NAG Library Routine Document

F07ADF (DGETRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07ADF (DGETRF) computes the LU factorization of a real m by n matrix.

2 Specification

```
SUBROUTINE F07ADF (M, N, A, LDA, IPIV, INFO)

INTEGER            M, N, LDA, IPIV(min(M,N)), INFO
REAL (KIND=nag_wp) A(LDA,*)
```

The routine may be called by its LAPACK name ***dgetrf***.

3 Description

F07ADF (DGETRF) forms the LU factorization of a real m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square $(m = n)$, and both L and U are triangular. The routine uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: M – INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: M ≥ 0.

2: N – INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the m by n matrix A.

On exit: the factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07ADF (DGETRF) is called.

Constraint: LDA $\geq \max(1, \mathrm{M})$.

5: IPIV(min(M, N)) – INTEGER array *Output*

On exit: the pivot indices that define the permutation matrix. At the ith step, if IPIV$(i) > i$ then row i of the matrix A was interchanged with row IPIV(i), for $i = 1, 2, \ldots, \min(m, n)$. IPIV$(i) \leq i$ indicates that, at the ith step, a row interchange was not required.

6: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}n^3$ if $m = n$ (the usual case), $\frac{1}{3}n^2(3m - n)$ if $m > n$ and $\frac{1}{3}m^2(3n - m)$ if $m < n$.

A call to this routine with $m = n$ may be followed by calls to the routines:

F07AEF (DGETRS) to solve $AX = B$ or $A^{\mathrm{T}}X = B$;

F07AGF (DGECON) to estimate the condition number of A;

F07AJF (DGETRI) to compute the inverse of A.

The complex analogue of this routine is F07ARF (ZGETRF).

9 Example

This example computes the LU factorization of the matrix A, where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix}.$$

9.1 Program Text

```
    Program f07adfe

!     F07ADF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dgetrf, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, m, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:)
      Integer, Allocatable             :: ipiv(:)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: min
!     .. Executable Statements ..
      Write (nout,*) 'F07ADF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) m, n
      lda = m
      Allocate (a(lda,n),ipiv(n))

!     Read A from data file

      Read (nin,*)(a(i,1:n),i=1,m)

!     Factorize A

!     The NAG name equivalent of dgetrf is f07adf
      Call dgetrf(m,n,a,lda,ipiv,info)

!     Print details of factorization

      Write (nout,*)
      Flush (nout)

!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04caf('General',' ',m,n,a,lda,'Details of factorization',ifail)

!     Print pivot indices

      Write (nout,*)
      Write (nout,*) 'IPIV'
      Write (nout,99999) ipiv(1:min(m,n))

      If (info/=0) Write (nout,*) 'The factor U is singular'

99999 Format ((3X,7I11))
    End Program f07adfe
```

9.2 Program Data

```
F07ADF Example Program Data
  4  4                         :Values of M and N
  1.80   2.88   2.05  -0.89
  5.25  -2.95  -0.95  -3.80
  1.58  -2.69  -2.90  -1.04
 -1.11  -0.66  -0.59   0.80   :End of matrix A
```

9.3 Program Results

```
 F07ADF Example Program Results

 Details of factorization
              1           2           3           4
 1       5.2500     -2.9500     -0.9500     -3.8000
 2       0.3429      3.8914      2.3757      0.4129
 3       0.3010     -0.4631     -1.5139      0.2948
 4      -0.2114     -0.3299      0.0047      0.1314

 IPIV
              2           2           3           4
```