# NAG Library Routine Document F01BUF 

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

## 1 Purpose

F01BUF performs a $U L D L^{\mathrm{T}} U^{\mathrm{T}}$ decomposition of a real symmetric positive definite band matrix.

## 2 Specification

```
SUBROUTINE FO1BUF (N, M1, K, A, LDA, W, IFAIL)
INTEGER N, M1, K, LDA, IFAIL
REAL (KIND=nag_wp) A(LDA,N), W(M1)
```


## 3 Description

The symmetric positive definite matrix $A$, of order $n$ and bandwidth $2 m+1$, is divided into the leading principal sub-matrix of order $k$ and its complement, where $m \leq k \leq n$. A $U D U^{\mathrm{T}}$ decomposition of the latter and an $L D L^{\mathrm{T}}$ decomposition of the former are obtained by means of a sequence of elementary transformations, where $U$ is unit upper triangular, $L$ is unit lower triangular and $D$ is diagonal. Thus if $k=n$, an $L D L^{\mathrm{T}}$ decomposition of $A$ is obtained.

This routine is specifically designed to precede F01BVF for the transformation of the symmetric-definite eigenproblem $A x=\lambda B x$ by the method of Crawford where $A$ and $B$ are of band form. In this context, $k$ is chosen to be close to $n / 2$ and the decomposition is applied to the matrix $B$.

## 4 References

Wilkinson J H (1965) The Algebraic Eigenvalue Problem Oxford University Press, Oxford
Wilkinson J H and Reinsch C (1971) Handbook for Automatic Computation II, Linear Algebra SpringerVerlag

## 5 Parameters

1: N - INTEGER
Input
On entry: $n$, the order of the matrix $A$.
2: M1 - INTEGER
Input
On entry: $m+1$, where $m$ is the number of nonzero superdiagonals in $A$. Normally $\mathrm{M} 1 \ll \mathrm{~N}$.
3: K - INTEGER Input
On entry: $k$, the change-over point in the decomposition.
Constraint: $\mathrm{M} 1-1 \leq \mathrm{K} \leq \mathrm{N}$.
4: $\quad \mathrm{A}(\mathrm{LDA}, \mathrm{N})-$ REAL (KIND=nag_wp) array
On entry: the upper triangle of the $n$ by $n$ symmetric band matrix $A$, with the diagonal of the matrix stored in the $(m+1)$ th row of the array, and the $m$ superdiagonals within the band stored in the first $m$ rows of the array. Each column of the matrix is stored in the corresponding column of the array. For example, if $n=6$ and $m=2$, the storage scheme is

| $*$ | $*$ | $a_{13}$ | $a_{24}$ | $a_{35}$ | $a_{46}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $*$ | $a_{12}$ | $a_{23}$ | $a_{34}$ | $a_{45}$ | $a_{56}$ |
| $a_{11}$ | $a_{22}$ | $a_{33}$ | $a_{44}$ | $a_{55}$ | $a_{66}$ |

Elements in the top left corner of the array are not used. The following code assigns the matrix elements within the band to the correct elements of the array:

```
    DO 20 J = 1, N
    DO 10 I = MAX(1,J-M1+1), J
        A(I-J+M1,J) = matrix(I,J)
    CONTINUE
CONTINUE
```

On exit: $A$ is overwritten by the corresponding elements of $L, D$ and $U$.
5: LDA - INTEGER
Input
On entry: the first dimension of the array A as declared in the (sub)program from which F01BUF is called.

Constraint: LDA $\geq$ M1.
6: W(M1) - REAL (KIND=nag_wp) array Workspace
7: IFAIL - INTEGER Input/Output
On entry: IFAIL must be set to $0,-1$ or 1 . If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0 . When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.

On exit: IFAIL $=0$ unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry IFAIL $=0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:
IFAIL $=1$
On entry, $\mathrm{K}<\mathrm{M} 1$ or $\mathrm{K}>\mathrm{N}$.
IFAIL $=2$
IFAIL $=3$
The matrix $A$ is not positive definite, perhaps as a result of rounding errors, giving an element of $D$ which is zero or negative. IFAIL $=3$ when the failure occurs in the leading principal sub-matrix of order K and $\operatorname{IFAIL}=2$ when it occurs in the complement.

## 7 Accuracy

The Cholesky decomposition of a positive definite matrix is known for its remarkable numerical stability (see Wilkinson (1965)). The computed $U, L$ and $D$ satisfy the relation $U L D L^{\mathrm{T}} U^{\mathrm{T}}=A+E$ where the 2-norms of $A$ and $E$ are related by $\|E\| \leq c(m+1)^{2} \epsilon\|A\|$ where $c$ is a constant of order unity and $\epsilon$ is the machine precision. In practice, the error is usually appreciably smaller than this.

## 8 Further Comments

The time taken by F01BUF is approximately proportional to $\mathrm{nm}^{2}+3 \mathrm{~nm}$.
This routine is specifically designed for use as the first stage in the solution of the generalized symmetric eigenproblem $A x=\lambda B x$ by Crawford's method which preserves band form in the transformation to a similar standard problem. In this context, for maximum efficiency, $k$ should be chosen as the multiple of $m$ nearest to $n / 2$.
The matrix $U$ is such that $U^{-1} A U^{-\mathrm{T}}$ is diagonal in its last $n-k$ rows and columns, $L$ is such that $L^{-1} U^{-1} A U^{-\mathrm{T}} L^{-\mathrm{T}}=D$ and $D$ is diagonal. To find $U, L$ and $D$ where $A=U L D L^{\mathrm{T}} U^{\mathrm{T}}$ requires $n m(m+3) / 2-m(m+1)(m+2) / 3$ multiplications and divisions which, is independent of $k$.

## 9 Example

This example finds a $U L D L^{\mathrm{T}} U^{\mathrm{T}}$ decomposition of the real symmetric positive definite matrix

$$
\left(\begin{array}{rrrrrrr}
3 & -9 & 6 & & & & \\
-9 & 31 & -2 & -4 & & & \\
6 & -2 & 123 & -66 & 15 & & \\
& -4 & -66 & 145 & -24 & 4 & \\
& & 15 & -24 & 61 & -74 & -18 \\
& & & 4 & -74 & 98 & 24 \\
& & & & -18 & 24 & 6
\end{array}\right)
$$

### 9.1 Program Text

```
Program f01bufe
    FO1BUF Example Program Text
    Mark 24 Release. NAG Copyright 2012.
    .. Use Statements ..
    Use nag_library, Only: f01buf, nag_wp
    .. Implicit None Statement ..
    Implicit None
    .. Parameters ..
    Integer, Parameter :: nin = 5, nout = 6
    .. Local Scalars ..
    Integer :: i, ifail, j, k, lda, m, ml, n
    .. Local Arrays ..
    Real (Kind=nag_wp), Allocatable :: a(:,:), w(:)
! .. Intrinsic Procedures ..
    Intrinsic :: max
    .. Executable Statements ..
    Write (nout,*) 'fO1BUF Example Program Results'
! Skip heading in data file
    Read (nin,*)
    Read (nin,*) n, m1
    lda = m1
    Allocate (a(lda,n),w(m1))
    Read (nin,*)((a(j,i),j=max(1,m1+1-i),m1),i=1,n)
    m = m1 - 1
    k = ((n+m)/(2*m))*m
    ifail: behaviour on error exit
                        =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
    ifail = 0
    Call f01buf(n,m1,k,a,lda,w,ifail)
    Write (nout,*)
    Write (nout,*) 'Computed upper triangular matrix'
    Write (nout,*)
    Do i = 1, n
```

```
            Write (nout,99999)(a(j,i),j=max(1,m1+1-i),m1)
            End Do
99999 Format (1X,8F9.4)
    End Program folbufe
```


### 9.2 Program Data

| F01BUF | Example Program | Data <br> 7 <br> 3 |  |
| ---: | ---: | ---: | :--- |
| 3 |  |  |  |
| -9 | 31 |  |  |
| 6 | -2 | 123 |  |
| -4 | -66 | 145 |  |
| 15 | -24 | 61 |  |
| 4 | -74 | 98 |  |
| -18 | 24 | 6 | $: a$ |

### 9.3 Program Results

F01BUF Example Program Results
Computed upper triangular matr

| 3.0000 |  |  |
| ---: | ---: | ---: |
| -3.0000 | 4.0000 |  |
| 2.0000 | 4.0000 | 2.0000 |
| -1.0000 | 5.0000 | 3.0000 |
| 3.0000 | -4.0000 | 5.0000 |
| 2.0000 | -1.0000 | 2.0000 |
| -3.0000 | 4.0000 | 6.0000 |

