NAG Library Routine Document

F08QKF (DTREVC)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08QKF (DTREVC) computes selected left and/or right eigenvectors of a real upper quasi-triangular matrix.

2 Specification

```
SUBROUTINE F08QKF (JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR,      &
                   MM, M, WORK, INFO)

INTEGER             N, LDT, LDVL, LDVR, MM, M, INFO
REAL (KIND=nag_wp)  T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(3*N)
LOGICAL             SELECT(*)
CHARACTER(1)        JOB, HOWMNY
```

The routine may be called by its LAPACK name ***dtrevc***.

3 Description

F08QKF (DTREVC) computes left and/or right eigenvectors of a real upper quasi-triangular matrix T in canonical Schur form. Such a matrix arises from the Schur factorization of a real general matrix, as computed by F08PEF (DHSEQR), for example.

The right eigenvector x, and the left eigenvector y, corresponding to an eigenvalue λ, are defined by:

$$Tx = \lambda x \quad \text{and} \quad y^{\mathrm{H}}T = \lambda y^{\mathrm{H}}\left(\text{or } T^{\mathrm{T}}y = \bar{\lambda}y\right).$$

Note that even though T is real, λ, x and y may be complex. If x is an eigenvector corresponding to a complex eigenvalue λ, then the complex conjugate vector $\bar{x}$ is the eigenvector corresponding to the complex conjugate eigenvalue $\bar{\lambda}$.

The routine can compute the eigenvectors corresponding to selected eigenvalues, or it can compute all the eigenvectors. In the latter case the eigenvectors may optionally be pre-multiplied by an input matrix Q. Normally Q is an orthogonal matrix from the Schur factorization of a matrix A as $A = QTQ^{\mathrm{T}}$; if x is a (left or right) eigenvector of T, then Qx is an eigenvector of A.

The eigenvectors are computed by forward or backward substitution. They are scaled so that, for a real eigenvector x, $\max(|x_i|) = 1$, and for a complex eigenvector, $\max(|\mathrm{Re}(x_i)| + |\mathrm{Im}(x_i)|) = 1$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: JOB – CHARACTER(1) *Input*

On entry: indicates whether left and/or right eigenvectors are to be computed.

JOB = 'R'
: Only right eigenvectors are computed.

JOB = 'L'
: Only left eigenvectors are computed.

JOB = 'B'
Both left and right eigenvectors are computed.

Constraint: JOB = 'R', 'L' or 'B'.

2: HOWMNY – CHARACTER(1) *Input*

On entry: indicates how many eigenvectors are to be computed.

HOWMNY = 'A'
All eigenvectors (as specified by JOB) are computed.

HOWMNY = 'B'
All eigenvectors (as specified by JOB) are computed and then pre-multiplied by the matrix Q (which is overwritten).

HOWMNY = 'S'
Selected eigenvectors (as specified by JOB and SELECT) are computed.

Constraint: HOWMNY = 'A', 'B' or 'S'.

3: SELECT($*$) – LOGICAL array *Input/Output*

Note: the dimension of the array SELECT must be at least $\max(1, \mathrm{N})$ if HOWMNY = 'S', and at least 1 otherwise.

On entry: specifies which eigenvectors are to be computed if HOWMNY = 'S'. To obtain the real eigenvector corresponding to the real eigenvalue λ_j, SELECT(j) must be set .TRUE.. To select the complex eigenvector corresponding to a complex conjugate pair of eigenvalues λ_j and λ_{j+1}, SELECT(j) and/or SELECT($j+1$) must be set .TRUE.; the eigenvector corresponding to the **first** eigenvalue in the pair is computed.

On exit: if a complex eigenvector was selected as specified above, then SELECT(j) is set to .TRUE. and SELECT($j+1$) to .FALSE..

If HOWMNY = 'A' or 'B', SELECT is not referenced.

4: N – INTEGER *Input*

On entry: n, the order of the matrix T.

Constraint: $\mathrm{N} \geq 0$.

5: T(LDT, $*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array T must be at least $\max(1, \mathrm{N})$.

On entry: the n by n upper quasi-triangular matrix T in canonical Schur form, as returned by F08PEF (DHSEQR).

6: LDT – INTEGER *Input*

On entry: the first dimension of the array T as declared in the (sub)program from which F08QKF (DTREVC) is called.

Constraint: $\mathrm{LDT} \geq \max(1, \mathrm{N})$.

7: VL(LDVL, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array VL must be at least $\max(1, \mathrm{MM})$ if JOB = 'L' or 'B' and at least 1 if JOB = 'R'.

On entry: if HOWMNY = 'B' and JOB = 'L' or 'B', VL must contain an n by n matrix Q (usually the matrix of Schur vectors returned by F08PEF (DHSEQR)).

If HOWMNY = 'A' or 'S', VL need not be set.

On exit: if JOB = 'L' or 'B', VL contains the computed left eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues. Corresponding to each real eigenvalue is a real eigenvector, occupying one column. Corresponding to each complex conjugate pair of eigenvalues, is a complex eigenvector occupying two columns; the first column holds the real part and the second column holds the imaginary part.

If JOB = 'R', VL is not referenced.

8: LDVL – INTEGER *Input*

On entry: the first dimension of the array VL as declared in the (sub)program from which F08QKF (DTREVC) is called.

Constraints:

if JOB = 'L' or 'B', LDVL $\geq$ N;
if JOB = 'R', LDVL ≥ 1.

9: VR(LDVR, $*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array VR must be at least $\max(1, \mathrm{MM})$ if JOB = 'R' or 'B' and at least 1 if JOB = 'L'.

On entry: if HOWMNY = 'B' and JOB = 'R' or 'B', VR must contain an n by n matrix Q (usually the matrix of Schur vectors returned by F08PEF (DHSEQR)).

If HOWMNY = 'A' or 'S', VR need not be set.

On exit: if JOB = 'R' or 'B', VR contains the computed right eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues. Corresponding to each real eigenvalue is a real eigenvector, occupying one column. Corresponding to each complex conjugate pair of eigenvalues, is a complex eigenvector occupying two columns; the first column holds the real part and the second column holds the imaginary part.

If JOB = 'L', VR is not referenced.

10: LDVR – INTEGER *Input*

On entry: the first dimension of the array VR as declared in the (sub)program from which F08QKF (DTREVC) is called.

Constraints:

if JOB = 'R' or 'B', LDVR $\geq$ N;
if JOB = 'L', LDVR ≥ 1.

11: MM – INTEGER *Input*

On entry: the number of columns in the arrays VL and/or VR. The precise number of columns required, m, is n if HOWMNY = 'A' or 'B'; if HOWMNY = 'S', m is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector (see SELECT), in which case $0 \leq m \leq n$.

Constraints:

if HOWMNY = 'A' or 'B', MM $\geq$ N;
otherwise MM $\geq m$.

12: M – INTEGER *Output*

On exit: m, the number of columns of VL and/or VR actually used to store the computed eigenvectors. If HOWMNY = 'A' or 'B', M is set to n.

13: WORK($3 \times$ N) – REAL (KIND=nag_wp) array *Workspace*

14: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO $= -11$

On entry, MM $= \langle value \rangle$ and N $= \langle value \rangle$.

Constraint: if HOWMNY $\neq$ 'S', MM $\geq$ N
else MM $\geq m$, where m is obtained by counting
1 for each selected real eigenvector and
2 for each selected complex eigenvector.

7 Accuracy

If x_i is an exact right eigenvector, and $\tilde{x}_i$ is the corresponding computed eigenvector, then the angle $\theta(\tilde{x}_i, x_i)$ between them is bounded as follows:

$$\theta(\tilde{x}_i, x_i) \leq \frac{c(n)\epsilon\|T\|_2}{sep_i}$$

where sep_i is the reciprocal condition number of x_i.

The condition number sep_i may be computed by calling F08QLF (DTRSNA).

8 Parallelism and Performance

F08QKF (DTREVC) is not threaded by NAG in any implementation.

F08QKF (DTREVC) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

For a description of canonical Schur form, see the document for F08PEF (DHSEQR).

The complex analogue of this routine is F08QXF (ZTREVC).

10 Example

See Section 10 in F08NHF (DGEBAL).