NAG Library Routine Document

G13CCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13CCF calculates the smoothed sample cross spectrum of a bivariate time series using one of four lag windows: rectangular, Bartlett, Tukey or Parzen.

2 Specification

```
SUBROUTINE G13CCF (NXY, MTXY, PXY, IW, MW, ISH, IC, NC, CXY, CYX, KC, L,        &
                   NXYG, XG, YG, NG, IFAIL)

INTEGER            NXY, MTXY, IW, MW, ISH, IC, NC, KC, L, NXYG, NG, IFAIL
REAL (KIND=nag_wp) PXY, CXY(NC), CYX(NC), XG(NXYG), YG(NXYG)
```

3 Description

The smoothed sample cross spectrum is a complex valued function of frequency ω, $f_{xy}(\omega) = cf(\omega) + iqf(\omega)$, defined by its real part or co-spectrum

$$cf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S)\cos(\omega k)$$

and imaginary part or quadrature spectrum

$$qf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S)\sin(\omega k)$$

where $w_k = w_{-k}$, for $k = 0, 1, \ldots, M-1$, is the smoothing lag window as defined in the description of G13CAF. The alignment shift S is recommended to be chosen as the lag k at which the cross-covariances $c_{xy}(k)$ peak, so as to minimize bias.

The results are calculated for frequency values

$$\omega_j = \frac{2\pi j}{L}, \qquad j = 0, 1, \ldots, [L/2],$$

where [] denotes the integer part.

The cross-covariances $c_{xy}(k)$ may be supplied by you, or constructed from supplied series $x_1, x_2, \ldots, x_n$; $y_1, y_2, \ldots, y_n$ as

$$c_{xy}(k) = \frac{\sum_{t=1}^{n-k} x_t y_{t+k}}{n}, \qquad k \geq 0$$

$$c_{xy}(k) = \frac{\sum_{t=1-k}^{n} x_t y_{t+k}}{n} = c_{yx}(-k), \qquad k < 0$$

this convolution being carried out using the finite Fourier transform.

The supplied series may be mean and trend corrected and tapered before calculation of the cross-covariances, in exactly the manner described in G13CAF for univariate spectrum estimation. The results are corrected for any bias due to tapering.

The bandwidth associated with the estimates is not returned. It will normally already have been calculated in previous calls of G13CAF for estimating the univariate spectra of y_t and x_t.

4 References

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden–Day

5 Parameters

1: NXY – INTEGER *Input*

On entry: n, the length of the time series x and y.

Constraint: NXY ≥ 1.

2: MTXY – INTEGER *Input*

On entry: if cross-covariances are to be calculated by the routine (IC $= 0$), MTXY must specify whether the data is to be initially mean or trend corrected.

MTXY $= 0$
For no correction.

MTXY $= 1$
For mean correction.

MTXY $= 2$
For trend correction.

If cross-covariances are supplied (IC $\neq 0$), MTXY is not used.

Constraint: if IC $= 0$, MTXY $= 0$, 1 or 2.

3: PXY – REAL (KIND=nag_wp) *Input*

On entry: if cross-covariances are to be calculated by the routine (IC $= 0$), PXY must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. A value of 0.0 implies no tapering.

If cross-covariances are supplied (IC $\neq 0$), PXY is not used.

Constraint: if IC $= 0$, $0.0 \leq$ PXY ≤ 1.0.

4: IW – INTEGER *Input*

On entry: the choice of lag window.

IW $= 1$
Rectangular.

IW $= 2$
Bartlett.

IW $= 3$
Tukey.

IW $= 4$
Parzen.

Constraint: $1 \leq$ IW ≤ 4.

5: MW – INTEGER *Input*

On entry: M, the 'cut-off' point of the lag window, relative to any alignment shift that has been applied. Windowed cross-covariances at lags $(-\text{MW} + \text{ISH})$ or less, and at lags $(\text{MW} + \text{ISH})$ or greater are zero.

Constraints:

MW ≥ 1;
MW + |ISH| $\leq$ NXY.

6: ISH – INTEGER *Input*

On entry: S, the alignment shift between the x and y series. If x leads y, the shift is positive.

Constraint: $-\text{MW} < \text{ISH} < \text{MW}$.

7: IC – INTEGER *Input*

On entry: indicates whether cross-covariances are to be calculated in the routine or supplied in the call to the routine.

IC $= 0$
Cross-covariances are to be calculated.

IC $\neq 0$
Cross-covariances are to be supplied.

8: NC – INTEGER *Input*

On entry: the number of cross-covariances to be calculated in the routine or supplied in the call to the routine.

Constraint: MW + |ISH| $\leq$ NC $\leq$ NXY.

9: CXY(NC) – REAL (KIND=nag_wp) array *Input/Output*

On entry: if IC $\neq 0$, CXY must contain the NC cross-covariances between values in the y series and earlier values in time in the x series, for lags from 0 to $(\text{NC} - 1)$.

If IC $= 0$, CXY need not be set.

On exit: if IC $= 0$, CXY will contain the NC calculated cross-covariances.

If IC $\neq 0$, the contents of CXY will be unchanged.

10: CYX(NC) – REAL (KIND=nag_wp) array *Input/Output*

On entry: if IC $\neq 0$, CYX must contain the NC cross-covariances between values in the y series and later values in time in the x series, for lags from 0 to $(\text{NC} - 1)$.

If IC $= 0$, CYX need not be set.

On exit: if IC $= 0$, CYX will contain the NC calculated cross-covariances.

If IC $\neq 0$, the contents of CYX will be unchanged.

11: KC – INTEGER *Input*

On entry: if IC $= 0$, KC must specify the order of the fast Fourier transform (FFT) used to calculate the cross-covariances. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq n + \text{NC}$.

If IC $\neq 0$, that is if covariances are supplied, KC is not used.

Constraint: KC $\geq$ NXY + NC. The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

12: L – INTEGER *Input*

On entry: L, the frequency division of the spectral estimates as $\frac{2\pi}{L}$. Therefore it is also the order of the FFT used to construct the sample spectrum from the cross-covariances. L should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2M - 1$.

Constraint: $\mathrm{L} \geq 2 \times \mathrm{MW} - 1$. The largest prime factor of L must not exceed 19, and the total number of prime factors of L, counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

13: NXYG – INTEGER *Input*

On entry: the dimension of the arrays XG and YG as declared in the (sub)program from which G13CCF is called.

Constraints:

if $\mathrm{IC} = 0$, $\mathrm{NXYG} \geq \max(\mathrm{KC}, \mathrm{L})$;
if $\mathrm{IC} \neq 0$, $\mathrm{NXYG} \geq \mathrm{L}$.

14: XG(NXYG) – REAL (KIND=nag_wp) array *Input/Output*

On entry: if the cross-covariances are to be calculated, then XG must contain the NXY data points of the x series. If covariances are supplied, XG need not be set.

On exit: contains the real parts of the NG complex spectral estimates in elements $\mathrm{XG}(1)$ to $\mathrm{XG}(\mathrm{NG})$, and $\mathrm{XG}(\mathrm{NG}+1)$ to $\mathrm{XG}(\mathrm{NXYG})$ contain 0.0. The y series leads the x series.

15: YG(NXYG) – REAL (KIND=nag_wp) array *Input/Output*

On entry: if cross-covariances are to be calculated, YG must contain the NXY data points of the y series. If covariances are supplied, YG need not be set.

On exit: contains the imaginary parts of the NG complex spectral estimates in elements $\mathrm{YG}(1)$ to $\mathrm{YG}(\mathrm{NG})$, and $\mathrm{YG}(\mathrm{NG}+1)$ to $\mathrm{YG}(\mathrm{NXYG})$ contain 0.0. The y series leads the x series.

16: NG – INTEGER *Output*

On exit: the number, $[\mathrm{L}/2] + 1$, of complex spectral estimates, whose separate parts are held in XG and YG.

17: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: $\mathrm{IFAIL} = 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, NXY < 1,
or MTXY < 0 and IC $= 0$,
or MTXY > 2 and IC $= 0$,
or PXY < 0.0 and IC $= 0$,
or PXY > 1.0 and IC $= 0$,
or IW ≤ 0,
or IW > 4,
or MW < 1,
or MW $+$ |ISH| $>$ NXY,
or |ISH| $\geq$ MW,
or NC $<$ MW $+$ |ISH|,
or NC $>$ NXY,
or NXYG $< \max(\mathrm{KC}, \mathrm{L})$ and IC $= 0$,
or NXYG $<$ L and IC $\neq 0$.

IFAIL $= 2$

On entry, KC $<$ NXY $+$ NC,
or KC has a prime factor exceeding 19,
or KC has more than 20 prime factors, counting repetitions.

This error only occurs when IC $= 0$.

IFAIL $= 3$

On entry, L $< 2 \times$ MW $- 1$,
or L has a prime factor exceeding 19,
or L has more than 20 prime factors, counting repetitions.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CCF carries out two FFTs of length KC to calculate the sample cross-covariances and one FFT of length L to calculate the sample spectrum. The timing of G13CCF is therefore dependent on the choice of these values. The time taken for an FFT of length n is approximately proportional to $n \log n$ (but see Section 8 in C06PAF for further details).

9 Example

This example reads two time series of length 296. It then selects mean correction, a 10% tapering proportion, the Parzen smoothing window and a cut-off point of 35 for the lag window. The alignment shift is set to 3 and 50 cross-covariances are chosen to be calculated. The program then calls G13CCF to calculate the cross spectrum and then prints the cross-covariances and cross spectrum.

9.1 Program Text

```
    Program g13ccfe

!     G13CCF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: g13ccf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: pxy
      Integer                          :: i, ic, ifail, ii, ish, iw, kc, l,    &
                                          lxg, lyg, mtxy, mw, nc, ng, nxy, nxyg
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: cxy(:), cyx(:), xg(:), yg(:)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: max, min
!     .. Executable Statements ..
      Write (nout,*) 'G13CCF Example Program Results'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Read in the problem size
      Read (nin,*) nxy, nc, ic

!     Read in control parameters
      Read (nin,*) mtxy, pxy
      Read (nin,*) iw, mw
      Read (nin,*) ish, kc, l

      If (ic==0) Then
        nxyg = max(kc,l)
      Else
        nxyg = l
      End If
      lxg = max(nxyg,nxy)
      lyg = max(nxyg,nxy)
      Allocate (xg(lxg),yg(lyg),cxy(nc),cyx(nc))

      If (ic==0) Then
        Read (nin,*)(xg(i),i=1,nxy)
        Read (nin,*)(yg(i),i=1,nxy)
      Else
        Read (nin,*)(cxy(i),i=1,nc)
        Read (nin,*)(cyx(i),i=1,nc)
      End If

      ifail = 0
      Call g13ccf(nxy,mtxy,pxy,iw,mw,ish,ic,nc,cxy,cyx,kc,l,nxyg,xg,yg,ng, &
        ifail)

!     Display results
      Write (nout,*) '                 Returned cross covariances'
      Write (nout,*)
      Write (nout,*) &
        'Lag     XY       YX   Lag     XY       YX   Lag     XY       YX'
      Do i = 1, nc, 3
        Write (nout,99999)(ii-1,cxy(ii),cyx(ii),ii=i,min(i+2,nc))
      End Do
      Write (nout,*)
      Write (nout,*) '                     Returned sample spectrum'
      Write (nout,*)
      Write (nout,*) &
        '       Real  Imaginary       Real  Imaginary       Real  Imaginary'
```

```
      Write (nout,*) &
        'Lag    part     part  Lag    part     part  Lag    part     part'
      Do i = 1, ng, 3
        Write (nout,99999)(ii-1,xg(ii),yg(ii),ii=i,min(i+2,ng))
      End Do

99999 Format (1X,I3,2F9.4,I4,2F9.4,I4,2F9.4)
    End Program g13ccfe
```

9.2 Program Data

```
G13CCF Example Program Data
   296 50 0                                                    :: NXY,NC,IC
1  0.1                                                         :: MTXY,PXY
4  35                                                          :: IW,MW
3  350  80                                                     :: ISH,KC,L
 -0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
  0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
 -0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
  0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
  1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
  0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
  1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
  0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
  1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
 -1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
 -0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
 -0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
  1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
 -0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
 -2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
 -0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
  0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
  0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
  0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
  0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
  0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
 -1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
  0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
  0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
  0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
 -2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
 -0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
 -1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
 -1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
 -0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
 -1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
 -0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
  0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
  1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
 -0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
  0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
  0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262 :: End of XG
 53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0
 52.4 53.0 54.0 54.9 56.0 56.8 56.8 56.4 55.7 55.0 54.3
 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9 47.6
 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2
 50.0 48.3 47.0 45.8 45.6 46.0 46.9 47.8 48.2 48.3 47.9
 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5 50.1 49.8
 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0
 56.9 57.5 57.3 56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4
 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1 51.6 51.0 50.5
 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8
 53.3 53.0 52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4
 58.4 57.6 56.9 56.4 56.0 55.7 55.3 55.0 54.4 53.7 52.8
 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9
 50.7 50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0
 53.6 53.2 53.0 52.8 52.3 51.9 51.6 51.6 51.4 51.2 50.7
 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
 54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9
```

```
 55.2 54.4 53.7 53.6 53.6 53.2 52.5 52.0 51.4 51.0 50.9
 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7 59.0
 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3
 57.4 57.0 56.4 55.9 55.5 55.3 55.2 55.4 56.0 56.5 57.1
 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2 57.8 58.3
 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4
 56.0 55.2 54.0 53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0
 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2 52.0 52.8 53.8
 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0
 54.3 56.0 57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0       :: End of YG
```

9.3 Program Results

```
G13CCF Example Program Results

                Returned cross covariances

Lag     XY       YX   Lag     XY       YX   Lag     XY       YX
  0  -1.6700  -1.6700    1  -2.0581  -1.3606    2  -2.4859  -1.1383
  3  -2.8793  -0.9926    4  -3.1473  -0.9009    5  -3.2239  -0.8382
  6  -3.0929  -0.7804    7  -2.7974  -0.7074    8  -2.4145  -0.6147
  9  -2.0237  -0.5080   10  -1.6802  -0.4032   11  -1.4065  -0.3159
 12  -1.2049  -0.2554   13  -1.0655  -0.2250   14  -0.9726  -0.2238
 15  -0.9117  -0.2454   16  -0.8658  -0.2784   17  -0.8180  -0.3081
 18  -0.7563  -0.3257   19  -0.6750  -0.3315   20  -0.5754  -0.3321
 21  -0.4701  -0.3308   22  -0.3738  -0.3312   23  -0.3023  -0.3332
 24  -0.2665  -0.3384   25  -0.2645  -0.3506   26  -0.2847  -0.3727
 27  -0.3103  -0.3992   28  -0.3263  -0.4152   29  -0.3271  -0.4044
 30  -0.3119  -0.3621   31  -0.2837  -0.2919   32  -0.2568  -0.2054
 33  -0.2427  -0.1185   34  -0.2490  -0.0414   35  -0.2774   0.0227
 36  -0.3218   0.0697   37  -0.3705   0.1039   38  -0.4083   0.1356
 39  -0.4197   0.1805   40  -0.3920   0.2460   41  -0.3241   0.3319
 42  -0.2273   0.4325   43  -0.1216   0.5331   44  -0.0245   0.6199
 45   0.0528   0.6875   46   0.1074   0.7329   47   0.1448   0.7550
 48   0.1713   0.7544   49   0.1943   0.7349

                    Returned sample spectrum

       Real  Imaginary       Real  Imaginary       Real  Imaginary
Lag    part      part  Lag    part      part  Lag    part      part
  0  -6.5500   0.0000    1  -5.4267  -1.9842    2  -3.1323  -2.7307
  3  -1.2649  -2.3998    4  -0.2102  -1.7520    5   0.3411  -1.1903
  6   0.6063  -0.7420    7   0.6178  -0.3586    8   0.4391  -0.1008
  9   0.2422   0.0061   10   0.1233   0.0409   11   0.0574   0.0529
 12   0.0174   0.0452   13  -0.0008   0.0289   14  -0.0058   0.0161
 15  -0.0051   0.0084   16  -0.0027   0.0040   17  -0.0010   0.0015
 18  -0.0006   0.0006   19  -0.0005   0.0003   20  -0.0003   0.0003
 21  -0.0003   0.0004   22  -0.0003   0.0003   23  -0.0003   0.0002
 24  -0.0004   0.0001   25  -0.0004  -0.0000   26  -0.0003  -0.0001
 27  -0.0002  -0.0001   28  -0.0001   0.0001   29  -0.0002   0.0003
 30  -0.0003   0.0002   31  -0.0002   0.0001   32  -0.0001   0.0000
 33  -0.0000  -0.0000   34   0.0001  -0.0001   35   0.0001  -0.0002
 36   0.0001  -0.0001   37   0.0001  -0.0001   38   0.0001  -0.0001
 39   0.0001  -0.0001   40   0.0001   0.0000
```