NAG Library Routine Document

F08QXF (ZTREVC)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1   Purpose

F08QXF (ZTREVC) computes selected left and/or right eigenvectors of a complex upper triangular matrix.

2   Specification

```
SUBROUTINE F08QXF (JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,     &
                   MM, M, WORK, RWORK, INFO)

INTEGER              N, LDT, LDVL, LDVR, MM, M, INFO
REAL (KIND=nag_wp)   RWORK(N)
COMPLEX (KIND=nag_wp) T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(2*N)
LOGICAL              SELECT(*)
CHARACTER(1)         JOB, HOWMNY
```

The routine may be called by its LAPACK name *ztrevc*.

3   Description

F08QXF (ZTREVC) computes left and/or right eigenvectors of a complex upper triangular matrix T. Such a matrix arises from the Schur factorization of a complex general matrix, as computed by F08PSF (ZHSEQR), for example.

The right eigenvector x, and the left eigenvector y, corresponding to an eigenvalue λ, are defined by:

$$Tx = \lambda x \quad \text{and} \quad y^{\mathrm{H}}T = \lambda y^{\mathrm{H}}\left(\text{or } T^{\mathrm{H}}y = \bar{\lambda}y\right).$$

The routine can compute the eigenvectors corresponding to selected eigenvalues, or it can compute all the eigenvectors. In the latter case the eigenvectors may optionally be pre-multiplied by an input matrix Q. Normally Q is a unitary matrix from the Schur factorization of a matrix A as $A = QTQ^{\mathrm{H}}$; if x is a (left or right) eigenvector of T, then Qx is an eigenvector of A.

The eigenvectors are computed by forward or backward substitution. They are scaled so that $\max |\mathrm{Re}(x_i)| + |\mathrm{Im}\, x_i| = 1$.

4   References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5   Parameters

1:    JOB – CHARACTER(1)   *Input*

On entry: indicates whether left and/or right eigenvectors are to be computed.

JOB = 'R'
    Only right eigenvectors are computed.

JOB = 'L'
    Only left eigenvectors are computed.

JOB = 'B'
> Both left and right eigenvectors are computed.

Constraint: JOB = 'R', 'L' or 'B'.

2:     HOWMNY – CHARACTER(1)   *Input*

On entry: indicates how many eigenvectors are to be computed.

HOWMNY = 'A'
> All eigenvectors (as specified by JOB) are computed.

HOWMNY = 'B'
> All eigenvectors (as specified by JOB) are computed and then pre-multiplied by the matrix Q (which is overwritten).

HOWMNY = 'S'
> Selected eigenvectors (as specified by JOB and SELECT) are computed.

Constraint: HOWMNY = 'A', 'B' or 'S'.

3:     SELECT(∗) – LOGICAL array   *Input*

Note: the dimension of the array SELECT must be at least N if HOWMNY = 'S', and at least 1 otherwise.

On entry: specifies which eigenvectors are to be computed if HOWMNY = 'S'. To obtain the eigenvector corresponding to the eigenvalue λ_j, SELECT(j) must be set .TRUE..

If HOWMNY = 'A' or 'B', SELECT is not referenced.

4:     N – INTEGER   *Input*

On entry: n, the order of the matrix T.

Constraint: N ≥ 0.

5:     T(LDT, ∗) – COMPLEX (KIND=nag_wp) array   *Input/Output*

Note: the second dimension of the array T must be at least N.

On entry: the n by n upper triangular matrix T, as returned by F08PSF (ZHSEQR).

On exit: is used as internal workspace prior to being restored and hence is unchanged.

6:     LDT – INTEGER   *Input*

On entry: the first dimension of the array T as declared in the (sub)program from which F08QXF (ZTREVC) is called.

Constraint: LDT ≥ max(1, N).

7:     VL(LDVL, ∗) – COMPLEX (KIND=nag_wp) array   *Input/Output*

Note: the second dimension of the array VL must be at least MM if JOB = 'L' or 'B' and at least 1 if JOB = 'R'.

On entry: if HOWMNY = 'B' and JOB = 'L' or 'B', VL must contain an n by n matrix Q (usually the matrix of Schur vectors returned by F08PSF (ZHSEQR)).

If HOWMNY = 'A' or 'S', VL need not be set.

On exit: if JOB = 'L' or 'B', VL contains the computed left eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues.

If JOB = 'R', VL is not referenced.

8:   LDVL – INTEGER   *Input*

   On entry: the first dimension of the array VL as declared in the (sub)program from which
   F08QXF (ZTREVC) is called.

   Constraints:

      if JOB = 'L' or 'B', LDVL ≥ N;
      if JOB = 'R', LDVL ≥ 1.

9:   VR(LDVR, ∗) – COMPLEX (KIND=nag_wp) array   *Input/Output*

   Note: the second dimension of the array VR must be at least MM if JOB = 'R' or 'B' and at least 1
   if JOB = 'L'.

   On entry: if HOWMNY = 'B' and JOB = 'R' or 'B', VR must contain an n by n matrix Q (usually
   the matrix of Schur vectors returned by F08PSF (ZHSEQR)).

   If HOWMNY = 'A' or 'S', VR need not be set.

   On exit: if JOB = 'R' or 'B', VR contains the computed right eigenvectors (as specified by
   HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array,
   in the same order as their eigenvalues.

   If JOB = 'L', VR is not referenced.

10: LDVR – INTEGER   *Input*

   On entry: the first dimension of the array VR as declared in the (sub)program from which
   F08QXF (ZTREVC) is called.

   Constraints:

      if JOB = 'R' or 'B', LDVR ≥ N;
      if JOB = 'L', LDVR ≥ 1.

11: MM – INTEGER   *Input*

   On entry: the number of columns in the arrays VL and/or VR. The precise number of columns
   required, m, is n if HOWMNY = 'A' or 'B'; if HOWMNY = 'S', m is the number of selected
   eigenvectors (see SELECT), in which case $0 \le m \le n$.

   Constraints:

      if HOWMNY = 'A' or 'B', MM ≥ N;
      otherwise MM ≥ m.

12: M – INTEGER   *Output*

   On exit: m, the number of selected eigenvectors. If HOWMNY = 'A' or 'B', M is set to n.

13: WORK(2 × N) – COMPLEX (KIND=nag_wp) array   *Workspace*

14: RWORK(N) – REAL (KIND=nag_wp) array   *Workspace*

15: INFO – INTEGER   *Output*

   On exit: INFO = 0 unless the routine detects an error (see Section 6).

6   Error Indicators and Warnings

INFO < 0

   If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution
   of the program is terminated.

7 Accuracy

If x_i is an exact right eigenvector, and $\tilde{x}_i$ is the corresponding computed eigenvector, then the angle $\theta(\tilde{x}_i, x_i)$ between them is bounded as follows:

$$\theta(\tilde{x}_i, x_i) \leq \frac{c(n)\epsilon \|T\|_2}{sep_i}$$

where sep_i is the reciprocal condition number of x_i.

The condition number sep_i may be computed by calling F08QYF (ZTRSNA).

8 Parallelism and Performance

F08QXF (ZTREVC) is not threaded by NAG in any implementation.

F08QXF (ZTREVC) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The real analogue of this routine is F08QKF (DTREVC).

10 Example

See Section 10 in F08NVF (ZGEBAL).