# NAG Library Routine Document

# F08QXF (ZTREVC)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

## 1 Purpose

F08QXF (ZTREVC) computes selected left and/or right eigenvectors of a complex upper triangular matrix.

## 2 Specification

```
SUBROUTINE F08QXF (JOB, HOWMNY, SELECT, N, T, LDT, VL, LDVL, VR, LDVR, MM,      &
                   M, WORK, RWORK, INFO)

INTEGER                 N, LDT, LDVL, LDVR, MM, M, INFO
REAL (KIND=nag_wp)      RWORK(N)
COMPLEX (KIND=nag_wp) T(LDT,*), VL(LDVL,*), VR(LDVR,*), WORK(2*N)
LOGICAL                 SELECT(*)
CHARACTER(1)            JOB, HOWMNY
```

The routine may be called by its LAPACK name ***ztrevc***.

## 3 Description

F08QXF (ZTREVC) computes left and/or right eigenvectors of a complex upper triangular matrix $T$. Such a matrix arises from the Schur factorization of a complex general matrix, as computed by F08PSF (ZHSEQR), for example.

The right eigenvector $x$, and the left eigenvector $y$, corresponding to an eigenvalue $\lambda$, are defined by:

$$Tx = \lambda x \qquad \text{and} \qquad y^{\mathrm{H}}T = \lambda y^{\mathrm{H}} \big(\text{or } T^{\mathrm{H}}y = \bar{\lambda}y\big).$$

The routine can compute the eigenvectors corresponding to selected eigenvalues, or it can compute all the eigenvectors. In the latter case the eigenvectors may optionally be pre-multiplied by an input matrix $Q$. Normally $Q$ is a unitary matrix from the Schur factorization of a matrix $A$ as $A = QTQ^{\mathrm{H}}$; if $x$ is a (left or right) eigenvector of $T$, then $Qx$ is an eigenvector of $A$.

The eigenvectors are computed by forward or backward substitution. They are scaled so that $\max |\mathrm{Re}(x_i)| + |\mathrm{Im}\, x_i| = 1$.

## 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

1: JOB – CHARACTER(1) *Input*

*On entry*: indicates whether left and/or right eigenvectors are to be computed.

JOB = 'R'
Only right eigenvectors are computed.

JOB = 'L'
Only left eigenvectors are computed.

JOB = 'B'
Both left and right eigenvectors are computed.

*Constraint*: JOB = 'R', 'L' or 'B'.

2: HOWMNY – CHARACTER(1) *Input*

*On entry*: indicates how many eigenvectors are to be computed.

HOWMNY = 'A'
All eigenvectors (as specified by JOB) are computed.

HOWMNY = 'B' or 'O'
All eigenvectors (as specified by JOB) are computed and then pre-multiplied by the matrix $Q$ (which is overwritten).

HOWMNY = 'S'
Selected eigenvectors (as specified by JOB and SELECT) are computed.

*Constraint*: HOWMNY = 'A', 'B', 'O' or 'S'.

3: SELECT($*$) – LOGICAL array *Input*

**Note**: the dimension of the array SELECT must be at least $\max(1, \mathrm{N})$ if HOWMNY = 'S', and at least 1 otherwise.

*On entry*: specifies which eigenvectors are to be computed if HOWMNY = 'S'. To obtain the eigenvector corresponding to the eigenvalue $\lambda_j$, SELECT($j$) must be set .TRUE..

If HOWMNY = 'A', 'O' or 'B', SELECT is not referenced.

4: N – INTEGER *Input*

*On entry*: $n$, the order of the matrix $T$.

*Constraint*: $\mathrm{N} \geq 0$.

5: T(LDT,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

**Note**: the second dimension of the array T must be at least $\max(1, \mathrm{N})$.

*On entry*: the $n$ by $n$ upper triangular matrix $T$, as returned by F08PSF (ZHSEQR).

*On exit*: is used as internal workspace prior to being restored and hence is unchanged.

6: LDT – INTEGER *Input*

*On entry*: the first dimension of the array T as declared in the (sub)program from which F08QXF (ZTREVC) is called.

*Constraint*: $\mathrm{LDT} \geq \max(1, \mathrm{N})$.

7: VL(LDVL,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

**Note**: the second dimension of the array VL must be at least $\max(1, \mathrm{MM})$ if JOB = 'L' or 'B' and at least 1 if JOB = 'R'.

*On entry*: if HOWMNY = 'O' or 'B' and JOB = 'L' or 'B', VL must contain an $n$ by $n$ matrix $Q$ (usually the matrix of Schur vectors returned by F08PSF (ZHSEQR)).

If HOWMNY = 'A' or 'S', VL need not be set.

*On exit*: if JOB = 'L' or 'B', VL contains the computed left eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues.

If JOB = 'R', VL is not referenced.

8: LDVL – INTEGER *Input*

*On entry*: the first dimension of the array VL as declared in the (sub)program from which F08QXF (ZTREVC) is called.

*Constraints*:

if JOB = 'L' or 'B', LDVL $\geq \max(1, \mathrm{N})$;
if JOB = 'R', LDVL $\geq 1$.

9: VR(LDVR,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

**Note**: the second dimension of the array VR must be at least $\max(1, \mathrm{MM})$ if JOB = 'R' or 'B' and at least 1 if JOB = 'L'.

*On entry*: if HOWMNY = 'O' or 'B' and JOB = 'R' or 'B', VR must contain an $n$ by $n$ matrix $Q$ (usually the matrix of Schur vectors returned by F08PSF (ZHSEQR)).

If HOWMNY = 'A' or 'S', VR need not be set.

*On exit*: if JOB = 'R' or 'B', VR contains the computed right eigenvectors (as specified by HOWMNY and SELECT). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues.

If JOB = 'L', VR is not referenced.

10: LDVR – INTEGER *Input*

*On entry*: the first dimension of the array VR as declared in the (sub)program from which F08QXF (ZTREVC) is called.

*Constraints*:

if JOB = 'R' or 'B', LDVR $\geq \max(1, \mathrm{N})$;
if JOB = 'L', LDVR $\geq 1$.

11: MM – INTEGER *Input*

*On entry*: the number of columns in the arrays VL and/or VR. The precise number of columns required, $m$, is $n$ if HOWMNY = 'A', 'O' or 'B'; if HOWMNY = 'S', $m$ is the number of selected eigenvectors (see SELECT), in which case $0 \leq m \leq n$.

*Constraint*: MM $\geq m$.

12: M – INTEGER *Output*

*On exit*: $m$, the number of selected eigenvectors. If HOWMNY = 'A', 'O' or 'B', M is set to $n$.

13: WORK($2 \times$ N) – COMPLEX (KIND=nag_wp) array *Workspace*

14: RWORK(N) – REAL (KIND=nag_wp) array *Workspace*

15: INFO – INTEGER *Output*

*On exit*: INFO = 0 unless the routine detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO $< 0$

If INFO $= -i$, argument $i$ had an illegal value. An explanatory message is output, and execution of the program is terminated.

## 7 Accuracy

If $x_i$ is an exact right eigenvector, and $\tilde{x}_i$ is the corresponding computed eigenvector, then the angle $\theta(\tilde{x}_i, x_i)$ between them is bounded as follows:

$$\theta(\tilde{x}_i, x_i) \leq \frac{c(n)\epsilon\|T\|_2}{sep_i}$$

where $sep_i$ is the reciprocal condition number of $x_i$.

The condition number $sep_i$ may be computed by calling F08QYF (ZTRSNA).

## 8 Further Comments

The real analogue of this routine is F08QKF (DTREVC).

## 9 Example

See Section 9 in F08NVF (ZGEBAL).