW – INTEGER *Input*  
*On entry:* the choice of lag window.  
 IW = 1  
     Rectangular.  
 IW = 2  
     Bartlett.  
 IW = 3  
     Tukey.  
 IW = 4  
     Parzen.  
*Constraint:*  $1 \leq IW \leq 4$ .

- 5: MW – INTEGER *Input*  
*On entry:*  $M$ , the ‘cut-off’ point of the lag window, relative to any alignment shift that has been applied. Windowed cross-covariances at lags  $(-MW + ISH)$  or less, and at lags  $(MW + ISH)$  or greater are zero.  
*Constraints:*  
 $MW \geq 1;$   
 $MW + |ISH| \leq NXY.$
- 6: ISH – INTEGER *Input*  
*On entry:*  $S$ , the alignment shift between the  $x$  and  $y$  series. If  $x$  leads  $y$ , the shift is positive.  
*Constraint:*  $-MW < ISH < MW.$
- 7: IC – INTEGER *Input*  
*On entry:* indicates whether cross-covariances are to be calculated in the routine or supplied in the call to the routine.  
 $IC = 0$   
 Cross-covariances are to be calculated.  
 $IC \neq 0$   
 Cross-covariances are to be supplied.
- 8: NC – INTEGER *Input*  
*On entry:* the number of cross-covariances to be calculated in the routine or supplied in the call to the routine.  
*Constraint:*  $MW + |ISH| \leq NC \leq NXY.$
- 9: CXY(NC) – REAL (KIND=nag\_wp) array *Input/Output*  
*On entry:* if  $IC \neq 0$ , CXY must contain the NC cross-covariances between values in the  $y$  series and earlier values in time in the  $x$  series, for lags from 0 to  $(NC - 1)$ .  
 If  $IC = 0$ , CXY need not be set.  
*On exit:* if  $IC = 0$ , CXY will contain the NC calculated cross-covariances.  
 If  $IC \neq 0$ , the contents of CXY will be unchanged.
- 10: CYX(NC) – REAL (KIND=nag\_wp) array *Input/Output*  
*On entry:* if  $IC \neq 0$ , CYX must contain the NC cross-covariances between values in the  $y$  series and later values in time in the  $x$  series, for lags from 0 to  $(NC - 1)$ .  
 If  $IC = 0$ , CYX need not be set.  
*On exit:* if  $IC = 0$ , CYX will contain the NC calculated cross-covariances.  
 If  $IC \neq 0$ , the contents of CYX will be unchanged.
- 11: KC – INTEGER *Input*  
*On entry:* if  $IC = 0$ , KC must specify the order of the fast Fourier transform (FFT) used to calculate the cross-covariances. KC should be a product of small primes such as  $2^m$  where  $m$  is the smallest integer such that  $2^m \geq n + NC$ .  
 If  $IC \neq 0$ , that is if covariances are supplied, KC is not used.  
*Constraint:*  $KC \geq NXY + NC$ . The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

12: L – INTEGER *Input*

*On entry:* L, the frequency division of the spectral estimates as  $\frac{2\pi}{L}$ . Therefore it is also the order of the FFT used to construct the sample spectrum from the cross-covariances. L should be a product of small primes such as  $2^m$  where  $m$  is the smallest integer such that  $2^m \geq 2M - 1$ .

*Constraint:*  $L \geq 2 \times MW - 1$ . The largest prime factor of L must not exceed 19, and the total number of prime factors of L, counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

13: NXYG – INTEGER *Input*

*On entry:* the dimension of the arrays XG and YG as declared in the (sub)program from which G13CCF is called.

*Constraints:*

if IC = 0,  $NXYG \geq \max(KC, L)$ ;  
otherwise  $NXYG \geq L$ .

14: XG(NXYG) – REAL (KIND=nag\_wp) array *Input/Output*

*On entry:* if the cross-covariances are to be calculated, then XG must contain the NXY data points of the  $x$  series. If covariances are supplied, XG need not be set.

*On exit:* contains the real parts of the NG complex spectral estimates in elements XG(1) to XG(NG), and XG(NG + 1) to XG(NXYG) contain 0.0. The  $y$  series leads the  $x$  series.

15: YG(NXYG) – REAL (KIND=nag\_wp) array *Input/Output*

*On entry:* if cross-covariances are to be calculated, YG must contain the NXY data points of the  $y$  series. If covariances are supplied, YG need not be set.

*On exit:* contains the imaginary parts of the NG complex spectral estimates in elements YG(1) to YG(NG), and YG(NG + 1) to YG(NXYG) contain 0.0. The  $y$  series leads the  $x$  series.

16: NG – INTEGER *Output*

*On exit:* the number,  $[L/2] + 1$ , of complex spectral estimates, whose separate parts are held in XG and YG.

17: IFAIL – INTEGER *Input/Output*

*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

*On exit:* IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry  $IFAIL = 0$  or  $-1$ , explanatory error messages are output on the current error message unit (as defined by  $X04AAF$ ).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry,  $NXY < 1$ ,  
 or  $MTXY < 0$  and  $IC = 0$ ,  
 or  $MTXY > 2$  and  $IC = 0$ ,  
 or  $PXY < 0.0$  and  $IC = 0$ ,  
 or  $PXY > 1.0$  and  $IC = 0$ ,  
 or  $IW \leq 0$ ,  
 or  $IW > 4$ ,  
 or  $MW < 1$ ,  
 or  $MW + |ISH| > NXY$ ,  
 or  $|ISH| \geq MW$ ,  
 or  $NC < MW + |ISH|$ ,  
 or  $NC > NXY$ ,  
 or  $NXYG < \max(KC, L)$  and  $IC = 0$ ,  
 or  $NXYG < L$  and  $IC \neq 0$ .

$IFAIL = 2$

On entry,  $KC < NXY + NC$ ,  
 or  $KC$  has a prime factor exceeding 19,  
 or  $KC$  has more than 20 prime factors, counting repetitions.

This error only occurs when  $IC = 0$ .

$IFAIL = 3$

On entry,  $L < 2 \times MW - 1$ ,  
 or  $L$  has a prime factor exceeding 19,  
 or  $L$  has more than 20 prime factors, counting repetitions.

$IFAIL = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

$IFAIL = -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

$IFAIL = -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

## 7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

## 8 Parallelism and Performance

Not applicable.

## 9 Further Comments

G13CCF carries out two FFTs of length  $KC$  to calculate the sample cross-covariances and one FFT of length  $L$  to calculate the sample spectrum. The timing of G13CCF is therefore dependent on the choice of these values. The time taken for an FFT of length  $n$  is approximately proportional to  $n \log(n)$  (but see Section 9 in C06PAF for further details).

## 10 Example

This example reads two time series of length 296. It then selects mean correction, a 10% tapering proportion, the Parzen smoothing window and a cut-off point of 35 for the lag window. The alignment shift is set to 3 and 50 cross-covariances are chosen to be calculated. The program then calls G13CCF to calculate the cross spectrum and then prints the cross-covariances and cross spectrum.

### 10.1 Program Text

```

Program g13ccfe

!      G13CCF Example Program Text

!      Mark 25 Release. NAG Copyright 2014.

!      .. Use Statements ..
Use nag_library, Only: g13ccf, nag_wp
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Real (Kind=nag_wp)         :: pxy
Integer                    :: i, ic, ifail, ii, ish, iw, kc, l,      &
                             lxg, lyg, mtxy, mw, nc, ng, nxy, nxyg
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: cxy(:), cyx(:), xg(:), yg(:)
!      .. Intrinsic Procedures ..
Intrinsic                   :: max, min
!      .. Executable Statements ..
Write (nout,*) 'G13CCF Example Program Results'
Write (nout,*)

!      Skip heading in data file
Read (nin,*)

!      Read in the problem size
Read (nin,*) nxy, nc, ic

!      Read in control parameters
Read (nin,*) mtxy, pxy
Read (nin,*) iw, mw
Read (nin,*) ish, kc, l

If (ic==0) Then
  nxyg = max(kc,l)
Else
  nxyg = 1
End If
lxg = max(nxyg,nxy)
lyg = max(nxyg,nxy)
Allocate (xg(lxg),yg(lyg),cxy(nc),cyx(nc))

If (ic==0) Then
  Read (nin,*)(xg(i),i=1,nxy)
  Read (nin,*)(yg(i),i=1,nxy)
Else
  Read (nin,*)(cxy(i),i=1,nc)
  Read (nin,*)(cyx(i),i=1,nc)
End If

```

```

ifail = 0
Call g13ccf(nxy,mtxy,pxy,iw,mw,ish,ic,nc,cxy,cyx,kc,l,nxyg,xg,yg,ng, &
ifail)

!   Display results
    Write (nout,*) '                Returned cross covariances'
    Write (nout,*)
    Write (nout,*) &
      'Lag      XY      YX      Lag      XY      YX      Lag      XY      YX'
    Do i = 1, nc, 3
      Write (nout,99999)(ii-1,cxy(ii),cyx(ii),ii=i,min(i+2,nc))
    End Do
    Write (nout,*)
    Write (nout,*) '                Returned sample spectrum'
    Write (nout,*)
    Write (nout,*) &
      '      Real Imaginary      Real Imaginary      Real Imaginary'
    Write (nout,*) &
      'Lag      part      part Lag      part      part Lag      part      part'
    Do i = 1, ng, 3
      Write (nout,99999)(ii-1,xg(ii),yg(ii),ii=i,min(i+2,ng))
    End Do

99999 Format (1X,I3,2F9.4,I4,2F9.4,I4,2F9.4)
End Program g13ccfe

```

## 10.2 Program Data

G13CCF Example Program Data

```

296 50 0
1 0.1
4 35
3 350 80
-0.109 0.000 0.178 0.339 0.373 0.441 0.461 0.348
0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193 0.088 0.435 0.771 0.866 0.875 0.891
0.987 1.263 1.775 1.976 1.934 1.866 1.832 1.767
1.608 1.265 0.790 0.360 0.115 0.088 0.331 0.645
0.960 1.409 2.670 2.834 2.812 2.483 1.929 1.485
1.214 1.239 1.608 1.905 2.023 1.815 0.535 0.122
0.009 0.164 0.671 1.019 1.146 1.155 1.112 1.121
1.223 1.257 1.157 0.913 0.620 0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524 0.040 0.788 0.943 0.930 1.006 1.137 1.198
1.054 0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007 0.254 0.330 0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
-0.876 -0.885 -0.800 -0.544 -0.416 -0.271 0.000 0.403
0.841 1.285 1.607 1.746 1.683 1.485 0.993 0.648
0.577 0.577 0.632 0.747 0.999 0.993 0.968 0.790
0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049 0.060
0.161 0.301 0.517 0.566 0.560 0.573 0.592 0.671
0.933 1.337 1.460 1.353 0.772 0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676 0.033 0.556 0.643 0.484
0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
0.063 0.084 0.000 0.001 0.209 0.556 0.782 0.858
0.918 0.862 0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395 0.185 0.662 0.709 0.605 0.501 0.603
0.943 1.223 1.249 0.824 0.102 0.025 0.382 0.922
1.032 0.866 0.527 0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033 0.102 0.251
0.280 0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204

```

```

:: NXY,NC,IC
:: MTXY,PXY
:: IW,MW
:: ISH,KC,L

```

```

0.034 0.204 0.253 0.195 0.131 0.017 -0.182 -0.262 :: End of XG
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0
52.4 53.0 54.0 54.9 56.0 56.8 56.8 56.4 55.7 55.0 54.3
53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9 47.6
47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2
50.0 48.3 47.0 45.8 45.6 46.0 46.9 47.8 48.2 48.3 47.9
47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5 50.1 49.8
49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0
56.9 57.5 57.3 56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4
58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1 51.6 51.0 50.5
50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8
53.3 53.0 52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4
58.4 57.6 56.9 56.4 56.0 55.7 55.3 55.0 54.4 53.7 52.8
51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9
50.7 50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0
53.6 53.2 53.0 52.8 52.3 51.9 51.6 51.6 51.4 51.2 50.7
50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9
55.2 54.4 53.7 53.6 53.6 53.2 52.5 52.0 51.4 51.0 50.9
52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7 59.0
57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3
57.4 57.0 56.4 55.9 55.5 55.3 55.2 55.4 56.0 56.5 57.1
57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2 57.8 58.3
58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4
56.0 55.2 54.0 53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0
52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2 52.0 52.8 53.8
54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0
54.3 56.0 57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0 :: End of YG

```

### 10.3 Program Results

G13CCF Example Program Results

Returned cross covariances

Lag	XY	YX	Lag	XY	YX	Lag	XY	YX
0	-1.6700	-1.6700	1	-2.0581	-1.3606	2	-2.4859	-1.1383
3	-2.8793	-0.9926	4	-3.1473	-0.9009	5	-3.2239	-0.8382
6	-3.0929	-0.7804	7	-2.7974	-0.7074	8	-2.4145	-0.6147
9	-2.0237	-0.5080	10	-1.6802	-0.4032	11	-1.4065	-0.3159
12	-1.2049	-0.2554	13	-1.0655	-0.2250	14	-0.9726	-0.2238
15	-0.9117	-0.2454	16	-0.8658	-0.2784	17	-0.8180	-0.3081
18	-0.7563	-0.3257	19	-0.6750	-0.3315	20	-0.5754	-0.3321
21	-0.4701	-0.3308	22	-0.3738	-0.3312	23	-0.3023	-0.3332
24	-0.2665	-0.3384	25	-0.2645	-0.3506	26	-0.2847	-0.3727
27	-0.3103	-0.3992	28	-0.3263	-0.4152	29	-0.3271	-0.4044
30	-0.3119	-0.3621	31	-0.2837	-0.2919	32	-0.2568	-0.2054
33	-0.2427	-0.1185	34	-0.2490	-0.0414	35	-0.2774	0.0227
36	-0.3218	0.0697	37	-0.3705	0.1039	38	-0.4083	0.1356
39	-0.4197	0.1805	40	-0.3920	0.2460	41	-0.3241	0.3319
42	-0.2273	0.4325	43	-0.1216	0.5331	44	-0.0245	0.6199
45	0.0528	0.6875	46	0.1074	0.7329	47	0.1448	0.7550
48	0.1713	0.7544	49	0.1943	0.7349			

Returned sample spectrum

Lag	Real part	Imaginary part	Lag	Real part	Imaginary part	Lag	Real part	Imaginary part
0	-6.5500	0.0000	1	-5.4267	-1.9842	2	-3.1323	-2.7307
3	-1.2649	-2.3998	4	-0.2102	-1.7520	5	0.3411	-1.1903
6	0.6063	-0.7420	7	0.6178	-0.3586	8	0.4391	-0.1008
9	0.2422	0.0061	10	0.1233	0.0409	11	0.0574	0.0529
12	0.0174	0.0452	13	-0.0008	0.0289	14	-0.0058	0.0161
15	-0.0051	0.0084	16	-0.0027	0.0040	17	-0.0010	0.0015
18	-0.0006	0.0006	19	-0.0005	0.0003	20	-0.0003	0.0003
21	-0.0003	0.0004	22	-0.0003	0.0003	23	-0.0003	0.0002
24	-0.0004	0.0001	25	-0.0004	-0.0000	26	-0.0003	-0.0001



27	-0.0002	-0.0001	28	-0.0001	0.0001	29	-0.0002	0.0003
30	-0.0003	0.0002	31	-0.0002	0.0001	32	-0.0001	0.0000
33	-0.0000	-0.0000	34	0.0001	-0.0001	35	0.0001	-0.0002
36	0.0001	-0.0001	37	0.0001	-0.0001	38	0.0001	-0.0001
39	0.0001	-0.0001	40	0.0001	0.0000			

---