

NAG Library Routine Document

F08VSF (ZGGSVP)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08VSF (ZGGSVP) uses unitary transformations to simultaneously reduce the m by n matrix A and the p by n matrix B to upper triangular form. This factorization is usually used as a preprocessing step for computing the generalized singular value decomposition (GSVD).

2 Specification

```

SUBROUTINE F08VSF (JOBV, JOBV, JOBQ, M, P, N, A, LDA, B, LDB, TOLA,      &
                  TOLB, K, L, U, LDU, V, LDV, Q, LDQ, IWORK, RWORK,    &
                  TAU, WORK, INFO)
INTEGER              M, P, N, LDA, LDB, K, L, LDU, LDV, LDQ, IWORK(N),  &
                  INFO
REAL (KIND=nag_wp)  TOLA, TOLB, RWORK(2*N)
COMPLEX (KIND=nag_wp) A(LDA,*), B(LDB,*), U(LDU,*), V(LDV,*),        &
                  Q(LDQ,*), TAU(N), WORK(max(3*N,M,P))
CHARACTER(1)        JOBV, JOBV, JOBQ

```

The routine may be called by its LAPACK name **zggsvp**.

3 Description

F08VSF (ZGGSVP) computes unitary matrices U , V and Q such that

$$U^H A Q = \begin{cases} \begin{matrix} & n-k-l & k & l \\ & k \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & 0 \end{pmatrix} \\ m-k-l \end{matrix}, & \text{if } m-k-l \geq 0; \\ \begin{matrix} & n-k-l & k & l \\ & k \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \end{pmatrix} \\ m-k \end{matrix}, & \text{if } m-k-l < 0; \end{cases}$$

$$V^H B Q = \begin{matrix} & n-k-l & k & l \\ l \begin{pmatrix} 0 & 0 & B_{13} \\ 0 & 0 & 0 \end{pmatrix} \\ p-l \end{matrix}$$

where the k by k matrix A_{12} and l by l matrix B_{13} are nonsingular upper triangular; A_{23} is l by l upper triangular if $m-k-l \geq 0$ and is $(m-k)$ by l upper trapezoidal otherwise. $(k+l)$ is the effective numerical rank of the $(m+p)$ by n matrix $(A^H \ B^H)^H$.

This decomposition is usually used as the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see routine F08VNF (ZGGSVD).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: JOBU – CHARACTER(1) *Input*
On entry: if JOBU = 'U', the unitary matrix U is computed.
 If JOBU = 'N', U is not computed.
Constraint: JOBU = 'U' or 'N'.
- 2: JOBV – CHARACTER(1) *Input*
On entry: if JOBV = 'V', the unitary matrix V is computed.
 If JOBV = 'N', V is not computed.
Constraint: JOBV = 'V' or 'N'.
- 3: JOBQ – CHARACTER(1) *Input*
On entry: if JOBQ = 'Q', the unitary matrix Q is computed.
 If JOBQ = 'N', Q is not computed.
Constraint: JOBQ = 'Q' or 'N'.
- 4: M – INTEGER *Input*
On entry: m , the number of rows of the matrix A .
Constraint: $M \geq 0$.
- 5: P – INTEGER *Input*
On entry: p , the number of rows of the matrix B .
Constraint: $P \geq 0$.
- 6: N – INTEGER *Input*
On entry: n , the number of columns of the matrices A and B .
Constraint: $N \geq 0$.
- 7: A(LDA,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the m by n matrix A .
On exit: contains the triangular (or trapezoidal) matrix described in Section 3.
- 8: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F08VSF (ZGGSVP) is called.
Constraint: $LDA \geq \max(1, M)$.

- 9: B(LDB,*) – COMPLEX (KIND=nag_wp) array Input/Output
Note: the second dimension of the array B must be at least $\max(1, N)$.
On entry: the p by n matrix B .
On exit: contains the triangular matrix described in Section 3.
- 10: LDB – INTEGER Input
On entry: the first dimension of the array B as declared in the (sub)program from which F08VSF (ZGGSVP) is called.
Constraint: $LDB \geq \max(1, P)$.
- 11: TOLA – REAL (KIND=nag_wp) Input
 12: TOLB – REAL (KIND=nag_wp) Input
On entry: TOLA and TOLB are the thresholds to determine the effective numerical rank of matrix B and a subblock of A . Generally, they are set to
- $$\begin{aligned} \text{TOLA} &= \max(M, N) \|A\| \epsilon, \\ \text{TOLB} &= \max(P, N) \|B\| \epsilon, \end{aligned}$$
- where ϵ is the *machine precision*.
 The size of TOLA and TOLB may affect the size of backward errors of the decomposition.
- 13: K – INTEGER Output
 14: L – INTEGER Output
On exit: K and L specify the dimension of the subblocks k and l as described in Section 3; $(k + l)$ is the effective numerical rank of $(A^T \ B^T)^T$.
- 15: U(LDU,*) – COMPLEX (KIND=nag_wp) array Output
Note: the second dimension of the array U must be at least $\max(1, M)$ if $\text{JOB} = \text{'U'}$, and at least 1 otherwise.
On exit: if $\text{JOB} = \text{'U'}$, U contains the unitary matrix U .
 If $\text{JOB} = \text{'N'}$, U is not referenced.
- 16: LDU – INTEGER Input
On entry: the first dimension of the array U as declared in the (sub)program from which F08VSF (ZGGSVP) is called.
Constraints:
 if $\text{JOB} = \text{'U'}$, $LDU \geq \max(1, M)$;
 otherwise $LDU \geq 1$.
- 17: V(LDV,*) – COMPLEX (KIND=nag_wp) array Output
Note: the second dimension of the array V must be at least $\max(1, P)$ if $\text{JOB} = \text{'V'}$, and at least 1 otherwise.
On exit: if $\text{JOB} = \text{'V'}$, V contains the unitary matrix V .
 If $\text{JOB} = \text{'N'}$, V is not referenced.
- 18: LDV – INTEGER Input
On entry: the first dimension of the array V as declared in the (sub)program from which F08VSF (ZGGSVP) is called.

Constraints:

if $\text{JOBV} = \text{'V'}$, $\text{LDV} \geq \max(1, P)$;
 otherwise $\text{LDV} \geq 1$.

19: $Q(\text{LDQ}, *)$ – COMPLEX (KIND=nag_wp) array *Output*

Note: the second dimension of the array Q must be at least $\max(1, N)$ if $\text{JOBQ} = \text{'Q'}$, and at least 1 otherwise.

On exit: if $\text{JOBQ} = \text{'Q'}$, Q contains the unitary matrix Q .

If $\text{JOBQ} = \text{'N'}$, Q is not referenced.

20: LDQ – INTEGER *Input*

On entry: the first dimension of the array Q as declared in the (sub)program from which F08VSF (ZGGSVP) is called.

Constraints:

if $\text{JOBQ} = \text{'Q'}$, $\text{LDQ} \geq \max(1, N)$;
 otherwise $\text{LDQ} \geq 1$.

21: $\text{IWORK}(N)$ – INTEGER array *Workspace*

22: $\text{RWORK}(2 \times N)$ – REAL (KIND=nag_wp) array *Workspace*

23: $\text{TAU}(N)$ – COMPLEX (KIND=nag_wp) array *Workspace*

24: $\text{WORK}(\max(3 \times N, M, P))$ – COMPLEX (KIND=nag_wp) array *Workspace*

25: INFO – INTEGER *Output*

On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

$\text{INFO} < 0$

If $\text{INFO} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed factorization is nearly the exact factorization for nearby matrices $(A + E)$ and $(B + F)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2 \quad \text{and} \quad \|F\|_2 = O(\epsilon)\|B\|_2,$$

and ϵ is the *machine precision*.

8 Parallelism and Performance

F08VSF (ZGGSVP) is not threaded by NAG in any implementation.

F08VSF (ZGGSVP) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The real analogue of this routine is F08VEF (DGGSVP).

10 Example

This example finds the generalized factorization

$$A = U\Sigma_1(0 \ S)Q^H, \quad B = V\Sigma_2(0 \ T)Q^H,$$

of the matrix pair $(A \ B)$, where

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i & -0.05 + 0.41i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i & -0.81 + 0.56i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i & -1.11 + 0.60i \\ 0.37 + 0.38i & 0.19 - 0.54i & -0.98 - 0.36i & 0.22 - 0.20i \\ 0.83 + 0.51i & 0.20 + 0.01i & -0.17 - 0.46i & 1.47 + 1.59i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i & 0.26 + 0.26i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

10.1 Program Text

Program f08vsfe

```
!      F08VSF Example Program Text
!
!      Mark 25 Release. NAG Copyright 2014.
!
!      .. Use Statements ..
!      Use nag_library, Only: f06uaf, nag_wp, x02ajf, x04dbf, zggsvp
!      .. Implicit None Statement ..
!      Implicit None
!      .. Parameters ..
!      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
!      Real (Kind=nag_wp)         :: eps, tola, tolb
!      Integer                    :: i, ifail, info, irank, k, l, lda,    &
!                                ldb, ldq, ldu, ldv, m, n, p
!
!      .. Local Arrays ..
!      Complex (Kind=nag_wp), Allocatable :: a(:,,:), b(:,,:), q(:,,:), tau(:),    &
!                                u(:,,:), v(:,,:), work(:)
!      Real (Kind=nag_wp), Allocatable  :: rwork(:)
!      Integer, Allocatable             :: iwork(:)
!      Character (1)                   :: clabs(1), rlabs(1)
!
!      .. Intrinsic Procedures ..
!      Intrinsic                      :: max, real
!
!      .. Executable Statements ..
!      Write (nout,*) 'F08VSF Example Program Results'
!      Write (nout,*)
!      Flush (nout)
!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) m, n, p
!      lda = m
!      ldb = p
!      ldq = n
!      ldu = m
!      ldv = p
!      Allocate (a(lda,n),b(ldb,n),q(ldq,n),tau(n),u(ldu,m),v(ldv,p), &
!              work(m+3*n+p),rwork(2*n),iwork(n))
!
!      Read the m by n matrix A and p by n matrix B from data file
```

```

Read (nin,*)(a(i,1:n),i=1,m)
Read (nin,*)(b(i,1:n),i=1,p)

!   Compute tola and tolB as
!       tola = max(m,n)*norm(A)*macheps
!       tolB = max(p,n)*norm(B)*macheps

eps = x02ajf()
tola = real(max(m,n),kind=nag_wp)*f06uaf('One-norm',m,n,a,lda,rwork)*eps
tolB = real(max(p,n),kind=nag_wp)*f06uaf('One-norm',p,n,b,ldb,rwork)*eps

!   Compute the factorization of (A, B)
!       (A = U*S*(Q**H), B = V*T*(Q**H))

!   The NAG name equivalent of zggsvp is f08vsf
Call zggsvp('U','V','Q',m,p,n,a,lda,b,ldb,tola,tolB,k,l,u,ldu,v,ldv,q, &
    ldq,iwork,rwork,tau,work,info)

!   Print solution

irank = k + 1
Write (nout,*) 'Numerical rank of (A**T B**T)**T (K+L)'
Write (nout,99999) irank

Write (nout,*)
Flush (nout)
If (m>=irank) Then

!       ifail: behaviour on error exit
!           =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
Call x04dbf('Upper','Non-unit',irank,irank,a(1,n-irank+1),lda, &
    'Bracketed','1P,E12.4','Upper triangular matrix S','Integer',rlabs, &
    'Integer',clabs,80,0,ifail)

Else

    ifail = 0
    Call x04dbf('Upper','Non-unit',m,irank,a(1,n-irank+1),lda,'Bracketed', &
        '1P,E12.4','Upper trapezoidal matrix S','Integer',rlabs,'Integer', &
        clabs,80,0,ifail)

End If
Write (nout,*)
Flush (nout)

ifail = 0
Call x04dbf('Upper','Non-unit',l,l,b(1,n-l+1),ldb,'Bracketed', &
    '1P,E12.4','Upper triangular matrix T','Integer',rlabs,'Integer', &
    clabs,80,0,ifail)

Write (nout,*)
Flush (nout)

ifail = 0
Call x04dbf('General',' ',m,m,u,ldu,'Bracketed','1P,E12.4', &
    'Orthogonal matrix U','Integer',rlabs,'Integer',clabs,80,0,ifail)

Write (nout,*)
Flush (nout)

ifail = 0
Call x04dbf('General',' ',p,p,v,ldv,'Bracketed','1P,E12.4', &
    'Orthogonal matrix V','Integer',rlabs,'Integer',clabs,80,0,ifail)

Write (nout,*)
Flush (nout)

ifail = 0

```

```

      Call x04dbf('General', ' ', n, n, q, ldq, 'Bracketed', '1P, E12.4', &
        'Orthogonal matrix Q', 'Integer', rlabs, 'Integer', clabs, 80, 0, ifail)

99999 Format (1X, I5)
      End Program f08vsfe

```

10.2 Program Data

F08VSF Example Program Data

```

      6              4              2              :Values of M, N and P

( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06) (-0.05, 0.41)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42) (-0.81, 0.56)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17) (-1.11, 0.60)
( 0.37, 0.38) ( 0.19,-0.54) (-0.98,-0.36) ( 0.22,-0.20)
( 0.83, 0.51) ( 0.20, 0.01) (-0.17,-0.46) ( 1.47, 1.59)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23) ( 0.26, 0.26) :End of matrix A

( 1.00, 0.00) ( 0.00, 0.00) (-1.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 1.00, 0.00) ( 0.00, 0.00) (-1.00, 0.00) :End of matrix B

```

10.3 Program Results

F08VSF Example Program Results

Numerical rank of (A**T B**T)**T (K+L)
4

Upper triangular matrix S

```

      1              2
1 ( -2.7118E+00,  0.0000E+00) ( -1.4390E+00, -1.0315E+00)
2              ( -1.8583E+00,  0.0000E+00)
3
4

      3              4
1 ( -1.0543E-01,  1.3176E+00) ( -3.9240E-01, -1.9504E-01)
2 ( -9.4529E-01,  1.9279E-01) (  1.4355E+00,  2.6313E-01)
3 (  2.9079E+00,  0.0000E+00) ( -2.3946E-01,  1.8856E-01)
4              ( -1.5759E+00,  0.0000E+00)

```

Upper triangular matrix T

```

      1              2
1 (  1.4142E+00,  0.0000E+00) (  0.0000E+00,  0.0000E+00)
2              (  1.4142E+00,  0.0000E+00)

```

Orthogonal matrix U

```

      1              2
1 ( -1.3038E-02, -3.2595E-01) ( -1.4039E-01, -2.6167E-01)
2 (  4.2764E-01, -6.2582E-01) (  8.6298E-02, -3.8174E-02)
3 ( -3.2595E-01,  1.6428E-01) (  3.8163E-01, -1.8219E-01)
4 (  1.5906E-01, -5.2151E-03) ( -2.8207E-01,  1.9732E-01)
5 ( -1.7210E-01, -1.3038E-02) ( -5.0942E-01, -5.0319E-01)
6 ( -2.6336E-01, -2.4772E-01) ( -1.0861E-01,  2.8474E-01)

      3              4
1 (  2.4357E-01, -7.7956E-01) ( -7.4007E-02, -2.7823E-01)
2 ( -3.2035E-01,  1.4475E-01) (  1.0740E-01,  1.8824E-01)
3 (  1.7217E-01, -1.4009E-03) ( -4.9770E-01,  1.7826E-01)
4 (  2.5307E-01,  1.9053E-01) ( -3.7794E-01,  2.6816E-01)
5 (  3.2057E-02,  1.8358E-01) (  2.0422E-01,  1.6601E-01)
6 (  1.4142E-01, -1.5707E-01) ( -8.7335E-02,  5.4683E-01)

      5              6
1 ( -4.5947E-02,  1.4052E-04) ( -5.2773E-02, -2.2492E-01)
2 ( -8.0311E-02, -4.3605E-01) ( -3.8117E-02, -2.1907E-01)
3 (  5.9714E-02, -5.8974E-01) ( -1.3850E-01, -9.0941E-02)
4 ( -4.6443E-02,  3.0864E-01) ( -3.7354E-01, -5.5148E-01)
5 (  5.7843E-01, -1.2439E-01) ( -1.8815E-02, -5.5686E-02)

```

6 (1.5763E-02, 4.7130E-02) (6.5007E-01, 4.9173E-03)

Orthogonal matrix V

1 (1.0000E+00, 0.0000E+00) (0.0000E+00, 0.0000E+00)
2 (0.0000E+00, 0.0000E+00) (1.0000E+00, 0.0000E+00)

Orthogonal matrix Q

1 (7.0711E-01, 0.0000E+00) (0.0000E+00, 0.0000E+00)
2 (0.0000E+00, 0.0000E+00) (7.0711E-01, 0.0000E+00)
3 (7.0711E-01, 0.0000E+00) (0.0000E+00, 0.0000E+00)
4 (0.0000E+00, 0.0000E+00) (7.0711E-01, 0.0000E+00)

1 (7.0711E-01, 0.0000E+00) (0.0000E+00, 0.0000E+00)
2 (0.0000E+00, 0.0000E+00) (7.0711E-01, 0.0000E+00)
3 (-7.0711E-01, 0.0000E+00) (0.0000E+00, 0.0000E+00)
4 (0.0000E+00, 0.0000E+00) (-7.0711E-01, 0.0000E+00)
