

## NAG Library Routine Document

### F07QVF (ZSPRFS)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

#### 1 Purpose

F07QVF (ZSPRFS) returns error bounds for the solution of a complex symmetric system of linear equations with multiple right-hand sides,  $AX = B$ , using packed storage. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

#### 2 Specification

SUBROUTINE F07QVF (UPLO, N, NRHS, AP, AFP, IPIV, B, LDB, X, LDX, FERR, &  
BERR, WORK, RWORK, INFO)

INTEGER N, NRHS, IPIV(\*), LDB, LDX, INFO  
REAL (KIND=nag\_wp) FERR(NRHS), BERR(NRHS), RWORK(N)  
COMPLEX (KIND=nag\_wp) AP(\*), AFP(\*), B(LDB,\*), X(LDX,\*), WORK(2\*N)  
CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zsprfs*.

#### 3 Description

F07QVF (ZSPRFS) returns the backward errors and estimated bounds on the forward errors for the solution of a complex symmetric system of linear equations with multiple right-hand sides  $AX = B$ , using packed storage. The routine handles each right-hand side vector (stored as a column of the matrix  $B$ ) independently, so we describe the function of F07QVF (ZSPRFS) in terms of a single right-hand side  $b$  and solution  $x$ .

Given a computed solution  $x$ , the routine computes the *component-wise backward error*  $\beta$ . This is the size of the smallest relative perturbation in each element of  $A$  and  $b$  such that  $x$  is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$

$$|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.$$

Then the routine estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where  $\hat{x}$  is the true solution.

For details of the method, see the F07 Chapter Introduction.

#### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

- 1: UPLO – CHARACTER(1) *Input*  
*On entry:* specifies whether the upper or lower triangular part of  $A$  is stored and how  $A$  is to be factorized.  
 UPLO = 'U'  
 The upper triangular part of  $A$  is stored and  $A$  is factorized as  $PU DU^T P^T$ , where  $U$  is upper triangular.  
 UPLO = 'L'  
 The lower triangular part of  $A$  is stored and  $A$  is factorized as  $PL DL^T P^T$ , where  $L$  is lower triangular.  
*Constraint:* UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:*  $N \geq 0$ .
- 3: NRHS – INTEGER *Input*  
*On entry:*  $r$ , the number of right-hand sides.  
*Constraint:* NRHS  $\geq 0$ .
- 4: AP(\*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array AP must be at least  $\max(1, N \times (N + 1)/2)$ .  
*On entry:* the  $n$  by  $n$  original symmetric matrix  $A$  as supplied to F07QRF (ZSPTRF).
- 5: AFP(\*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the dimension of the array AFP must be at least  $\max(1, N \times (N + 1)/2)$ .  
*On entry:* the factorization of  $A$  stored in packed form, as returned by F07QRF (ZSPTRF).
- 6: IPIV(\*) – INTEGER array *Input*  
**Note:** the dimension of the array IPIV must be at least  $\max(1, N)$ .  
*On entry:* details of the interchanges and the block structure of  $D$ , as returned by F07QRF (ZSPTRF).
- 7: B(LDB,\*) – COMPLEX (KIND=nag\_wp) array *Input*  
**Note:** the second dimension of the array B must be at least  $\max(1, \text{NRHS})$ .  
*On entry:* the  $n$  by  $r$  right-hand side matrix  $B$ .
- 8: LDB – INTEGER *Input*  
*On entry:* the first dimension of the array B as declared in the (sub)program from which F07QVF (ZSPRFS) is called.  
*Constraint:* LDB  $\geq \max(1, N)$ .
- 9: X(LDX,\*) – COMPLEX (KIND=nag\_wp) array *Input/Output*  
**Note:** the second dimension of the array X must be at least  $\max(1, \text{NRHS})$ .  
*On entry:* the  $n$  by  $r$  solution matrix  $X$ , as returned by F07QSF (ZSPTRS).  
*On exit:* the improved solution matrix  $X$ .

- 10: LDX – INTEGER *Input*  
*On entry:* the first dimension of the array X as declared in the (sub)program from which F07QVF (ZSPRFS) is called.  
*Constraint:*  $LDX \geq \max(1, N)$ .
- 11: FERR(NRHS) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* FERR(*j*) contains an estimated error bound for the *j*th solution vector, that is, the *j*th column of X, for  $j = 1, 2, \dots, r$ .
- 12: BERR(NRHS) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* BERR(*j*) contains the component-wise backward error bound  $\beta$  for the *j*th solution vector, that is, the *j*th column of X, for  $j = 1, 2, \dots, r$ .
- 13: WORK( $2 \times N$ ) – COMPLEX (KIND=nag\_wp) array *Workspace*
- 14: RWORK(N) – REAL (KIND=nag\_wp) array *Workspace*
- 15: INFO – INTEGER *Output*  
*On exit:* INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

INFO < 0

If INFO =  $-i$ , argument *i* had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

The bounds returned in FERR are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

## 8 Parallelism and Performance

F07QVF (ZSPRFS) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

F07QVF (ZSPRFS) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

For each right-hand side, computation of the backward error involves a minimum of  $16n^2$  real floating-point operations. Each step of iterative refinement involves an additional  $24n^2$  real operations. At most five steps of iterative refinement are performed, but usually only one or two steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form  $Ax = b$ ; the number is usually 5 and never more than 11. Each solution involves approximately  $8n^2$  real operations.

The real analogue of this routine is F07PHF (DSPRFS).

## 10 Example

This example solves the system of equations  $AX = B$  using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -55.64 + 41.22i & -19.09 - 35.97i \\ -48.18 + 66.00i & -12.08 - 27.02i \\ -0.49 - 1.47i & 6.95 + 20.49i \\ -6.43 + 19.24i & -4.59 - 35.53i \end{pmatrix}.$$

Here  $A$  is symmetric, stored in packed form, and must first be factorized by F07QRF (ZSPTRF).

### 10.1 Program Text

```

Program f07qvfe

!      F07QVF Example Program Text

!      Mark 25 Release. NAG Copyright 2014.

!      .. Use Statements ..
Use nag_library, Only: nag_wp, x04dbf, zsprfs, zsptrf, zpstrs
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Integer                     :: aplen, i, ifail, info, j, ldb, ldx, &
                             n, nrhs
Character (1)               :: uplo
!      .. Local Arrays ..
Complex (Kind=nag_wp), Allocatable :: afp(:), ap(:), b(:,,:), work(:), &
                                     x(:,,:)
Real (Kind=nag_wp), Allocatable  :: berr(:), ferr(:), rwork(:)
Integer, Allocatable             :: ipiv(:)
Character (1)                    :: clabs(1), rlabs(1)
!      .. Executable Statements ..
Write (nout,*) 'F07QVF Example Program Results'
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n, nrhs
ldb = n
ldx = n
aplen = n*(n+1)/2
Allocate (afp(aplen), ap(aplen), b(ldb,nrhs), work(2*n), x(ldx,n), &
         berr(nrhs), ferr(nrhs), rwork(n), ipiv(n))

!      Read A and B from data file, and copy A to AFP and B to X

Read (nin,*) uplo
If (uplo=='U') Then
  Read (nin,*)((ap(i+j*(j-1)/2), j=i, n), i=1, n)
Else If (uplo=='L') Then
  Read (nin,*)((ap(i+(2*n-j)*(j-1)/2), j=1, i), i=1, n)
End If
Read (nin,*)(b(i,1:nrhs), i=1, n)

afp(1:aplen) = ap(1:aplen)
x(1:n,1:nrhs) = b(1:n,1:nrhs)

!      Factorize A in the array AFP
!      The NAG name equivalent of zsptrf is f07qrf

```

```

      Call zsptrf(uplo,n,afp,ipiv,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute solution in the array X
!       The NAG name equivalent of zsptrs is f07qsf
!       Call zsptrs(uplo,n,nrhs,afp,ipiv,x,ldx,info)

!       Improve solution, and compute backward errors and
!       estimated bounds on the forward errors

!       The NAG name equivalent of zsprfs is f07qvf
!       Call zsprfs(uplo,n,nrhs,ap,afp,ipiv,b,ldb,x,ldx,ferr,berr,work,rwork, &
!       info)

!       Print solution

!       ifail: behaviour on error exit
!       =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04dbf('General',' ',n,nrhs,x,ldx,'Bracketed','F7.4', &
!       'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

      Write (nout,*)
      Write (nout,*) 'Backward errors (machine-dependent)'
      Write (nout,99999) berr(1:nrhs)
      Write (nout,*) 'Estimated forward error bounds (machine-dependent)'
      Write (nout,99999) ferr(1:nrhs)
      Else
!       Write (nout,*) 'The factor D is singular'
      End If

99999 Format ((5X,1P,4(E11.1,7X)))
      End Program f07qvfe

```

## 10.2 Program Data

F07QVF Example Program Data

```

  4  2                                     :Values of N and NRHS
  'L'                                     :Value of UPL0
(-0.39,-0.71)
( 5.14,-0.64) ( 8.86, 1.81)
(-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)
( 3.80, 0.92) ( 5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12) :End of matrix A
(-55.64, 41.22) (-19.09,-35.97)
(-48.18, 66.00) (-12.08,-27.02)
( -0.49, -1.47) ( 6.95, 20.49)
( -6.43, 19.24) ( -4.59,-35.53)                                     :End of matrix B

```

## 10.3 Program Results

F07QVF Example Program Results

Solution(s)

```

           1           2
1 ( 1.0000,-1.0000) (-2.0000,-1.0000)
2 (-2.0000, 5.0000) ( 1.0000,-3.0000)
3 ( 3.0000,-2.0000) ( 3.0000, 2.0000)
4 (-4.0000, 3.0000) (-1.0000, 1.0000)

```

Backward errors (machine-dependent)

```

      8.9E-17      7.3E-17

```

Estimated forward error bounds (machine-dependent)

```

      1.2E-14      1.2E-14

```

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