NAG Library Routine Document

H03ADF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

H03ADF finds the shortest path through a directed or undirected acyclic network using Dijkstra's algorithm.

2 Specification

```
SUBROUTINE H03ADF (N, NS, NE, DIRECT, NNZ, D, IROW, ICOL, SPLEN, PATH,          &
                   IWORK, WORK, IFAIL)

INTEGER            N, NS, NE, NNZ, IROW(NNZ), ICOL(NNZ), PATH(N),               &
                   IWORK(3*N+1), IFAIL
REAL (KIND=nag_wp) D(NNZ), SPLEN, WORK(2*N)
LOGICAL            DIRECT
```

3 Description

H03ADF attempts to find the shortest path through a **directed** or **undirected acyclic** network, which consists of a set of points called **vertices** and a set of curves called **arcs** that connect certain pairs of distinct vertices. An acyclic network is one in which there are no paths connecting a vertex to itself. An arc whose origin vertex is i and whose destination vertex is j can be written as $i \rightarrow j$. In an undirected network the arcs $i \rightarrow j$ and $j \rightarrow i$ are equivalent (i.e., $i \leftrightarrow j$), whereas in a directed network they are different. Note that the shortest path may not be unique and in some cases may not even exist (e.g., if the network is disconnected).

The network is assumed to consist of n vertices which are labelled by the integers $1, 2, \ldots, n$. The lengths of the arcs between the vertices are defined by the n by n **distance matrix** D, in which the element d_{ij} gives the length of the arc $i \rightarrow j$; $d_{ij} = 0$ if there is no arc connecting vertices i and j (as is the case for an acyclic network when $i = j$). Thus the matrix D is usually **sparse**. For example, if $n = 4$ and the network is directed, then

$$\mathrm{D} = \begin{pmatrix} 0 & d_{12} & d_{13} & d_{14} \\ d_{21} & 0 & d_{23} & d_{24} \\ d_{31} & d_{32} & 0 & d_{34} \\ d_{41} & d_{42} & d_{43} & 0 \end{pmatrix}.$$

If the network is undirected, D is **symmetric** since $d_{ij} = d_{ji}$ (i.e., the length of the arc $i \rightarrow j \equiv$ the length of the arc $j \rightarrow i$).

The method used by H03ADF is described in detail in Section 8.

4 References

Dijkstra E W (1959) A note on two problems in connection with graphs *Numer. Math.* **1** 269–271

5 Parameters

1: N – INTEGER *Input*

On entry: n, the number of vertices.

Constraint: $\mathrm{N} \geq 2$.

2: NS – INTEGER *Input*
3: NE – INTEGER *Input*

On entry: n_s and n_e, the labels of the first and last vertices, respectively, between which the shortest path is sought.

Constraints:

$1 \leq \text{NS} \leq \text{N}$;
$1 \leq \text{NE} \leq \text{N}$;
$\text{NS} \neq \text{NE}$.

4: DIRECT – LOGICAL *Input*

On entry: indicates whether the network is directed or undirected.

DIRECT = .TRUE.
The network is directed.

DIRECT = .FALSE.
The network is undirected.

5: NNZ – INTEGER *Input*

On entry: the number of nonzero elements in the distance matrix D.

Constraints:

if DIRECT = .TRUE., $1 \leq \text{NNZ} \leq \text{N} \times (\text{N} - 1)$;
if DIRECT = .FALSE., $1 \leq \text{NNZ} \leq \text{N} \times (\text{N} - 1)/2$.

6: D(NNZ) – REAL (KIND=nag_wp) array *Input*

On entry: the nonzero elements of the distance matrix D, ordered by increasing row index and increasing column index within each row. More precisely, $\text{D}(k)$ must contain the value of the nonzero element with indices $(\text{IROW}(k), \text{ICOL}(k))$; this is the length of the arc from the vertex with label $\text{IROW}(k)$ to the vertex with label $\text{ICOL}(k)$. Elements with the same row and column indices are not allowed. If DIRECT = .FALSE., then only those nonzero elements in the strict upper triangle of D need be supplied since $d_{ij} = d_{ji}$. (F11ZAF may be used to sort the elements of an arbitrarily ordered matrix into the required form. This is illustrated in Section 9.)

Constraint: $\text{D}(k) > 0.0$, for $k = 1, 2, \ldots, \text{NNZ}$.

7: IROW(NNZ) – INTEGER array *Input*
8: ICOL(NNZ) – INTEGER array *Input*

On entry: $\text{IROW}(k)$ and $\text{ICOL}(k)$ must contain the row and column indices, respectively, for the nonzero element stored in $\text{D}(k)$.

Constraints:

IROW and ICOL must satisfy the following constraints (which may be imposed by a call to F11ZAF):

$\text{IROW}(k - 1) < \text{IROW}(k)$;
$\text{IROW}(k - 1) = \text{IROW}(k)$ and $\text{ICOL}(k - 1) < \text{ICOL}(k)$, for $k = 2, 3, \ldots, \text{NNZ}$.

In addition, if DIRECT = .TRUE., $1 \leq \text{IROW}(k) \leq \text{N}$, $1 \leq \text{ICOL}(k) \leq \text{N}$ and $\text{IROW}(k) \neq \text{ICOL}(k)$;

if DIRECT = .FALSE., $1 \leq \text{IROW}(k) < \text{ICOL}(k) \leq \text{N}$.

9: SPLEN – REAL (KIND=nag_wp) *Output*

On exit: the length of the shortest path between the specified vertices n_s and n_e.

10: PATH(N) – INTEGER array *Output*

On exit: contains details of the shortest path between the specified vertices n_s and n_e. More precisely, NS = PATH(1) $\rightarrow$ PATH(2) $\rightarrow \ldots \rightarrow$ PATH(p) = NE for some $p \leq n$. The remaining $(n - p)$ elements are set to zero.

11: IWORK($3 \times \mathrm{N} + 1$) – INTEGER array *Workspace*

12: WORK($2 \times \mathrm{N}$) – REAL (KIND=nag_wp) array *Workspace*

13: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, N < 2,
or NS < 1,
or NS $>$ N,
or NE < 1,
or NE $>$ N,
or NS = NE.

IFAIL = 2

On entry, NNZ $>$ N $\times$ (N $-$ 1) when DIRECT = .TRUE.,
or NNZ $>$ N $\times$ (N $-$ 1)/2 when DIRECT = .FALSE.,
or NNZ < 1.

IFAIL = 3

On entry, IROW(k) < 1 or IROW(k) $>$ N or ICOL(k) < 1 or ICOL(k) $>$ N or IROW(k) = ICOL(k) for some k when DIRECT = .TRUE..

IFAIL = 4

On entry, IROW(k) < 1 or IROW(k) $\geq$ ICOL(k) or ICOL(k) $>$ N for some k when DIRECT = .FALSE..

IFAIL = 5

D(k) ≤ 0.0 for some k.

IFAIL = 6

On entry, IROW($k - 1$) $>$ IROW(k) or IROW($k - 1$) = IROW(k) and ICOL($k - 1$) $>$ ICOL(k) for some k.

IFAIL $= 7$

On entry, $\mathrm{IROW}(k-1) = \mathrm{IROW}(k)$ and $\mathrm{ICOL}(k-1) = \mathrm{ICOL}(k)$ for some k.

IFAIL $= 8$

No connected network exists between vertices NS and NE.

7 Accuracy

The results are exact, except for the obvious rounding errors in summing the distances in the length of the shortest path.

8 Further Comments

H03ADF is based upon Dijkstra's algorithm (see Dijkstra (1959)), which attempts to find a path $n_s \rightarrow n_e$ between two specified vertices n_s and n_e of shortest length $d(n_s, n_e)$.

The algorithm proceeds by assigning labels to each vertex, which may be **temporary** or **permanent**. A temporary label can be changed, whereas a permanent one cannot. For example, if vertex p has a permanent label (q, r), then r is the distance $d(n_s, r)$ and q is the previous vertex on a shortest length $n_s \rightarrow p$ path. If the label is temporary, then it has the same meaning but it refers only to the shortest $n_s \rightarrow p$ path found so far. A shorter one may be found later, in which case the label may become permanent.

The algorithm consists of the following steps.

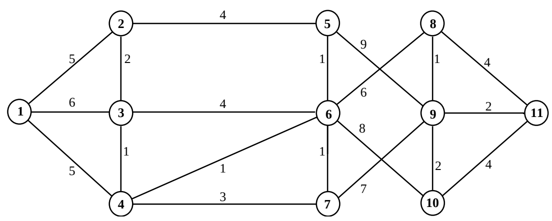
1. Assign the permanent label $(-, 0)$ to vertex n_s and temporary labels $(-, \infty)$ to every other vertex. Set $k = n_s$ and go to 2..
2. Consider each vertex y adjacent to vertex k with a temporary label in turn. Let the label at k be (p, q) and at $y(r, s)$. If $q + d_{ky} < s$, then a new temporary label $\left(k, q + d_{ky}\right)$ is assigned to vertex y; otherwise no change is made in the label of y. When all vertices y with temporary labels adjacent to k have been considered, go to 3..
3. From the set of temporary labels, select the one with the smallest second component and declare that label to be permanent. The vertex it is attached to becomes the new vertex k. If $k = n_e$ go to 4.. Otherwise go to 2. unless no new vertex can be found (e.g., when the set of temporary labels is 'empty' but $k \neq n_e$, in which case no connected network exists between vertices n_s and n_e).
4. To find the shortest path, let (y, z) denote the label of vertex n_e. The column label (z) gives $d(n_s, n_e)$ while the row label (y) then links back to the previous vertex on a shortest length $n_s \rightarrow n_e$ path. Go to vertex y. Suppose that the (permanent) label of vertex y is (w, x), then the next previous vertex is w on a shortest length $n_s \rightarrow y$ path. This process continues until vertex n_s is reached. Hence the shortest path is

$$n_s \rightarrow \ldots \rightarrow w \rightarrow y \rightarrow n_e,$$

which has length $d(n_s, n_e)$.

9 Example

This example finds the shortest path between vertices 1 and 11 for the undirected network



9.1 Program Text

```
    Program h03adfe

!     H03ADF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: f11zaf, h03adf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
      Character (1), Parameter         :: dup = 'F', zero = 'R'
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: splen
      Integer                          :: ifail, j, lenc, n, ne, nnz, ns
      Logical                          :: direct
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: d(:), work(:)
      Integer, Allocatable             :: icol(:), irow(:), iwork(:), path(:)
!     .. Executable Statements ..
      Write (nout,*) 'H03ADF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Read (nin,*) n, ns, ne, nnz, direct
      Allocate (d(nnz),work(2*n),icol(nnz),irow(nnz),iwork(3*n+1),path(n))

      Read (nin,*)(d(j),irow(j),icol(j),j=1,nnz)

!     Reorder the elements of D into the form required by H03ADF.

      ifail = 0
      Call f11zaf(n,nnz,d,irow,icol,dup,zero,iwork,iwork(n+2),ifail)

!     Find the shortest path between vertices NS and NE.

      ifail = 0
      Call h03adf(n,ns,ne,direct,nnz,d,irow,icol,splen,path,iwork,work,ifail)

!     Print details of shortest path.

      lenc = n

loop: Do j = 0, n - 1

        If (path(j+1)==0) Then
          lenc = j
          Exit loop
        End If

      End Do loop

      Write (nout,99999) 'Shortest path = ', (path(j),j=1,lenc)
      Write (nout,99998) 'Length of shortest path = ', splen

99999 Format (/1X,A,10(I2:'  to '))
99998 Format (/1X,A,G16.6)
    End Program h03adfe
```

9.2 Program Data

```
H03ADF Example Program Data
11  1  11  20  F   :Values of N, NS, NE, NNZ and DIRECT
6.0   6   8
1.0   8   9
2.0   9  11
```

```
4.0   2   5
1.0   3   4
6.0   1   3
4.0   3   6
1.0   4   6
2.0   2   3
3.0   4   7
5.0   1   2
7.0   6  10
1.0   5   6
4.0   8  11
9.0   5   9
1.0   6   7
8.0   7   9
4.0  10  11
2.0   9  10
5.0   1   4        :End of D, IROW, ICOL
```

9.3 Program Results

```
 H03ADF Example Program Results

 Shortest path =  1  to  4  to  6  to  8  to  9  to 11

 Length of shortest path =      15.0000
```