NAG Library Routine Document

G01EMF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EMF returns the probability associated with the lower tail of the distribution of the Studentized range statistic, via the routine name.

2 Specification

```
FUNCTION G01EMF (Q, V, IR, IFAIL)
REAL (KIND=nag_wp) G01EMF
INTEGER            IR, IFAIL
REAL (KIND=nag_wp) Q, V
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as:

$$q = \frac{\max x_i - \min x_i}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - qx)]^{r-1}\, dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

The above two-dimensional integral is evaluated using D01DAF with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - q)]^{r-1}\, dy.$$

This integral is evaluated using D01AMF.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32(2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

1: Q – REAL (KIND=nag_wp) *Input*

On entry: q, the Studentized range statistic.

Constraint: Q > 0.0.

2: V – REAL (KIND=nag_wp) *Input*

On entry: v, the number of degrees of freedom for the experimental error.

Constraint: V ≥ 1.0.

3: IR – INTEGER *Input*

On entry: r, the number of groups.

Constraint: IR ≥ 2.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

If on exit IFAIL $= 1$, then G01EMF returns to 0.0.

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: G01EMF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, Q ≤ 0.0,
or V < 1.0,
or IR < 2.

IFAIL $= 2$

There is some doubt as to whether full accuracy has been achieved.

IFAIL $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.8 in the Essential Introduction for further information.

IFAIL $= -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.7 in the Essential Introduction for further information.

IFAIL $= -999$

Dynamic memory allocation failed.

See Section 3.6 in the Essential Introduction for further information.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless IFAIL $= 2$. When IFAIL $= 2$ it is usual that the returned value will be a good estimate of the true value.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of q, ν and r.

10.1 Program Text

```
    Program g01emfe

!     G01EMF Example Program Text

!     Mark 25 Release. NAG Copyright 2014.

!     .. Use Statements ..
      Use nag_library, Only: g01emf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: q, v, valp
      Integer                          :: ifail, ir
!     .. Executable Statements ..
      Write (nout,*) 'G01EMF Example Program Results '
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Display titles
```

```
      Write (nout,*) '  Q       V    IR    Prob     '
      Write (nout,*)

d_lp: Do
        Read (nin,*,Iostat=ifail) q, v, ir
        If (ifail/=0) Then
          Exit d_lp
        End If

!       Calculate probability
        ifail = 0
        valp = g01emf(q,v,ir,ifail)

!       Display data
        Write (nout,99999) q, v, ir, valp
      End Do d_lp

99999 Format (1X,F7.4,2X,F4.1,1X,I3,1X,F10.4)
    End Program g01emfe
```

10.2 Program Data

```
G01EMF Example Program Data
4.6543  10.0   5
2.8099  60.0  12
4.2636   5.0   4
```

10.3 Program Results

```
 G01EMF Example Program Results

   Q       V    IR    Prob

  4.6543  10.0   5     0.9500
  2.8099  60.0  12     0.3000
  4.2636   5.0   4     0.9000
```