NAG Library Routine Document

E05SBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

Note: *this routine uses* **optional parameters** *to define choices in the problem specification and in the details of the algorithm. If you wish to use* default *settings for all of the optional parameters, you need only read Sections 1 to 9 of this document. If, however, you wish to reset some or all of the settings please refer to Section 10 for a detailed description of the algorithm and to Section 11 for a detailed description of the specification of the optional parameters*.

1 Purpose

E05SBF is designed to search for the global minimum or maximum of an arbitrary function, subject to general nonlinear constraints, using Particle Swarm Optimization (PSO). Derivatives are not required, although these may be used by an accompanying local minimization routine if desired. E05SBF is essentially identical to E05SAF, with an expert interface and various additional parameters added; otherwise most parameters are identical. In particular, E05SAF does not handle general constraints.

1.1 Information for users of the NAG Library for SMP & Multicore

E05SBF has been designed to be particularly effective on SMP systems, by allowing multiple threads to advance subiterations of the algorithm in a highly asynchronous manner. In doing this, the callback routines supplied to E05SBF are called simultaneously on multiple threads, and therefore must themselves be thread-safe. In particular, the arrays IUSER and RUSER provided are classified as OpenMP shared memory, and as such it is imperative that any operations performed on these arrays are done in an appropriate manner. Failure to ensure thread safety of the provided callback functions may result in unpredictable behaviour, including the callback functions returning completely wrong solutions to E05SBF – invalidating any solution returned.

When using an SMP parallel version of this routine, you must indicate that the callback routines are thread-safe by setting the optional argument **SMP Callback Thread Safe** before calling E05SBF in a multi-threaded environment. See Section 11.2 for more information on this and other SMP options.

Note: the stochastic method used in E05SBF will not produce repeatable answers when run on multiple threads.

2 Specification

```
SUBROUTINE E05SBF (NDIM, NCON, NPAR, XB, FB, CB, BL, BU, XBEST, FBEST,          &
                   CBEST, OBJFUN, CONFUN, MONMOD, IOPTS, OPTS, IUSER,           &
                   RUSER, ITT, INFORM, IFAIL)

INTEGER            NDIM, NCON, NPAR, IOPTS(*), IUSER(*), ITT(7), INFORM,        &
                   IFAIL
REAL (KIND=nag_wp) XB(NDIM), FB, CB(NCON), BL(NDIM+NCON), BU(NDIM+NCON),        &
                   XBEST(NDIM,NPAR), FBEST(NPAR), CBEST(NCON,NPAR),             &
                   OPTS(*), RUSER(*)
EXTERNAL           OBJFUN, CONFUN, MONMOD
```

Before calling E05SBF, E05ZKF **must** be called with OPTSTR set to '`Initialize = e05sbf`'. Optional parameters may also be specified by calling E05ZKF before the call to E05SBF.

3 Description

E05SBF uses a stochastic method based on Particle Swarm Optimization (PSO) to search for the global optimum of a nonlinear function F, subject to a set of bound constraints on the variables, and optionally a set of general nonlinear constraints. In the PSO algorithm (see Section 10), a set of particles is generated

in the search space, and advances each iteration to (hopefully) better positions using a heuristic velocity based upon *inertia*, *cognitive memory* and *global memory*. The inertia is provided by a decreasingly weighted contribution from a particles current velocity, the cognitive memory refers to the best candidate found by an individual particle and the global memory refers to the best candidate found by all the particles. This allows for a global search of the domain in question.

Further, this may be coupled with a selection of local minimization routines, which may be called during the iterations of the heuristic algorithm, the *interior* phase, to hasten the discovery of locally optimal points, and after the heuristic phase has completed to attempt to refine the final solution, the *exterior* phase. Different options may be set for the local optimizer in each phase.

Without loss of generality, the problem is assumed to be stated in the following form:

$$\underset{\mathbf{x}\in R^{ndim}}{\text{minimize}}\, F(\mathbf{x}) \qquad \text{subject to} \qquad \boldsymbol{\ell} \leq \begin{pmatrix} \mathbf{x} \\ \mathbf{c}(\mathbf{x}) \end{pmatrix} \leq \mathbf{u},$$

where the objective $F(\mathbf{x})$ is a scalar function, $\mathbf{c}(\mathbf{x})$ is a vector of scalar constraint functions, $\mathbf{x}$ is a vector in R^{ndim} and the vectors $\boldsymbol{\ell} \leq \mathbf{u}$ are lower and upper bounds respectively for the *ndim* variables and *ncon* constraints. Both the objective function and the *ncon* constraints may be nonlinear. Continuity of F, and the functions $\mathbf{c}(\mathbf{x})$, is not essential. For functions which are smooth and primarily unimodal, faster solutions will almost certainly be achieved by using Chapter E04 routines directly.

For functions which are smooth and multi-modal, gradient dependent local minimization routines may be coupled with E05SBF.

For multi-modal functions for which derivatives cannot be provided, particularly functions with a significant level of noise in their evaluation, E05SBF should be used either alone, or coupled with E04CBF.

For heavily constrained problems, E05SBF should either be used alone, or coupled with E04UCF/E04UCA provided the function and the constraints are sufficiently smooth.

The *ndim* lower and upper box bounds on the variable $\mathbf{x}$ are included to initialize the particle swarm into a finite hypervolume, although their subsequent influence on the algorithm is user determinable (see the option **Boundary** in Section 11). It is strongly recommended that sensible bounds are provided for all variables and constraints.

E05SBF may also be used to maximize the objective function, or to search for a feasible point satisfying the simple bounds and general constraints (see the option **Optimize**).

Due to the nature of global optimization, unless a predefined target is provided, there is no definitive way of knowing when to end a computation. As such several stopping heuristics have been implemented into the algorithm. If any of these is achieved, E05SBF will exit with IFAIL $= 1$, and the parameter INFORM will indicate which criteria was reached. See INFORM for more information.

In addition, you may provide your own stopping criteria through MONMOD, OBJFUN and CONFUN.

E05SAF provides a simpler interface, without the inclusion of general nonlinear constraints.

4 References

Gill P E, Murray W and Wright M H (1981) *Practical Optimization* Academic Press

Kennedy J and Eberhart R C (1995) Particle Swarm Optimization *Proceedings of the 1995 IEEE International Conference on Neural Networks* 1942–1948

Koh B, George A D, Haftka R T and Fregly B J (2006) Parallel Asynchronous Particle Swarm Optimization *International Journal for Numerical Methods in Engineering* **67(4)** 578–595

Vaz A I and Vicente L N (2007) A Particle Swarm Pattern Search Method for Bound Constrained Global Optimization *Journal of Global Optimization* **39(2)** 197–219 Kluwer Academic Publishers

5 Parameters

Note: for descriptions of the symbolic variables, see Section 10.

1: NDIM – INTEGER *Input*

On entry: *ndim*, the number of dimensions.

Constraint: NDIM ≥ 1.

2: NCON – INTEGER *Input*

On entry: *ncon*, the number of constraints, not including box constraints.

Constraint: NCON ≥ 0.

3: NPAR – INTEGER *Input*

On entry: *npar*, the number of particles to be used in the swarm. Assuming all particles remain within constraints, each complete iteration will perform at least NPAR function evaluations. Otherwise, significantly fewer objective function evaluations may be performed.

Suggested value: NPAR $= 10 \times$ NDIM.

Constraint: NPAR $\geq 5 \times$ **num_threads**, where **num_threads** is the value returned by the OpenMP environment variable `OMP_NUM_THREADS`, or **num_threads** is 1 for a serial version of this routine.

4: XB(NDIM) – REAL (KIND=nag_wp) array *Output*

On exit: the location of the best solution found, $\tilde{\mathbf{x}}$, in R^{ndim}.

5: FB – REAL (KIND=nag_wp) *Output*

On exit: the objective value of the best solution, $\tilde{f} = F(\tilde{\mathbf{x}})$.

6: CB(NCON) – REAL (KIND=nag_wp) array *Output*

On exit: the constraint violations of the best solution found, $\tilde{\mathbf{e}} = \mathbf{e}(\tilde{\mathbf{x}})$. These may have been deemed to be acceptable given the tolerance and scaling of the constraints. See Sections 10 and 11.

7: BL(NDIM + NCON) – REAL (KIND=nag_wp) array *Input*
8: BU(NDIM + NCON) – REAL (KIND=nag_wp) array *Input*

On entry: BL is $\boldsymbol{\ell}$, the array of lower bounds, BU is $\mathbf{u}$, the array of upper bounds. The first NDIM entries in BL and BU must contain the lower and upper simple (box) bounds of the variables respectively. These must be provided to initialize the sample population into a finite hypervolume, although their subsequent influence on the algorithm is user determinable (see the option **Boundary** in Section 11).

The next NCON entries must contain the lower and upper bounds for any general constraints respectively.

If BL(i) = BU(i) for any $i \in \{1, \ldots, \text{NDIM}\}$, variable i will remain locked to BL(i) regardless of the **Boundary** option selected.

It is strongly advised that you place sensible lower and upper bounds on all variables and constraints, even if your model allows for unbounded variables or constraints.

Constraints:

BL$(i) \leq$ BU(i), for $i = 1, 2, \ldots,$ NDIM + NCON;
BL$(i) \neq$ BU(i) for at least one $i \in \{1, \ldots, \text{NDIM}\}$.

9: XBEST(NDIM,NPAR) – REAL (KIND=nag_wp) array *Input/Output*

Note: the ith component of the best position of the jth particle, $\hat{x}_j(i)$, is stored in XBEST(i, j).

On entry: if using **Start** = WARM, the initial particle positions, $\hat{\mathbf{x}}_j^0$.

On exit: the best positions found, $\hat{\mathbf{x}}_j$, by the NPAR particles in the swarm.

10: FBEST(NPAR) – REAL (KIND=nag_wp) array *Input/Output*

On entry: if using **Start** = WARM, objective function values, $\hat{f}_j^0 = F\left(\hat{\mathbf{x}}_j^0\right)$, corresponding to the NPAR particle locations stored in XBEST.

On exit: objective function values, $\hat{f}_j = F\left(\hat{\mathbf{x}}_j\right)$, corresponding to the locations returned in XBEST.

11: CBEST(NCON,NPAR) – REAL (KIND=nag_wp) array *Input/Output*

Note: the kth constraint violation of the jth particle is stored in CBEST(k, j).

On entry: if using **Start** = WARM, the initial constraint violations, $\hat{\mathbf{e}}_j^0 = \mathbf{e}\left(\hat{\mathbf{x}}_j^0\right)$, corresponding to the NPAR particle locations.

On exit: the final constraint violations, $\hat{\mathbf{e}}_j$, corresponding to the locations returned in XBEST.

12: OBJFUN – SUBROUTINE, supplied by the user. *External Procedure*

OBJFUN must, depending on the value of MODE, calculate the objective function *and/or* calculate the gradient of the objective function for a *ndim*-variable vector **x**. Gradients are only required if a local minimizer has been chosen which requires gradients. See the option **Local Minimizer** for more information.

The specification of OBJFUN is:

```
SUBROUTINE OBJFUN (MODE, NDIM, X, OBJF, VECOUT, NSTATE, IUSER,          &
                   RUSER)

INTEGER            MODE, NDIM, NSTATE, IUSER(*)
REAL (KIND=nag_wp) X(NDIM), OBJF, VECOUT(NDIM), RUSER(*)
```

1: MODE – INTEGER *Input/Output*

On entry: indicates which functionality is required.

MODE = 0

$F(\mathbf{x})$ should be returned in OBJF. The value of OBJF on entry may be used as an upper bound for the calculation. Any expected value of $F(\mathbf{x})$ that is greater than OBJF may be approximated by this upper bound; that is OBJF can remain unaltered.

MODE = 1

Local Minimizer = 'E04UCF' only
First derivatives can be evaluated and returned in VECOUT. Any unaltered elements of VECOUT will be approximated using finite differences.

MODE = 2

Local Minimizer = 'E04UCF' only
$F(\mathbf{x})$ *must* be calculated and returned in OBJF, and available first derivatives can be evaluated and returned in VECOUT. Any unaltered elements of VECOUT will be approximated using finite differences.

MODE = 5

$F(\mathbf{x})$ *must* be calculated and returned in OBJF. The value of OBJF on entry may not be used as an upper bound.

MODE = 6

Local Minimizer = 'E04DGF' or 'E04KZF' only
All first derivatives *must* be evaluated and returned in VECOUT.

MODE = 7

Local Minimizer = 'E04DGF' or 'E04KZF' only
$F(\mathbf{x})$ *must* be calculated and returned in OBJF, and *all* first derivatives *must* be evaluated and returned in VECOUT.

On exit: if the value of MODE is set to be negative, then E05SBF will exit as soon as possible with IFAIL = 3 and INFORM = MODE.

2: NDIM – INTEGER *Input*

On entry: the number of dimensions.

3: X(NDIM) – REAL (KIND=nag_wp) array *Input*

On entry: **x**, the point at which the objective function and/or its gradient are to be evaluated.

4: OBJF – REAL (KIND=nag_wp) *Input/Output*

On entry: the value of OBJF passed to OBJFUN varies with the parameter MODE.

MODE = 0

OBJF is an upper bound for the value of $F(\mathbf{x})$, often equal to the best constraint penalised value of $F(\mathbf{x})$ found so far by a given particle if the objective function is strictly positive (see Section 10). Only objective function values less than the value of OBJF on entry will be used further. As such this upper bound may be used to stop further evaluation when this will only increase the objective function value above the upper bound.

MODE = 1, 2, 5, 6 or 7

OBJF is meaningless on entry.

On exit: the value of OBJF returned varies with the parameter MODE.

MODE = 0

OBJF must be the value of $F(\mathbf{x})$. Only values of $F(\mathbf{x})$ strictly less than OBJF on entry need be accurate.

MODE = 1 or 6

Need not be set.

MODE = 2, 5 or 7

$F(\mathbf{x})$ must be calculated and returned in OBJF. The entry value of OBJF may not be used as an upper bound.

5: VECOUT(NDIM) – REAL (KIND=nag_wp) array *Input/Output*

On entry: if **Local Minimizer** = E04UCF or E04UCA, the values of VECOUT are used internally to indicate whether a finite difference approximation is required. See E04UCF/E04UCA.

On exit: the required values of VECOUT returned to the calling routine depend on the value of MODE.

MODE = 0 or 5

The value of VECOUT need not be set.

MODE = 1 or 2

VECOUT can contain components of the gradient of the objective function $\frac{\partial F}{\partial x_i}$ for some $i = 1, 2, \ldots$ NDIM, or acceptable approximations. Any unaltered elements of VECOUT will be approximated using finite differences.

MODE $= 6$ or 7

VECOUT must contain the gradient of the objective function $\frac{\partial F}{\partial x_i}$ for all $i = 1, 2, \ldots$ NDIM. Approximation of the gradient is strongly discouraged, and no finite difference approximations will be performed internally (see E04DGF/E04DGA and E04KZF).

6: NSTATE – INTEGER *Input*

On entry: NSTATE indicates various stages of initialization throughout the routine. This allows for permanent global parameters to be initialized the least number of times. For example, you may initialize a random number generator seed.

NSTATE $= 3$

SMP users only. OBJFUN is called for the first time in a parallel region on a new thread other than the master thread. You may use this opportunity to set up any thread-dependent information in IUSER and RUSER.

NSTATE $= 2$

OBJFUN is called for the very first time. You may save computational time if certain data must be read or calculated only once.

NSTATE $= 1$

OBJFUN is called for the first time by a NAG local minimization routine. You may save computational time if certain data required for the local minimizer need only be calculated at the initial point of the local minimization.

NSTATE $= 0$

Used in all other cases.

7: IUSER$(*)$ – INTEGER array *User Workspace*
8: RUSER$(*)$ – REAL (KIND=nag_wp) array *User Workspace*

OBJFUN is called with the parameters IUSER and RUSER as supplied to E05SBF. You are free to use the arrays IUSER and RUSER to supply information to OBJFUN as an alternative to using COMMON global variables.

OBJFUN must either be a module subprogram USEd by, or declared as EXTERNAL in, the (sub)program from which E05SBF is called. Parameters denoted as *Input* must **not** be changed by this procedure.

13: CONFUN – SUBROUTINE, supplied by the NAG Library or the user. *External Procedure*

CONFUN must calculate any constraints other than the box constraints. If no constraints are required, CONFUN may be the dummy constraint routine E05SZM. (E05SZM is included in the NAG Library). For information on how a NAG local minimizer will use CONFUN see the documentation for E04UCA.

The specification of CONFUN is:

```
SUBROUTINE CONFUN (MODE, NCON, NDIM, LDCJ, NEEDC, X, C, CJAC,           &
                   NSTATE, IUSER, RUSER)

INTEGER            MODE, NCON, NDIM, LDCJ, NEEDC(NCON), NSTATE,         &
                   IUSER(*)
REAL (KIND=nag_wp) X(NDIM), C(NCON), CJAC(LDCJ,NDIM), RUSER(*)
```

1: MODE – INTEGER *Input/Output*

On entry: indicates which values must be assigned during each call of CONFUN. Only the following values need be assigned, for each value of $k \in \{1, \ldots, \text{NCON}\}$ such that NEEDC$(k) > 0$:

MODE $= 0$

the constraint values $c_k(\mathbf{x})$.

MODE $= 1$

rows of the constraint jacobian, $\frac{\partial c_k}{\partial x_i}(\mathbf{x})$, for $i = 1, 2, \ldots,$ NDIM.

MODE $= 2$

the constraint values $c_k(\mathbf{x})$ and the corresponding rows of the constraint jacobian, $\frac{\partial c_k}{\partial x_i}(\mathbf{x})$, for $i = 1, 2, \ldots,$ NDIM.

On exit: may be set to a negative value if you wish to terminate the solution to the current problem. In this case E05SBF will terminate with IFAIL $= 3$ and INFORM $=$ MODE as soon as possible.

2: NCON – INTEGER *Input*

On entry: the number of constraints, not including box bounds.

3: NDIM – INTEGER *Input*

On entry: the number of variables.

4: LDCJ – INTEGER *Input*

On entry: the first dimension of the array CJAC as declared in the (sub)program from which E05SBF is called.

5: NEEDC(NCON) – INTEGER array *Input*

On entry: the indices of the elements of C and/or CJAC that must be evaluated by CONFUN. If NEEDC$(k) > 0$, the kth element of C, corresponding to the values of the kth constraint, and/or the available elements of the kth row of CJAC, corresponding to the derivatives of the kth constraint, must be evaluated at **x** (see parameter MODE).

6: X(NDIM) – REAL (KIND=nag_wp) array *Input*

On entry: **x**, the vector of variables at which the constraint functions and/or the available elements of the constraint Jacobian are to be evaluated.

7: C(NCON) – REAL (KIND=nag_wp) array *Output*

On exit: if NEEDC$(k) > 0$ and MODE $= 0$ or 2, C(k) must contain the value of $c_k(\mathbf{x})$. The remaining elements of C, corresponding to the non-positive elements of NEEDC, need not be set.

8: CJAC(LDCJ,NDIM) – REAL (KIND=nag_wp) array *Input/Output*

Note: the derivative of the kth constraint with respect to the ith component, $\dfrac{\partial c_k}{\partial x_\mathrm{i}}$, is stored in CJAC$(k, i)$.

On entry: the elements of CJAC are set to special values which enable E05SBF to detect whether they are changed by CONFUN.

On exit: if NEEDC$(k) > 0$ and MODE $= 1$ or 2, the elements of CJAC corresponding to the kth row of the constraint jacobian should contain the available elements of the vector ∇c_k given by

$$\nabla c_k = \left(\frac{\partial c_k}{\partial x_1}, \frac{\partial c_k}{\partial x_2}, \ldots, \frac{\partial c_k}{\partial x_n}\right),$$

where $\dfrac{\partial c_k}{\partial x_i}$ is the partial derivative of the kth constraint with respect to the ith variable, evaluated at the point **x**; elements of CJAC that remain unaltered will be approximated

internally using finite differences. The remaining rows of CJAC, corresponding to non-positive elements of NEEDC, need not be set.

It must be emphasized that unassigned elements of CJAC are not treated as constant; they are estimated by finite differences, at nontrivial expense. An interval for each element of **x** is computed automatically at the start of the optimization. The automatic procedure can usually identify constant elements of CJAC, which are then computed once only by finite differences.

9: NSTATE – INTEGER *Input*

On entry: NSTATE indicates various stages of initialization throughout the routine. This allows for permanent global parameters to be initialized a minimum number of times. For example, you may initialize a random number generator seed. Note that unless the option **Optimize** = CONSTRAINTS has been set, OBJFUN will be called before CONFUN.

NSTATE = 3

SMP users only. OBJFUN is called for the first time in a parallel region on a new thread other than the master thread. You may use this opportunity to set up any thread-dependent information in IUSER and RUSER.

NSTATE = 2

CONFUN is called for the very first time. This parameter setting allows you to save computational time if certain data must be read or calculated only once.

NSTATE = 1

CONFUN is called for the first time during a NAG local minimization routine. This parameter setting allows you to save computational time if certain data required for the local minimizer need only be calculated at the initial point of the local minimization.

NSTATE = 0

Used in all other cases.

10: IUSER(∗) – INTEGER array *User Workspace*
11: RUSER(∗) – REAL (KIND=nag_wp) array *User Workspace*

CONFUN is called with the parameters IUSER and RUSER as supplied to E05SBF. You are free to use the arrays IUSER and RUSER to supply information to CONFUN as an alternative to using COMMON global variables.

CONFUN must either be a module subprogram USEd by, or declared as EXTERNAL in, the (sub)program from which E05SBF is called. Parameters denoted as *Input* must **not** be changed by this procedure.

CONFUN should be tested separately before being used in conjunction with E05SBF.

14: MONMOD – SUBROUTINE, supplied by the NAG Library or the user. *External Procedure*

A user-specified monitoring and modification function. MONMOD is called once every complete iteration after a finalization check. It may be used to modify the particle locations that will be evaluated at the next iteration. This permits the incorporation of algorithmic modifications such as including additional advection heuristics and genetic mutations. MONMOD is only called during the main loop of the algorithm, and as such will be unaware of any further improvement from the final local minimization. If no monitoring and/or modification is required, MONMOD may be the dummy monitoring routine E05SYM. (E05SYM is included in the NAG Library) .

The specification of MONMOD is:

```
SUBROUTINE MONMOD (NDIM, NCON, NPAR, X, XB, FB, CB, XBEST, FBEST,        &
                   CBEST, ITT, IUSER, RUSER, INFORM)

INTEGER            NDIM, NCON, NPAR, ITT(7), IUSER(*), INFORM
REAL (KIND=nag_wp) X(NDIM,NPAR), XB(NDIM), FB, CB(NCON),                 &
                   XBEST(NDIM,NPAR), FBEST(NPAR), CBEST(NCON,NPAR),      &
                   RUSER(*)
```

1: NDIM – INTEGER *Input*

On entry: the number of dimensions.

2: NCON – INTEGER *Input*

On entry: the number of constraints.

3: NPAR – INTEGER *Input*

On entry: the number of particles.

4: X(NDIM,NPAR) – REAL (KIND=nag_wp) array *Input/Output*

Note: the ith component of the jth particle, $x_j(i)$, is stored in $\mathrm{X}(i,j)$.

On entry: the NPAR particle locations, $\mathbf{x}_j$, which will currently be used during the next iteration unless altered in MONMOD.

On exit: the particle locations to be used during the next iteration.

5: XB(NDIM) – REAL (KIND=nag_wp) array *Input*

On entry: the location, $\tilde{\mathbf{x}}$, of the best solution yet found.

6: FB – REAL (KIND=nag_wp) *Input*

On entry: the objective value, $\tilde{f} = F(\tilde{\mathbf{x}})$, of the best solution yet found.

7: CB(NCON) – REAL (KIND=nag_wp) array *Input*

On entry: the constraint violations, $\tilde{\mathbf{e}} = \mathbf{e}(\tilde{\mathbf{x}})$, of the best solution yet found.

8: XBEST(NDIM,NPAR) – REAL (KIND=nag_wp) array *Input*

Note: the ith component of the position of the jth particle's cognitive memory, $\hat{x}_j(i)$, is stored in $\mathrm{XBEST}(i,j)$.

On entry: the locations currently in the cognitive memory, $\hat{\mathbf{x}}_j$, for $j = 1, 2, \ldots,$ NPAR (see Section 10).

9: FBEST(NPAR) – REAL (KIND=nag_wp) array *Input*

On entry: the objective values currently in the cognitive memory, $F\left(\hat{\mathbf{x}}_j\right)$, for $j = 1, 2, \ldots,$ NPAR.

10: CBEST(NCON,NPAR) – REAL (KIND=nag_wp) array *Input*

Note: the kth constraint violation of the jth particle's cognitive memory is stored in $\mathrm{CBEST}(k,j)$.

On entry: the constraint violations currently in the cognitive memory, $\hat{\mathbf{e}} = \mathbf{e}\left(\hat{\mathbf{x}}_j\right)$, for $j = 1, 2, \ldots,$ NPAR, evaluated at $\hat{\mathbf{x}}_j$.

11: ITT(7) – INTEGER array *Input*

On entry: iteration and function evaluation counters (see description of ITT below).

12: IUSER(∗) – INTEGER array *User Workspace*
13: RUSER(∗) – REAL (KIND=nag_wp) array *User Workspace*

MONMOD is called with the parameters IUSER and RUSER as supplied to E05SBF. You are free to use the arrays IUSER and RUSER to supply information to MONMOD as an alternative to using COMMON global variables.

14: INFORM – INTEGER *Input/Output*

On entry: INFORM = **thread_num**, where **thread_num** is the value returned by a call of the OpenMP function `OMP_GET_THREAD_NUM()`. If running in serial this will always be zero.

On exit: setting INFORM < 0 will cause near immediate exit from E05SBF. This value will be returned as INFORM with IFAIL $= 3$. You need not set INFORM unless you wish to force an exit.

MONMOD must either be a module subprogram USEd by, or declared as EXTERNAL in, the (sub)program from which E05SBF is called. Parameters denoted as *Input* must **not** be changed by this procedure.

15: IOPTS(∗) – INTEGER array *Communication Array*

Note: the contents of IOPTS **must not** have been altered between calls to E05ZKF, E05ZLF, E05SBF and the selected problem solving routine.

On entry: optional parameter array as generated and possibly modified by calls to E05ZKF. The contents of IOPTS **must not** be modified directly between calls to E05SBF, E05ZKF or E05ZLF.

16: OPTS(∗) – REAL (KIND=nag_wp) array *Communication Array*

Note: the contents of OPTS **must not** have been altered between calls to E05ZKF, E05ZLF, E05SBF and the selected problem solving routine.

On entry: optional parameter array as generated and possibly modified by calls to E05ZKF. The contents of OPTS **must not** be modified directly between calls to E05SBF, E05ZKF or E05ZLF.

17: IUSER(∗) – INTEGER array *User Workspace*

IUSER is not used by E05SBF, but is passed directly to OBJFUN, CONFUN and MONMOD and may be used to pass information to these routines as an alternative to using COMMON global variables.

With care, you may also write information back into IUSER. This might be useful, for example, should there be a need to preserve the state of a random number generator.

With SMP-enabled versions of E05SBF the array IUSER provided are classified as OpenMP shared memory. Use of IUSER has to take account of this in order to preserve thread safety whenever information is written back to either of these arrays.

18: RUSER(∗) – REAL (KIND=nag_wp) array *User Workspace*

RUSER is not used by E05SBF, but is passed directly to OBJFUN, CONFUN and MONMOD and may be used to pass information to these routines as an alternative to using COMMON global variables.

With care, you may also write information back into RUSER. This might be useful, for example, should there be a need to preserve the state of a random number generator.

With SMP-enabled versions of E05SBF the array RUSER provided are classified as OpenMP shared memory. Use of RUSER has to take account of this in order to preserve thread safety whenever information is written back to either of these arrays.

19: ITT(7) – INTEGER array *Output*

On exit: integer iteration counters for E05SBF.

ITT(1)
Number of complete iterations.

ITT(2)
Number of complete iterations without improvement to the current optimum.

ITT(3)
Number of particles converged to the current optimum.

ITT(4)
Number of improvements to the optimum.

ITT(5)
Number of function evaluations performed.

ITT(6)
Number of particles reset.

ITT(7)
Number of violated constraints at completion. Note this is always calculated using the L^1 norm and a nonzero result does not necessarily mean that the algorithm did not find a suitably constrained point with respect to the single norm used.

20: INFORM – INTEGER *Output*

On exit: indicates which finalization criterion was reached. The possible values of INFORM are:

INFORM	**Meaning**
< 0	Exit from a user-supplied subroutine.
0	E05SBF has detected an error and terminated.
1	The provided objective target has been achieved. (**Target Objective Value**).
2	The standard deviation of the location of all the particles is below the set threshold (**Swarm Standard Deviation**). If the solution returned is not satisfactory, you may try setting a smaller value of **Swarm Standard Deviation**, or try adjusting the options governing the repulsive phase (**Repulsion Initialize**, **Repulsion Finalize**).
3	The total number of particles converged (**Maximum Particles Converged**) to the current global optimum has reached the set limit. This is the number of particles which have moved to a distance less than **Distance Tolerance** from the optimum with regard to the L^2 norm. If the solution is not satisfactory, you may consider lowering the **Distance Tolerance**. However, this may hinder the global search capability of the algorithm.
4	The maximum number of iterations without improvement (**Maximum Iterations Static**) has been reached, and the required number of particles (**Maximum Iterations Static Particles**) have converged to the current optimum. Increasing either of these options will allow the algorithm to continue searching for longer. Alternatively if the solution is not satisfactory, re-starting the application several times with **Repeatability** = OFF may lead to an improved solution.
5	The maximum number of iterations (**Maximum Iterations Completed**) has been reached. If the number of iterations since improvement is small, then a better solution may be found by increasing this limit, or by using the option **Local Minimizer** with corresponding exterior options. Otherwise if the solution is not satisfactory, you may try re-running the application several times with **Repeatability** = OFF and a lower iteration limit, or adjusting the options governing the repulsive phase (**Repulsion Initialize**, **Repulsion Finalize**).

6 The maximum allowed number of function evaluations (**Maximum Function Evaluations**) has been reached. As with INFORM = 5, increasing this limit if the number of iterations without improvement is small, or decreasing this limit and running the algorithm multiple times with **Repeatability** = OFF, may provide a superior result.

7 A feasible point has been found. The objective has not been minimized, although it has been evaluated at the final solutions given in XB and XBEST (**Optimize** = CONSTRAINTS).

If you wish to continue from the final position gained from a previous simulation with adjusted options, you may set the option **Start** = WARM, and pass back in the returned arrays XBEST, FBEST, and CBEST. You should either record the returned values of XB, FB and CB for comparison, as these will not be re-used by the algorithm, or include them in XBEST, FBEST and CBEST respectively by overwriting the entries corresponding to one particle with the relevant information.

21: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: the most common exit will be IFAIL = 1.

For this reason, the value −1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended; otherwise, the recommended value is −1. **When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.**

E05SBF returns IFAIL = 0 if and only if a finalization criterion has been reached which can guarantee success. This may only happen if:

(i) The option **Target Objective Value** has been set and has been reached at a sufficiently constrained point within the search domain.

(ii) The option **Optimize** = CONSTRAINTS has been set, and a sufficiently constrained point has been found within the search domain.

These finalization criteria are not active using default option settings, and must be explicitly set using E05ZKF if required.

E05SBF will return IFAIL = 1 if no error has been detected, and a finalization criterion has been achieved which cannot guarantee success. This does not indicate that the routine has failed, merely that the returned solution cannot be guaranteed to be the true global optimum.

The value of INFORM should be examined to determine which finalization criterion was reached.

Other positive values of IFAIL indicate that either an error or a warning has been triggered. See Sections 6, 7 and 10 for more information.

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Note: E05SBF may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the routine:

IFAIL = 1

The algorithm has reached a finalization criterion that does not guarantee success. You should investigate the returned value of INFORM for more information on why this occurred.

IFAIL $= 2$

If the option **Target Warning** has been activated, this indicates that the **Target Objective Value** has been achieved to specified tolerances at a sufficiently constrained point, either during the initialization phase, or during the first two iterations of the algorithm. While this is not necessarily an error, it may occur if:

(i) The target was achieved at the first point sampled by the routine. This will be the mean of the lower and upper bounds.

(ii) The target may have been achieved at a randomly generated sample point. This will always be a possibility provided that the domain under investigation contains a point with a target objective value which is sufficiently constrained.

(iii) If the **Local Minimizer** has been set, then a sample point may have been inside the basin of attraction of a satisfactory point. If this occurs repeatedly when the routine is called, it may imply that the objective is largely unimodal, and that it may be more efficient to use the routine selected as the **Local Minimizer** directly.

(iv) If **Start** = WARM has been set, a particle supplied in XBEST with corresponding cognitive memory supplied in FBEST and CBEST satisfies the required target. If IFAIL $= -1$ on entry, the array index corresponding to the first particle to satisfy the target will be returned in the error message. You should verify that the provided values are correct, and if required set a lower **Target Objective Value**, or lower **Constraint Tolerance**.

Assuming that OBJFUN and CONFUN are correct, you may wish to set a better **Target Objective Value**, a stricter **Constraint Tolerance**, or a stricter **Target Objective Tolerance**.

IFAIL $= 3$

You requested an exit from either OBJFUN, CONFUN or MONMOD. The exit flag you provided will be returned in INFORM.

IFAIL $= 4$

After the algorithm had completed, the best point found did not satisfy the general constraints to the requested scaled **Constraint Tolerance**. This exit may be suppressed using the option **Constraint Warning**.

IFAIL $= 11$

On entry, NDIM < 1.

IFAIL $= 12$

On entry, NPAR $< 5 \times$ **num_threads**, where **num_threads** is the value returned by the OpenMP environment variable `OMP_NUM_THREADS`, or **num_threads** is 1 for a serial version of this routine.

IFAIL $= 13$

On entry, NCON < 0.

IFAIL $= 14$

On entry, at least one lower bound BL$(i) >$ BU(i), for $i = 1, 2, \ldots,$ NDIM $+$ NCON, or all of the bounds BL$(i) =$ BU(i), for $i = 1, 2, \ldots,$ NDIM.

IFAIL $= 17$

On entry, NCON > 0 and the subroutine E05SZM has been passed to E05SBF as CONFUN.

IFAIL $= 18$

On entry, NCON $= 0$ and the option **Optimize** = CONSTRAINTS has been set.

IFAIL = 21

On entry, either the option arrays IOPTS and OPTS have not been initialized, or they have been corrupted. Re-initialize the arrays using E05ZKF.

Alternatively, both **Advance Cognitive** and **Advance Global** may have been set to 0.0.

IFAIL = 32

The gradient check has indicated an error may be present in the objective gradient, or the gradients of the constraints. These checks are not infallible. If you are sure your gradients are correct then the gradient checks may be disabled by setting **Verify Gradients** = OFF.

IFAIL = 51

(SMP parallel version only). Multiple threads have been detected, however you have not set the option **SMP Callback Thread Safe** to declare that the provided callback routines are thread-safe, or that you want the algorithm to run in serial. See Section 11.2 for more information.

7 Accuracy

If IFAIL = 0 (or IFAIL = 2) or IFAIL = 1 on exit, a criterion will have been reached depending on user selected options. As with all global optimization software, the solution achieved may not be the true global optimum. Various options allow for either greater search diversity or faster convergence to a (local) optimum (See Sections 10 and 11).

Provided the objective function and constraints are sufficiently well behaved, if a local minimizer is used in conjunction with E05SBF, then it is more likely that the final result will at least be in the near vicinity of a local optimum, and due to the global search characteristics of the particle swarm, this solution should be superior to many other local optima.

Caution should be used in accelerating the rate of convergence, as with faster convergence, less of the domain will remain searchable by the swarm, making it increasingly difficult for the algorithm to detect the basins of attraction of superior local optima. Using the options **Repulsion Initialize** and **Repulsion Finalize** described in Section 11 will help to overcome this, by causing the swarm to diverge away from the current optimum once no more local improvement is likely.

On successful exit with guaranteed success, IFAIL = 0 (or IFAIL = 2). This may happen if a **Target Objective Value** is assigned and is reached by the algorithm at a satisfactorily constrained point. It will also occur if a constrained point is found when **Optimize** = CONSTRAINTS is set.

On successful exit without guaranteed success, IFAIL = 1 is returned. This will happen if another finalization criterion is achieved without the detection of an error.

In both cases, the value of INFORM provides further information as to the cause of the exit.

8 Further Comments

The memory used by E05SBF is relatively static throughout. Indeed, most of the memory required is used to store the current particle locations, the cognitive particle memories, the particle velocities and the particle weights. As such, E05SBF may be used in problems with high dimension number (NDIM $>$ 100) without the concern of computational resource exhaustion, although the probability of successfully locating the global optimum will decrease dramatically with the increase in dimensionality.

Due to the stochastic nature of the algorithm, the result will vary over multiple runs. This is particularly true if parameters and options are chosen to accelerate convergence at the expense of the global search. However, the option **Repeatability** = ON may be set to initialize the internal random number generator using a preset seed, which will result in identical solutions being obtained.

(For SMP users only) The option **Repeatability** = ON will use preset seeds to initialize the random number generator on each thread, however due to the unpredictable nature of parallel communication, this cannot ensure repeatable results when running on multiple threads, even with **SMP Thread Overrun** set to force synchronization every iteration.

9 Example

This example uses a particle swarm to find the global minimum of the 2 dimensional Schwefel function:

$$\underset{\mathbf{x}\in R^2}{\text{minimize}} f = \sum_{j=1}^{2} x_j \sin\left(\sqrt{|x_j|}\right)$$

subject to the constraints:

$$\begin{gathered} 3.0x_1 - 2.0x_2 < 10.0, \\ -1.0 < x_1^2 - x_2^2 + 3.0x_1x_2 < 50000.0, \\ -0.9 < \cos\left((x_1/200)^2 + (x_2/100)\right) < 0.9, \\ -500 \leq x_1 \leq 500, \\ -500 \leq x_2 \leq 500. \end{gathered}$$

The global optimum has an objective value of $f_{\min} = -731.707$, located at $\mathbf{x} = (-394.15, -433.48)$. Only the third constraint is active at this point.

The example demonstrates how to initialize and set the options arrays using E05ZKF, how to query options using E05ZLF, and finally how to search for the global optimum using E05SBF. The problem is solved twice, first using E05SBF alone, and secondly by coupling E05SBF with E04UCF/E04UCA as a dedicated local minimizer. In both cases the default option **Repeatability** = ON is used to produce repeatable solutions.

Note: for users of the NAG Library for SMP & Multicore the following example program does not include the setting of the optional parameter **SMP Callback Thread Safe**, and as such if run on multiple threads it will issue an error message.

9.1 Program Text

```
!   E05SBF Example Program Text
!   Mark 24 Release. NAG Copyright 2012.

    Module e05sbfe_mod

!     E05SBF Example Program Module:
!            Parameters and User-defined Routines

!     .. Use Statements ..
      Use nag_library, Only: nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Real (Kind=nag_wp), Parameter        ::                                &
                                     f_target_c = -731.70709230672696_nag_wp
      Real (Kind=nag_wp), Parameter        ::                                &
                                      f_target_u = -837.9657745448674_nag_wp
      Real (Kind=nag_wp), Parameter        ::                                &
                                        x_target = -420.9687463599820_nag_wp
      Real (Kind=nag_wp), Parameter        :: zero = 0.0_nag_wp
      Integer, Parameter                   :: detail_level = 0, liopts = 100,  &
                                              liuser = 1, lopts = 100,         &
                                              lruser = 1, ncon = 3, ndim = 2,  &
                                              nout = 6, npar = 20,             &
                                              report_freq = 100
      Real (Kind=nag_wp), Parameter        :: c_scale(ncon) = (/2490.0_nag_wp, &
                                              750000.0_nag_wp,0.1_nag_wp/)
      Real (Kind=nag_wp), Parameter        ::                                &
                                     c_target_c(ncon) = (/zero,zero,zero/)
      Real (Kind=nag_wp), Parameter        :: c_target_u(ncon) = (/zero,       &
                           31644.05623568455_nag_wp,0.07574889943398055_nag_wp/)
      Real (Kind=nag_wp), Parameter        ::                                &
    x_target_c(ndim) = (/-394.1470221120988_nag_wp,-433.48214189947606_nag_wp/)
      Real (Kind=nag_wp), Parameter        ::                                &
                                   x_target_u(ndim) = (/x_target,x_target/)
    Contains
```

```
      Subroutine objfun_schwefel(mode,ndim,x,objf,vecout,nstate,iuser,ruser)
!       Objfun routine for the Schwefel function for E05SBF.

!       .. Scalar Arguments ..
        Real (Kind=nag_wp), Intent (Inout)   :: objf
        Integer, Intent (Inout)              :: mode
        Integer, Intent (In)                 :: ndim, nstate
!       .. Array Arguments ..
        Real (Kind=nag_wp), Intent (Inout)   :: ruser(*), vecout(ndim)
        Real (Kind=nag_wp), Intent (In)      :: x(ndim)
        Integer, Intent (Inout)              :: iuser(*)
!       .. Local Scalars ..
        Logical                              :: evalobjf, evalobjg
!       .. Intrinsic Procedures ..
        Intrinsic                            :: abs, cos, sin, sqrt, sum
!       .. Executable Statements ..
!       Test NSTATE to indicate what stage of computation has been reached.
        Select Case (nstate)
        Case (2)
!       OBJFUN is called for the very first time.
        Case (1)
!       OBJFUN is called on entry to a NAG local minimizer.
        Case (0)
!       This will be the normal value of NSTATE.
        End Select

!       Test MODE to determine whether to calculate OBJF and/or OBJGRD.
        evalobjf = .False.
        evalobjg = .False.
        Select Case (mode)
        Case (0,5)
!         Only the value of the objective function is needed.
          evalobjf = .True.
        Case (1,6)
!         Only the values of the NDIM gradients are required.
          evalobjg = .True.
        Case (2,7)
!         Both the objective function and the NDIM gradients are required.
          evalobjf = .True.
          evalobjg = .True.
        End Select

        If (evalobjf) Then
!         Evaluate the objective function.
          objf = sum(x(1:ndim)*sin(sqrt(abs(x(1:ndim)))))
        End If

        If (evalobjg) Then
!         Calculate the gradient of the objective function.
          vecout = sqrt(abs(x))
          vecout = sin(vecout) + 0.5E0_nag_wp*vecout*cos(vecout)
        End If

        Return

      End Subroutine objfun_schwefel
      Subroutine confun_non_linear(mode,ncon,ndim,ldcj,needc,x,c,cjac,nstate, &
        iuser,ruser)
!       Subroutine used to supply constraints

!       .. Scalar Arguments ..
        Integer, Intent (In)                 :: ldcj, ncon, ndim, nstate
        Integer, Intent (Inout)              :: mode
!       .. Array Arguments ..
        Real (Kind=nag_wp), Intent (Out)     :: c(ncon)
        Real (Kind=nag_wp), Intent (Inout)   :: cjac(ldcj,ndim), ruser(*)
        Real (Kind=nag_wp), Intent (In)      :: x(ndim)
        Integer, Intent (Inout)              :: iuser(*)
        Integer, Intent (In)                 :: needc(ncon)
!       .. Local Scalars ..
        Integer                              :: k
```

```
        Logical                               :: evalc, evalcjac
!       .. Intrinsic Procedures ..
        Intrinsic                             :: cos
!       .. Executable Statements ..
!       Test NSTATE to determine whether the local minimizer is being called
!       for the first time from a new start point
        If (nstate==1) Then
!         Set any constant elements of the Jacobian matrix.
          cjac(1,1) = 3.0_nag_wp
          cjac(1,2) = -2.0_nag_wp
        End If

!       MODE: are constraints, derivatives, or both are required?
        evalc = mode == 0 .Or. mode == 2
        evalcjac = mode == 1 .Or. mode == 2



loop_constraints: Do k = 1, ncon
!         Only those for which needc is non-zero need be set.
          If (needc(k)<=0) Then
            Cycle loop_constraints
          End If

          If (evalc) Then
!           Constraint values are required.
            Select Case (k)
            Case (1)
              c(k) = 3.0_nag_wp*x(1) - 2.0_nag_wp*x(2)
            Case (2)
              c(k) = x(1)**2 - x(2)**2 + 3.0_nag_wp*x(1)*x(2)
            Case (3)
              c(k) = cos((x(1)/200.0_nag_wp)**2+(x(2)/100.0_nag_wp))
            Case Default
              c(k) = zero
            End Select
          End If

          If (evalcjac) Then
!           Constraint derivatives (CJAC) are required.
            Select Case (k)
            Case (1)
!             Constant derivatives set when NSTATE=1 remain throughout
!             the local minimization.
              Continue
            Case (2)
!             If the constraint derivatives are known and are readily
!             calculated, populate CJAC when required.
              cjac(k,1) = 2.0_nag_wp*x(1) + 3.0_nag_wp*x(2)
              cjac(k,2) = -2.0_nag_wp*x(2) + 3.0_nag_wp*x(1)
            Case Default
!             Any elements of CJAC left unaltered will be approximated
!             using finite differences when required.
              Continue
            End Select
          End If

        End Do loop_constraints

!       If an immediate exit is required, return MODE<0
        mode = 0
        Return

      End Subroutine confun_non_linear
      Subroutine monmod(ndim,ncon,npar,x,xb,fb,cb,xbest,fbest,cbest,itt,iuser, &
        ruser,inform)

!       .. Scalar Arguments ..
        Real (Kind=nag_wp), Intent (In)      :: fb
        Integer, Intent (Inout)              :: inform
        Integer, Intent (In)                 :: ncon, ndim, npar
```

```
!       .. Array Arguments ..
        Real (Kind=nag_wp), Intent (In)      :: cb(ncon), cbest(ncon,npar),    &
                                                fbest(npar), xb(ndim),         &
                                                xbest(ndim,npar)
        Real (Kind=nag_wp), Intent (Inout)   :: ruser(*), x(ndim,npar)
        Integer, Intent (In)                 :: itt(7)
        Integer, Intent (Inout)              :: iuser(*)
!       .. Local Scalars ..
        Integer                              :: k
!       .. Intrinsic Procedures ..
        Intrinsic                            :: modulo
!       .. Executable Statements ..
        If (detail_level>=2) Then
!         Report on the first iteration, and every report_freq iterations.
          If (itt(1)==1 .Or. modulo(itt(1),report_freq)==0) Then
            Write (nout,*)
            Write (nout,*) '* Current global optimum candidate:'
            Do k = 1, ndim
              Write (nout,99999) k, xb(k)
            End Do
            Write (nout,*) '* Current global optimum value:'
            Write (nout,99998) fb
            Write (nout,99997)
            Do k = 1, ncon
              Write (nout,99996) k, cb(k)
            End Do
          End If
        End If

!       If required set INFORM<0 to force exit
        inform = 0

        Return
99999   Format (1X,'* xb(',I3,') = ',F9.2)
99998   Format (1X,'* fb = ',F9.5)
99997   Format ('** Current global optimum constraint violations **')
99996   Format (1X,'* cb(',I3,') = ',F9.2)
      End Subroutine monmod
      Subroutine display_option(optstr,optype,ivalue,rvalue,cvalue)
!       Subroutine to query optype and print the appropriate option values

!       .. Scalar Arguments ..
        Real (Kind=nag_wp), Intent (In)      :: rvalue
        Integer, Intent (In)                 :: ivalue, optype
        Character (*), Intent (In)           :: cvalue, optstr
!       .. Executable Statements ..
        Select Case (optype)
        Case (1)
          Write (nout,99999) optstr, ivalue
        Case (2)
          Write (nout,99998) optstr, rvalue
        Case (3)
          Write (nout,99997) optstr, cvalue
        Case (4)
          Write (nout,99996) optstr, ivalue, cvalue
        Case (5)
          Write (nout,99995) optstr, rvalue, cvalue
        Case Default
        End Select

        Flush (nout)

        Return
99999   Format (3X,A39,' : ',I13)
99998   Format (3X,A39,' : ',F13.4)
99997   Format (3X,A39,' : ',16X,A16)
99996   Format (3X,A39,' : ',I13,3X,A16)
99995   Format (3X,A39,' : ',F13.4,3X,A16)
      End Subroutine display_option

      Subroutine display_result(ndim,ncon,xb,fb,cb,itt,inform)
```

```
!       Display final results in comparison to known global optimum.

!       .. Use Statements ..
        Use nag_library, Only: x04cbf
!       .. Parameters ..
        Integer, Parameter                  :: indent = 1, ncols = 79
        Character (11), Parameter           :: clabs(1:6) = (/ 'x_target_u ', &
                     'x_target_c ','xb         ','c_target_u ','c_target_c ', &
                                               'cb          ' /)
        Character (1), Parameter            :: diag = 'D', labcol = 'C',      &
                                               labrow = 'I', matrix = 'G'
        Character (5), Parameter            :: fmtc = 'f12.5', fmtx = 'f12.2'
!       .. Scalar Arguments ..
        Real (Kind=nag_wp), Intent (In)     :: fb
        Integer, Intent (In)                :: inform, ncon, ndim
!       .. Array Arguments ..
        Real (Kind=nag_wp), Intent (In)     :: cb(ncon), xb(ndim)
        Integer, Intent (In)                :: itt(7)
!       .. Local Scalars ..
        Integer                             :: ifail, ldcom
        Character (ncols)                   :: titlec, titlex
!       .. Local Arrays ..
        Real (Kind=nag_wp), Allocatable     :: ccom(:,:), xcom(:,:)
!       .. Executable Statements ..
!       Display final counters.
        Write (nout,*) ' Algorithm Statistics'
        Write (nout,*) ' ---------------------'
        Write (nout,99994) 'Total complete iterations               ', itt(1)
        Write (nout,99994) 'Complete iterations since improvement   ', itt(2)
        Write (nout,99994) 'Total particles converged to xb         ', itt(3)
        Write (nout,99994) 'Total improvements to global optimum    ', itt(4)
        Write (nout,99994) 'Total function evaluations              ', itt(5)
        Write (nout,99994) 'Total particles re-initialized          ', itt(6)
        Write (nout,99994) 'Total constraints violated              ', itt(7)

!       Display why finalization occurred.
        Write (nout,*)
        Select Case (inform)
        Case (1)
          Write (nout,99999) 'Target value achieved'
        Case (2)
          Write (nout,99999) 'Minimum swarm standard deviation obtained'
        Case (3)
          Write (nout,99999) 'Sufficient particles converged'
        Case (4)
          Write (nout,99999) 'No improvement in preset iteration limit'
        Case (5)
          Write (nout,99999) 'Maximum complete iterations attained'
        Case (6)
          Write (nout,99999) 'Maximum function evaluations exceeded'
        Case (7)
          Write (nout,99999) 'Constrained point located'
        Case (:-1)
          Write (nout,99998) inform
        Case Default
        End Select

!       Display final objective value and location.
        Write (nout,*)
        Write (nout,99997) f_target_u
        Write (nout,99996) f_target_c
        Write (nout,99995) fb
        Flush (nout)

        ldcom = ndim
        Allocate (xcom(ldcom,3))
        xcom(1:ndim,1) = x_target_u(1:ndim)
        xcom(1:ndim,2) = x_target_c(1:ndim)
        xcom(1:ndim,3) = xb(1:ndim)

        Write (nout,*)
```

```
        titlex = 'Comparison between known and achieved optima.'
        ifail = 0
        Call x04cbf(matrix,diag,ndim,3,xcom,ldcom,fmtx,titlex,labrow,clabs, &
          labcol,clabs,ncols,indent,ifail)

        Deallocate (xcom)

        If (ncon>0) Then
          ldcom = ncon
          Allocate (ccom(ldcom,3))
          ccom(1:ncon,1) = c_target_u(1:ncon)/c_scale(1:ncon)
          ccom(1:ncon,2) = c_target_c(1:ncon)/c_scale(1:ncon)
          ccom(1:ncon,3) = cb(1:ncon)/c_scale(1:ncon)

          Write (nout,*)
          Flush (nout)
          titlec = 'Comparison between scaled constraint violations.'
          ifail = 0
          Call x04cbf(matrix,diag,ncon,3,ccom,ldcom,fmtc,titlec,labrow,clabs, &
            labcol,clabs(4:6),ncols,indent,ifail)

          Deallocate (ccom)
        End If

        Write (nout,*)

        Return
99999   Format (2X,'Solution Status : ',A38)
99998   Format ('  User termination case                    : ',I13)
99997   Format ('  Known unconstrained objective minimum    : ',F13.3)
99996   Format ('  Best Known constrained objective minimum : ',F13.3)
99995   Format ('  Achieved objective value                 : ',F13.3)
99994   Format (2X,A40,' :',I13)
      End Subroutine display_result
    End Module e05sbfe_mod
    Program e05sbfe
!     E05SBF Example Main Program

!     This example program demonstrates how to use E05SBF in standard
!       execution, and with E04UCF as a coupled local minimizer.
!     The non-default option 'REPEATABILITY ON' is used here, giving
!       repeatable results.

!     .. Use Statements ..
      Use nag_library, Only: e05sbf, e05zkf, e05zlf, nag_wp
      Use e05sbfe_mod, Only: confun_non_linear, display_option,               &
                             display_result, f_target_c, liopts, liuser,      &
                             lopts, lruser, monmod, ncon, ndim, nout, npar,   &
                             objfun_schwefel, zero
!     .. Implicit None Statement ..
      Implicit None
!     .. Local Scalars ..
      Real (Kind=nag_wp)                  :: fb, rvalue
      Integer                             :: ifail, inform, ivalue, optype
      Character (16)                      :: cvalue
      Character (80)                      :: optstr
!     .. Local Arrays ..
      Real (Kind=nag_wp)                  :: bl(ndim+ncon), bu(ndim+ncon),    &
                                             cb(ncon), cbest(ncon,npar),      &
                                             fbest(ndim,npar), opts(lopts),   &
                                             ruser(lruser), xb(ndim),         &
                                             xbest(ndim,npar)
      Integer                             :: iopts(liopts), itt(7),           &
                                             iuser(liuser)
!     .. Executable Statements ..
!     Print advisory information.
      Write (nout,*) 'E05SBF Example Program Results'
      Write (nout,*)
      Write (nout,*) 'Minimization of the Schwefel function.'
      Write (nout,*) 'Subject to one linear and two nonlinear constraints.'
      Write (nout,*)
```

```
      xbest = zero
      fbest = zero
      cbest = zero

!     Set problem specific values.
!     Set box bounds.
      bl(1:ndim) = -500.0_nag_wp
      bu(1:ndim) = 500.0_nag_wp
!     Set constraint bounds.
      bl((ndim+1):(ndim+ncon)) = (/-1.0E6_nag_wp,-1.0_nag_wp,-0.9_nag_wp/)
      bu((ndim+1):(ndim+ncon)) = (/10.0_nag_wp,5.0E5_nag_wp,0.9_nag_wp/)

!     Initialize the option arrays for E05SBF.
      ifail = 0
      Call e05zkf('Initialize = E05SBF',iopts,liopts,opts,lopts,ifail)

!     Query some default option values.
      Write (nout,*) '  Default Option Queries:'
      Write (nout,*)
      ivalue = 0
      rvalue = 0.0_nag_wp
      ifail = 0
      optstr = 'Constraint Norm'
      Call e05zlf(optstr,ivalue,rvalue,cvalue,optype,iopts,opts,ifail)
      Call display_option(optstr,optype,ivalue,rvalue,cvalue)

      ifail = 0
      optstr = 'Maximum Iterations Completed'
      Call e05zlf(optstr,ivalue,rvalue,cvalue,optype,iopts,opts,ifail)
      Call display_option(optstr,optype,ivalue,rvalue,cvalue)

      ifail = 0
      optstr = 'Distance Tolerance'
      Call e05zlf(optstr,ivalue,rvalue,cvalue,optype,iopts,opts,ifail)
      Call display_option(optstr,optype,ivalue,rvalue,cvalue)

!     ------------------------------------------------------------------
      Write (nout,*)
      Write (nout,*) '1. Solution without using coupled local minimizer'
      Write (nout,*)
!     Set various options to non-default values if required.
      ifail = 0
      Write (optstr,99999) 'Distance Tolerance', rvalue*0.1_nag_wp
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Write (optstr,99999) 'Constraint Tolerance', 1.0E-4_nag_wp
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Call e05zkf('Constraint Norm = Euclidean',iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Call e05zkf('Repeatability = On',iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Write (optstr,99999) 'Target Objective Value', f_target_c
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Write (optstr,99999) 'Target Objective Tolerance', 1.0E-4_nag_wp
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)

!     Call E05SBF to search for the global optimum.
!     Non-zero IFAIL expected on exit here, so use IFAIL=1 (quiet) on entry.
      ifail = 1
      Call e05sbf(ndim,ncon,npar,xb,fb,cb,bl,bu,xbest,fbest,cbest, &
        objfun_schwefel,confun_non_linear,monmod,iopts,opts,iuser,ruser,itt, &
        inform,ifail)

!     It is essential to test IFAIL on exit.
      Select Case (ifail)
      Case (0,1)
!       No errors, best found optimum at xb returned in fb.
        Call display_result(ndim,ncon,xb,fb,cb,itt,inform)
```

```
      Case (3)
!       Exit flag set in OBJFUN, CONFUN or MONMOD and returned in INFORM.
        Call display_result(ndim,ncon,xb,fb,cb,itt,inform)
      Case Default
!       An error was detected. Print message since IFAIL=1 on entry.
        Write (nout,99998) '** E05SBF returned with an error, IFAIL = ', ifail
        Continue
      End Select

!     ------------------------------------------------------------------
      Write (nout,*) '2. Solution using coupled local minimizer E04UCF'
      Write (nout,*)

!     Set the local minimizer to be E04UCF and set corresponding options.
      ifail = 0
      Call e05zkf('Local Minimizer = E04UCF',iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Call e05zkf('Local Interior Major Iterations = 15',iopts,liopts,opts, &
        lopts,ifail)
      ifail = 0
      Call e05zkf('Local Interior Minor Iterations = 5',iopts,liopts,opts, &
        lopts,ifail)
      ifail = 0
      Call e05zkf('Local Exterior Major Iterations = 50',iopts,liopts,opts, &
        lopts,ifail)
      ifail = 0
      Call e05zkf('Local Exterior Minor Iterations = 15',iopts,liopts,opts, &
        lopts,ifail)

!     Query the option Distance Tolerance
      ifail = 0
      optstr = 'Distance Tolerance'
      Call e05zlf(optstr,ivalue,rvalue,cvalue,optype,iopts,opts,ifail)
!     Adjust Distance Tolerance dependent upon its current value
      Write (optstr,99999) 'Distance Tolerance', rvalue*10.0_nag_wp
      ifail = 0
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Write (optstr,99999) 'Local Interior Tolerance', rvalue
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)
      ifail = 0
      Write (optstr,99999) 'Local Exterior Tolerance', rvalue*1.0E-4_nag_wp
      Call e05zkf(optstr,iopts,liopts,opts,lopts,ifail)

!     Call E05SBF to search for the global optimum.
      ifail = 1
      Call e05sbf(ndim,ncon,npar,xb,fb,cb,bl,bu,xbest,fbest,cbest, &
        objfun_schwefel,confun_non_linear,monmod,iopts,opts,iuser,ruser,itt, &
        inform,ifail)

!     It is essential to test IFAIL on exit.
      Select Case (ifail)
      Case (0,1)
!       E05SBF encountered no errors during operation,
!       and will have returned the best found optimum.
        Call display_result(ndim,ncon,xb,fb,cb,itt,inform)
      Case (3)
!       Exit flag set in OBJFUN, CONFUN or MONMOD and returned in INFORM.
        Call display_result(ndim,ncon,xb,fb,cb,itt,inform)
      Case Default
!       An error was detected. Print message since IFAIL=1 on entry.
        Write (nout,99998) '** E05SBF returned with an error, IFAIL = ', ifail
        Continue
      End Select

99999 Format (A,' = ',E32.16)
99998 Format (1X,A,I6)
    End Program e05sbfe
```

9.2 Program Data

None.

9.3 Program Results

```
 E05SBF Example Program Results

 Minimization of the Schwefel function.
 Subject to one linear and two nonlinear constraints.

   Default Option Queries:

   Constraint Norm                        :                 L1
   Maximum Iterations Completed           :          1000   DEFAULT
   Distance Tolerance                     :        0.0001

 1. Solution without using coupled local minimizer

  Algorithm Statistics
  --------------------
  Total complete iterations               :          277
  Complete iterations since improvement   :            1
  Total particles converged to xb         :            0
  Total improvements to global optimum    :          117
  Total function evaluations              :         4222
  Total particles re-initialized          :            0
  Total constraints violated              :            0

  Solution Status :                  Target value achieved

  Known unconstrained objective minimum    :      -837.966
  Best Known constrained objective minimum :      -731.707
  Achieved objective value                 :      -731.708

  Comparison between known and achieved optima.
      x_target_u  x_target_c          xb
 1       -420.97     -394.15     -394.17
 2       -420.97     -433.48     -433.53

  Comparison between scaled constraint violations.
      c_target_u  c_target_c          cb
 1       0.00000     0.00000     0.00000
 2       0.04219     0.00000     0.00000
 3       0.75749     0.00000     0.00002

 2. Solution using coupled local minimizer E04UCF

  Algorithm Statistics
  --------------------
  Total complete iterations               :            4
  Complete iterations since improvement   :            1
  Total particles converged to xb         :            0
  Total improvements to global optimum    :            7
  Total function evaluations              :          155
  Total particles re-initialized          :            0
  Total constraints violated              :            0

  Solution Status :                  Target value achieved

  Known unconstrained objective minimum    :      -837.966
  Best Known constrained objective minimum :      -731.707
  Achieved objective value                 :      -731.706

  Comparison between known and achieved optima.
      x_target_u  x_target_c          xb
 1       -420.97     -394.15     -394.15
 2       -420.97     -433.48     -433.49
```

```
 Comparison between scaled constraint violations.
     c_target_u  c_target_c          cb
 1      0.00000     0.00000     0.00000
 2      0.04219     0.00000     0.00000
 3      0.75749     0.00000     0.00000
```

10 Algorithmic Details

The following pseudo-code describes the algorithm used with the repulsion mechanism.

INITIALIZE for $j = 1, n_p$

$\mathbf{x}_j = \mathbf{R} \in U(\boldsymbol{\ell}_{\text{box}}, \mathbf{u}_{\text{box}})$

$\hat{\mathbf{x}}_j = \begin{cases} \mathbf{R} \in U(\boldsymbol{\ell}_{\text{box}}, \mathbf{u}_{\text{box}}) & \textbf{Start} = \text{COLD} \\ \hat{\mathbf{x}}_j^0 & \textbf{Start} = \text{WARM} \end{cases}$

$\mathbf{v}_j = \mathbf{R} \in U(-\mathbf{V}_{\max}, \mathbf{V}_{\max})$

$\hat{f}_j = \begin{cases} F(\hat{\mathbf{x}}_j) & \textbf{Start} = \text{COLD} \\ \hat{f}_j^0 & \textbf{Start} = \text{WARM} \end{cases}$

$\hat{\mathbf{e}}_j = \begin{cases} \mathbf{e}(\hat{\mathbf{x}}_j) & \textbf{Start} = \text{COLD} \\ \hat{\mathbf{e}}_j^0 & \textbf{Start} = \text{WARM} \end{cases}$

$w_j = \begin{cases} W_{\max} & \textbf{Weight Initialize} = \text{MAXIMUM} \\ W_{\text{ini}} & \textbf{Weight Initialize} = \text{INITIAL} \\ R \in U(W_{\min}, W_{\max}) & \textbf{Weight Initialize} = \text{RANDOMIZED} \end{cases}$

end for

$\tilde{\mathbf{x}} = \frac{1}{2}(\boldsymbol{\ell}_{\text{box}} + \mathbf{u}_{\text{box}})$

$\tilde{f} = F(\tilde{\mathbf{x}})$

$\tilde{\mathbf{e}} = \mathbf{e}(\tilde{\mathbf{x}})$

$I_c = I_s = 0$

SWARM while (not finalized),

$I_c = I_c + 1$

for $j = 1, n_p$

$\mathbf{x}_j = \text{BOUNDARY}(\mathbf{x}_j, \boldsymbol{\ell}_{\text{box}}, \mathbf{u}_{\text{box}})$

$f_j = F(\mathbf{x}_j)$

$\mathbf{e}_j = \mathbf{e}(\mathbf{x}_j)$

if $\left(f_j/f_{\text{scale}} + \phi(w_j)\|\mathbf{e}_j\| < \hat{f}_j/f_{\text{scale}} + \phi(w_j)\|\hat{\mathbf{e}}_j\|\right)$

$\hat{f}_j = f_j$, $\hat{\mathbf{x}}_j = \mathbf{x}_j$

if $((\|\mathbf{e}_j\| < \|\tilde{\mathbf{e}}\|)$ or $(\|\mathbf{e}_j\| \approx \|\tilde{\mathbf{e}}\|$ and $f_j < \tilde{f}))$

$\tilde{f} = f_j$, $\tilde{\mathbf{x}} = \mathbf{x}_j$

end for

if $(\text{new}(\tilde{f}))$

LOCMIN $(\tilde{\mathbf{x}}, \tilde{f}, \tilde{\mathbf{e}}, O_i)$, $I_s = 0$

[see note on repulsion below for code insertion]

else

$I_s = I_s + 1$

for $j = 1, n_p$

$\mathbf{v}_j = w_j\mathbf{v}_j + C_s\mathbf{D}_1(\hat{\mathbf{x}}_j - \mathbf{x}_j) + C_g\mathbf{D}_2(\tilde{\mathbf{x}} - \mathbf{x}_j)$

$\mathbf{x}_j = \mathbf{x}_j + \mathbf{v}_j$

if $(\|\mathbf{x}_j - \tilde{\mathbf{x}}\| < dtol)$

reset $\mathbf{x}_j$, $\mathbf{v}_j$, w_j; $\hat{\mathbf{x}}_j = \mathbf{x}_j$

else

update (w_j)

end for

if (target achieved or termination criterion satisfied)

finalized = true

MONMOD$(\mathbf{x}_j)$

end

LOCMIN $(\tilde{\mathbf{x}}, \tilde{f}, \tilde{\mathbf{e}}, O_e)$

The definition of terms used in the above pseudo-code are as follows.

n_p the number of particles, NPAR

$\boldsymbol{\ell}_{\text{box}}$ array of NDIM lower box bounds

$\mathbf{u}_{\text{box}}$ array of NDIM upper box bounds

$\mathbf{x}_j$ position of particle j

$\hat{\mathbf{x}}_j$ best position found by particle j

$\tilde{\mathbf{x}}$ best position found by any particle

f_j $F(\mathbf{x}_j)$

$\hat{f}_j$ $F(\hat{\mathbf{x}}_j)$, best value found by particle j

$\tilde{f}$ $F(\tilde{\mathbf{x}})$, best value found by any particle

$e_k(\mathbf{x})$ kth (scaled) constraint violation at $\mathbf{x}$, evaluated as $\min(c_k(\mathbf{x}) - l_{\text{NDIM}+k}, 0.0) + \max(c_k(\mathbf{x}) - u_{\text{NDIM}+k}, 0.0)$; this may be scaled by the maximum kth constraint found thus far

$\mathbf{e}(\mathbf{x})$ the array of NCON constraint violations, $e_k(\mathbf{x})$, for $k = 1, 2, \ldots, \text{NCON}$, at a point $\mathbf{x}$

$\mathbf{e}_j$ $\mathbf{e}(\mathbf{x}_j)$, the array of constraint violations evaluated at $\mathbf{x}_j$

$\hat{\mathbf{e}}_j$ $\mathbf{e}(\hat{\mathbf{x}}_j)$, the array of constraint violations evaluated at $\hat{\mathbf{x}}_j$

$\tilde{\mathbf{e}}$ $\mathbf{e}(\tilde{\mathbf{x}})$, the array of constraint violations evaluated at $\tilde{\mathbf{x}}$

$\mathbf{v}_j$ velocity of particle j

w_j weight on $\mathbf{v}_j$ for velocity update, decreasing according to **Weight Decrease**

$\mathbf{V}_{\text{max}}$ maximum absolute velocity, dependent upon **Maximum Variable Velocity**

I_c swarm iteration counter

I_s iterations since $\tilde{\mathbf{x}}$ was updated

f_{scale} objective function scaling defined by the options **Constraint Scaling**, **Objective Scaling** and **Objective Scale**.

$\mathbf{D}_1$,$\mathbf{D}_2$ diagonal matrices with random elements in range $(0, 1)$

C_s the cognitive advance coefficient which weights velocity towards $\hat{\mathbf{x}}_j$, adjusted using **Advance Cognitive**

C_g the global advance coefficient which weights velocity towards $\tilde{\mathbf{x}}$, adjusted using **Advance Global**

dtol the **Distance Tolerance** for resetting a converged particle

$\mathbf{R} \in U(\boldsymbol{\ell}_{\text{box}}, \mathbf{u}_{\text{box}})$
an array of random numbers whose i-th element is drawn from a uniform distribution in the range $(\boldsymbol{\ell}_{\text{box}i}, \mathbf{u}_{\text{box}i})$, for $i = 1, 2, \ldots, \text{NDIM}$

O_i local optimizer interior options

O_e local optimizer exterior options

$\phi(w_j)$ a function of w_j designed to increasingly weight towards minimizing constraint violations as w_j decreases

LOCMIN $(\mathbf{x}, f, \mathbf{e}, O)$
apply local optimizer using the set of options O using the solution $(\mathbf{x}, f, \mathbf{e})$ as the starting point, if used (not default)

MONMOD monitor progress and possibly modify $\mathbf{x}_j$

BOUNDARY
apply required behaviour for $\mathbf{x}_j$ outside bounding box, (see **Boundary**)

new $(\tilde{f})$ true if $\tilde{\mathbf{x}}$, $\tilde{\mathbf{c}}$, $\tilde{f}$ were updated at this iteration

Additionally a repulsion phase can be introduced by changing from the default values of options **Repulsion Finalize** (r_f), **Repulsion Initialize** (r_i) and **Repulsion Particles** (r_p). If the number of static particles is denoted n_s then the following can be inserted after the new($\tilde{f}$) check in the pseudo-code above.

```
else  if  (n_s ≥ r_p and r_i ≤ I_s ≤ r_i + r_f)
          LOCMIN (x̃, f̃, ẽ, O_i)
          use −C_g instead of C_g in velocity updates
      if  (I_s = r_i + r_f)
          I_s = 0
```

10.1 Details of SMP parallelization

The algorithm has been parallelized to allow for a high degree of asynchronicity between threads. Each thread is assigned a static number of the NPAR particles requested, and performs a sub-iteration using these particles and a private copy of $\tilde{\mathbf{x}}$. The thread only updates this private copy if a superior solution is found.

Once a thread has completed a sub-iteration, it enters a brief critical section where it compares this private $\tilde{\mathbf{x}}$ to a globally accessible version. If either is superior, the inferior version is updated and the thread continues into a new sub-iteration.

Parallelizing the algorithm in this way allows for individual threads to continue searching even if other threads are completing sub-iterations in inferior times. The optional argument **SMP Thread Overrun** allows you to force a synchronization across the team of threads once one thread completes sufficiently more sub-iterations than the slowest thread. In particular, this may be used to force synchronization after every sub-iteration if so desired.

11 Optional Parameters

This section can be skipped if you wish to use the default values for all optional parameters, otherwise, the following is a list of the optional parameters available and a full description of each optional parameter is provided in Section 11.1.

Advance Cognitive
Advance Global
Boundary
Constraint Norm
Constraint Scale Maximum
Constraint Scaling
Constraint Superiority
Constraint Tolerance
Constraint Warning
Distance Scaling
Distance Tolerance
Function Precision
Local Boundary Restriction
Local Exterior Iterations
Local Exterior Major Iterations
Local Exterior Minor Iterations
Local Exterior Tolerance
Local Interior Iterations

Local Interior Major Iterations
Local Interior Minor Iterations
Local Interior Tolerance
Local Minimizer
Maximum Function Evaluations
Maximum Iterations Completed
Maximum Iterations Static
Maximum Iterations Static Particles
Maximum Particles Converged
Maximum Particles Reset
Maximum Variable Velocity
Objective Scale
Objective Scaling
Optimize
Repeatability
Repulsion Finalize
Repulsion Initialize
Repulsion Particles
SMP Callback Thread Safe
SMP Local Minimizer External
SMP Monitor
SMP Monmod
SMP Subswarm
SMP Thread Overrun
Start
Swarm Standard Deviation
Target Objective
Target Objective Safeguard
Target Objective Tolerance
Target Objective Value
Target Warning
Verify Gradients
Weight Decrease
Weight Initial
Weight Initialize
Weight Maximum
Weight Minimum
Weight Reset
Weight Value

11.1 Description of the Optional Parameters

For each option, we give a summary line, a description of the optional parameter and details of constraints.

The summary line contains:

the keywords;

a parameter value, where the letters a, i and r denote options that take character, integer and real values respectively;

the default value, where the symbol ϵ is a generic notation for ***machine precision*** (see X02AJF), and *Imax* represents the largest representable integer value (see X02BBF).

All options accept the value 'DEFAULT' in order to return single options to their default states.

Keywords and character values are case insensitive, however they must be separated by at least one space.

For E05SBF the maximum length of the parameter CVALUE used by E05ZLF is 15.

Advance Cognitive r Default $= 2.0$

The cognitive advance coefficient, C_s. When larger than the global advance coefficient, this will cause particles to be attracted toward their previous best positions. Setting $r = 0.0$ will cause E05SBF to act predominantly as a local optimizer. Setting $r > 2.0$ may cause the swarm to diverge, and is generally inadvisable. At least one of the global and cognitive coefficients must be nonzero.

Advance Global r Default $= 2.0$

The global advance coefficient, C_g. When larger than the cognitive coefficient this will encourage convergence toward the best solution yet found. Values $r \in (0, 1)$ will inhibit particles overshooting the optimum. Values $r \in [1, 2)$ cause particles to fly over the optimum some of the time. Larger values can prohibit convergence. Setting $r = 0.0$ will remove any attraction to the current optimum, effectively generating a Monte–Carlo multi-start optimization algorithm. At least one of the global and cognitive coefficients must be nonzero.

Boundary a Default $=$ FLOATING

Determines the behaviour if particles leave the domain described by the box bounds. This only affects the general PSO algorithm, and will not pass down to any NAG local minimizers chosen.

This option is only effective in those dimensions for which $\mathrm{BL}(i) \neq \mathrm{BU}(i)$, $i = 1, 2, \ldots, \mathrm{NDIM}$.

IGNORE
: The box bounds are ignored. The objective function is still evaluated at the new particle position.

RESET
: The particle is re-initialized inside the domain. $\hat{\mathbf{x}}_j$, $\hat{f}_j$ and $\hat{\mathbf{e}}_j$ are not affected.

FLOATING
: The particle position remains the same, however the objective function will not be evaluated at the next iteration. The particle will probably be advected back into the domain at the next advance due to attraction by the cognitive and global memory.

HYPERSPHERICAL
: The box bounds are wrapped around an *ndim*-dimensional hypersphere. As such a particle leaving through a lower bound will immediately re-enter through the corresponding upper bound and vice versa. The standard distance between particles is also modified accordingly.

FIXED
: The particle rests on the boundary, with the corresponding dimensional velocity set to 0.0.

Constraint Norm a Default $=$ L1

Determines with respect to which norm the constraint residuals should be constructed. These are automatically scaled with respect to NCON as stated. For the set of (scaled) violations $\mathbf{e}$, these may be,

L1
: The L^1 norm will be used, $\|\mathbf{e}\|_1 = \frac{1}{\mathrm{NCON}} \sum_{1}^{\mathrm{NCON}} |e_k|$

L2

The L^2 norm will be used, $\|\mathbf{e}\|_2 = \frac{1}{\text{NCON}}\sqrt{\sum_{1}^{\text{NCON}} e_k^2}$

L2SQ

The square of the L^2 norm will be used, $\|\mathbf{e}\|_{2^2} = \frac{1}{\text{NCON}}\sum_{1}^{\text{NCON}} e_k^2$

LMAX

The L^∞ norm will be used, $\|\mathbf{e}\|_\infty = \max_{0<k\leq\text{NCON}}(|e_k|)$

Constraint Scale Maximum r Default $= 1.0\text{E}6$

Internally, each constraint violation is scaled with respect to the maximum violation yet achieved for that constraint. This option acts as a ceiling for this scale.

Constraint: $r > 1.0$.

Constraint Scaling a Default $=$ INITIAL

Determines whether to scale the constraints and objective function when constructing the penalty function.

OFF

Neither the constraint violations nor the objective will be scaled automatically. This should only be used if the constraints and objective are similarly scaled everywhere throughout the domain.

INITIAL

The maximum of the initial cognitive memories, $\hat{f}_j$ and $\hat{\mathbf{e}}_j$, will be used to scale the objective function and constraint violations respectively.

ADAPTIVE

Initially, the maximum of the initial cognitive memories, $\hat{f}_j$ and $\hat{\mathbf{e}}_j$, will be used to scale the objective function and constraint violations respectively. If a significant change is detected in the behaviour of the constraints or the objective, these will be rescaled with respect to the current state of the cognitive memory.

Constraint Superiority r Default $= 0.01$

The minimum scaled improvement in the constraint violation for a location to be immediately superior to that in memory, regardless of the objective value.

Constraint: $r > 0.0$.

Constraint Tolerance r Default $= 10^{-4}$

The maximum scaled violation of the constraints for which a sample particle is considered comparable to the current global optimum. Should this not be exceeded, then the current global optimum will be updated if the value of the objective function of the sample particle is superior.

Constraint Warning a Default $=$ ON

Activates or deactivates the error exit associated with the inability to completely satisfy all constraints, IFAIL $= 4$. It is advisable to deactivate this option if IFAIL $= 0$ on entry and the satisfaction of all constraints is not program critical.

OFF

No error will be returned.

ON

An error will be returned if any constraints are sufficiently violated at the end of the simulation.

Distance Scaling a Default = ON

Determines whether distances should be scaled by box widths.

ON

When a distance is calculated between **x** and **y**, a scaled L^2 norm is used.

$$L^2(\mathbf{x},\mathbf{y}) = \left(\sum_{\{i|\mathbf{u}_i \neq \boldsymbol{\ell}_i, i \leq ndim\}} \left(\frac{x_i - y_i}{\mathbf{u}_i - \boldsymbol{\ell}_i} \right)^2 \right)^{\frac{1}{2}}.$$

OFF

Distances are calculated as the standard L^2 norm without any rescaling.

$$L^2(\mathbf{x},\mathbf{y}) = \left(\sum_{i=1}^{ndim} (x_i - y_i)^2 \right)^{\frac{1}{2}}.$$

Distance Tolerance r Default = 10^{-4}

This is the distance, *dtol* between particles and the global optimum which must be reached for the particle to be considered converged, i.e., that any subsequent movement of such a particle cannot significantly alter the global optimum. Once achieved the particle is reset into the box bounds to continue searching.

Constraint: $r > 0.0$.

Function Precision r Default = $\epsilon^{0.9}$

The parameter defines ϵ_r, which is intended to be a measure of the accuracy with which the problem function $F(\mathbf{x})$ can be computed. If $r < \epsilon$ or $r \geq 1$, the default value is used.

The value of ϵ_r should reflect the relative precision of $1 + |F(\mathbf{x})|$; i.e., ϵ_r acts as a relative precision when $|F|$ is large, and as an absolute precision when $|F|$ is small. For example, if $F(\mathbf{x})$ is typically of order 1000 and the first six significant digits are known to be correct, an appropriate value for ϵ_r would be 10^{-6}. In contrast, if $F(\mathbf{x})$ is typically of order 10^{-4} and the first six significant digits are known to be correct, an appropriate value for ϵ_r would be 10^{-10}. The choice of ϵ_r can be quite complicated for badly scaled problems; see Chapter 8 of Gill *et al.* (1981) for a discussion of scaling techniques. The default value is appropriate for most simple functions that are computed with full accuracy. However when the accuracy of the computed function values is known to be significantly worse than full precision, the value of ϵ_r should be large enough so that no attempt will be made to distinguish between function values that differ by less than the error inherent in the calculation.

Local Boundary Restriction r Default = 0.5

Contracts the box boundaries used by a box constrained local minimizer to, $[\beta_l, \beta_u]$, containing the start point x, where

$$\begin{aligned} \partial_i &= r \times (\mathbf{u}_i - \boldsymbol{\ell}_i) \\ \beta_l^i &= \max\left(\boldsymbol{\ell}_i, x_i - \tfrac{\partial_i}{2}\right) \\ \beta_u^i &= \min\left(\mathbf{u}_i, x_i + \tfrac{\partial_i}{2}\right), \qquad i = 1, \ldots, \text{NDIM}. \end{aligned}$$

Smaller values of r thereby restrict the size of the domain exposed to the local minimizer, possibly reducing the amount of work done by the local minimizer.

Constraint: $0.0 \leq r \leq 1.0$.

Local Interior Iterations i_1
Local Interior Major Iterations i_1
Local Exterior Iterations i_2
Local Exterior Major Iterations i_2

The maximum number of iterations or function evaluations the chosen local minimizer will perform inside (outside) the main loop if applicable. For the NAG minimizers these correspond to:

Minimizer	**Parameter/option**	**Default Interior**	**Default Exterior**
E04CBF	MAXCAL	NDIM + 10	2 × NDIM + 15
E04DGF/E04DGA	**Iteration Limit**	max(30, 3 × NDIM)	max(50, 5 × NDIM)
E04UCF/E04UCA	**Major Iteration Limit**	max(10, 2 × NDIM)	max(30, 3 × NDIM)

Unless set, these are functions of the parameters passed to E05SBF.

Setting $i = 0$ will disable the local minimizer in the corresponding algorithmic region. For example, setting **Local Interior Iterations** $= 0$ and **Local Exterior Iterations** $= 30$ will cause the algorithm to perform no local minimizations inside the main loop of the algorithm, and a local minimization with upto 30 iterations after the main loop has been exited.

Note: currently E04JYF or E04KZF are restricted to using 400 × NDIM and 50 × NDIM as function evaluation limits respectively. This applies to both local minimizations inside and outside the main loop. They may still be deactivated in either phase by setting $i = 0$, and may subsequently be reactivated in either phase by setting $i > 0$.

Constraint: $i_1 \geq 0$, $i_2 \geq 0$.

Local Interior Tolerance r_1 Default $= 10^{-4}$
Local Exterior Tolerance r_2 Default $= 10^{-4}$

This is the tolerance provided to a local minimizer in the interior (exterior) of the main loop of the algorithm.

Constraint: $r_1 > 0.0$, $r_2 > 0.0$.

Local Interior Minor Iterations i_1
Local Exterior Minor Iterations i_2

Where applicable, the secondary number of iterations the chosen local minimizer will use inside (outside) the main loop. Currently the relevant default values are:

Minimizer	**Parameter/option**	**Default Interior**	**Default Exterior**
E04UCF/E04UCA	**Minor Iteration Limit**	max(10, 2 × NDIM)	max(30, 3 × NDIM)

Constraint: $i_1 \geq 0$, $i_2 \geq 0$.

Local Minimizer a Default = OFF

Allows for a choice of Chapter E04 routines to be used as a coupled, dedicated local minimizer.

OFF
: No local minimization will be performed in either the INTERIOR or EXTERIOR sections of the algorithm.

E04CBF
: Use E04CBF as the local minimizer. This does not require the calculation of derivatives.

  On a call to OBJFUN during a local minimization, MODE = 5.

E04KZF
: Use E04KZF as the local minimizer. This requires the calculation of derivatives in OBJFUN, as indicated by MODE.

  The box bounds forwarded to this routine from E05SBF will have been acted upon by **Local Boundary Restriction**. As such, the domain exposed may be greatly smaller than that provided to E05SBF.

  Accurate derivatives must be provided to this routine, and will not be approximated internally. Each iteration of this local minimizer also requires the calculation of both the objective function and its derivative. Hence on a call to OBJFUN during a local minimization, MODE = 7.

E04JYF
: Use E04JYF as the local minimizer. This does not require the calculation of derivatives.

  On a call to OBJFUN during a local minimization, MODE = 5.

The box bounds forwarded to this routine from E05SBF will have been acted upon by **Local Boundary Restriction**. As such, the domain exposed may be greatly smaller than that provided to E05SBF.

E04DGF
E04DGA
Use E04DGA as the local minimizer.

Accurate derivatives must be provided, and will not be approximated internally. Additionally, each call to OBJFUN during a local minimization will require either the objective to be evaluated alone, or both the objective and its gradient to be evaluated. Hence on a call to OBJFUN, MODE = 5 or 7.

E04UCF
E04UCA
Use E04UCA as the local minimizer. This operates such that any derivatives of either the objective function or the constraint Jacobian, which you cannot supply, will be approximated internally using finite differences.

Either, the objective, objective gradient, or both may be requested during a local minimization, and as such on a call to OBJFUN, MODE = 1, 2 or 5.

The box bounds forwarded to this routine from E05SBF will have been acted upon by **Local Boundary Restriction**. As such, the domain exposed may be greatly smaller than that provided to E05SBF.

Maximum Function Evaluations i Default = *Imax*

The maximum number of evaluations of the objective function. When reached this will return IFAIL = 1 and INFORM = 6.

Constraint: $i > 0$.

Maximum Iterations Completed i Default = 1000 × NDIM

The maximum number of complete iterations that may be performed. Once exceeded E05SBF will exit with IFAIL = 1 and INFORM = 5.

Unless set, this adapts to the parameters passed to E05SBF.

Constraint: $i \geq 1$.

Maximum Iterations Static i Default = 100

The maximum number of iterations without any improvement to the current global optimum. If exceeded E05SBF will exit with IFAIL = 1 and INFORM = 4. This exit will be hindered by setting **Maximum Iterations Static Particles** to larger values.

Constraint: $i \geq 1$.

Maximum Iterations Static Particles i Default = 0

The minimum number of particles that must have converged to the current optimum before the routine may exit due to **Maximum Iterations Static** with IFAIL = 1 and INFORM = 4.

Constraint: $i \geq 0$.

Maximum Particles Converged i Default = *Imax*

The maximum number of particles that may converge to the current optimum. When achieved, E05SBF will exit with IFAIL = 1 and INFORM = 3. This exit will be hindered by setting '**Repulsion**' options, as these cause the swarm to re-expand.

Constraint: $i > 0$.

Maximum Particles Reset i Default $=$ *Imax*

The maximum number of particles that may be reset after converging to the current optimum. Once achieved no further particles will be reset, and any particles within **Distance Tolerance** of the global optimum will continue to evolve as normal.

Constraint: $i > 0$.

Maximum Variable Velocity r Default $= 0.25$

Along any dimension j, the absolute velocity is bounded above by $\left|\mathbf{v}_j\right| \leq r \times \left(\mathbf{u}_j - \boldsymbol{\ell}_j\right) = \mathbf{V}_{\max}$. Very low values will greatly increase convergence time. There is no upper limit, although larger values will allow more particles to be advected out of the box bounds, and values greater than 4.0 may cause significant and potentially unrecoverable swarm divergence.

Constraint: $r > 0.0$.

Objective Scale r Default $= 1.0$

The initial scale for the objective function. This will remain fixed if **Objective Scaling** $=$ USER is selected.

Objective Scaling a Default $=$ MAXIMUM

The method of (re)scaling applied to the objective function when the routine detects a significant difference between the scale and the global and cognitive memory ($\tilde{f}$ and $\hat{f}_j$). This only has an effect when NCON > 0 and **Constraint Scaling** is active.

MAXIMUM
: The objective is rescaled with respect to the maximum absolute value of the objective in the cognitive and global memory.

MEAN
: The objective is rescaled with respect to the mean absolute value of the objective in the cognitive and global memory.

USER
: The scale remains fixed at the value set using **Objective Scale**.

Optimize a Default $=$ MINIMIZE

Determines whether to maximize or minimize the objective function, or ignore the objective and search for a constrained point.

MINIMIZE
: The objective function will be minimized.

MAXIMIZE
: The objective function will be maximized. This is accomplished by minimizing the negative of the objective.

CONSTRAINTS
: The objective function will be ignored, and the algorithm will attempt to find a feasible point given the provided constraints. The objective function will be evaluated at the best point found with regards to constraint violations, and the final positions returned in XBEST. The objective will be calculated at the best point found in terms of constraints only. Should a constrained point be found, E05SBF will exit with IFAIL $= 0$ and INFORM $= 6$.

Constraint: if **Optimize** $=$ CONSTRAINTS, NCON > 0 is required.

Repeatability a Default = OFF

Allows for the same random number generator seed to be used for every call to E05SBF. **Repeatability** = OFF is recommended in general.

OFF
: The internal generation of random numbers will be nonrepeatable.

ON
: The same seed will be used.

Repulsion Finalize i Default = *Imax*

The number of iterations performed in a repulsive phase before re-contraction. This allows a re-diversified swarm to contract back toward the current optimum, allowing for a finer search of the near optimum space.

Constraint: $i \geq 2$.

Repulsion Initialize i Default = *Imax*

The number of iterations without any improvement to the global optimum before the algorithm begins a repulsive phase. This phase allows the particle swarm to re-expand away from the current optimum, allowing more of the domain to be investigated. The repulsive phase is automatically ended if a superior optimum is found.

Constraint: $i \geq 2$.

Repulsion Particles i Default = 0

The number of particles required to have converged to the current optimum before any repulsive phase may be initialized. This will prevent repulsion before a satisfactory search of the near optimum area has been performed, which may happen for large dimensional problems.

Constraint: $i \geq 0$.

Start a Default = COLD

Used to affect the initialization of the routine.

COLD
: The random number generators and all initialization data will be generated internally. The variables XBEST, FBEST and CBEST need not be set.

WARM
: You must supply the initial best location, function and constraint violation values XBEST, FBEST and CBEST. This option is recommended if you already have a data set you wish to improve upon.

Swarm Standard Deviation r Default = 0.1

The target standard deviation of the particle distances from the current optimum. Once the standard deviation is below this level, E05SBF will exit with IFAIL = 1 and INFORM = 2. This criterion will be penalized by the use of '**Repulsion**' options, as these cause the swarm to re-expand, increasing the standard deviation of the particle distances from the best point.

In SMP parallel implementations of E05SBF, the standard deviation will be calculated based only on the particles local to the particular thread that checks for finalization. Considerably fewer particles may be used in this calculation than when the algorithm is run in serial. It is therefore recommended that you provide a smaller value of **Swarm Standard Deviation** when running in parallel than when running in serial.

Constraint: $r \geq 0.0$.

Target Objective a Default = OFF
Target Objective Value r Default = 0.0

Activate or deactivate the use of a target value as a finalization criterion. If active, then once the supplied target value for the objective function is found (beyond the first iteration if **Target Warning** is active)

E05SBF will exit with IFAIL $= 0$ and INFORM $= 1$. Other than checking for feasibility only (**Optimize** = CONSTRAINTS), this is the only finalization criterion that guarantees that the algorithm has been successful. If the target value was achieved at the initialization phase or first iteration and **Target Warning** is active, E05SBF will exit with IFAIL $= 2$. This option may take any real value r, or the character ON/OFF as well as DEFAULT. If this option is queried using E05ZLF, the current value of r will be returned in RVALUE, and CVALUE will indicate whether this option is ON or OFF. The behaviour of the option is as follows:

r
: Once a point is found with an objective value within the **Target Objective Tolerance** of r, E05SBF will exit successfully with IFAIL $= 0$ and INFORM $= 1$.

OFF
: The current value of r will remain stored, however it will not be used as a finalization criterion.

ON
: The current value of r stored will be used as a finalization criterion.

DEFAULT
: The stored value of r will be reset to its default value (0.0), and this finalization criterion will be deactivated.

Target Objective Safeguard r Default $= 10.0\epsilon$

If you have given a target objective value to reach in *objval* (the value of the optional parameter **Target Objective Value**), *objsfg* sets your desired safeguarded termination tolerance, for when *objval* is close to zero.

Constraint: *objsfg* $\geq 2\epsilon$.

Target Objective Tolerance r Default $= 0.0$

The optional tolerance to a user-specified target value.

Constraint: $r \geq 0.0$.

Target Warning a Default $=$ OFF

Activates or deactivates the error exit associated with the target value being achieved before entry into the main loop of the algorithm, IFAIL $= 2$.

OFF
: No error will be returned, and the routine will exit normally.

ON
: An error will be returned if the target objective is reached prematurely, and the routine will exit with IFAIL $= 2$.

Verify Gradients a Default $=$ ON

Adjusts the level of gradient checking performed when gradients are required. Gradient checks are only performed on the first call to the chosen local minimizer if it requires gradients. There is no guarantee that the gradient check will be correct, as the finite differences used in the gradient check are themselves subject to inaccuracies.

OFF
: No gradient checking will be performed.

ON
: A cheap gradient check will be performed on both the gradients corresponding to the objective through OBJFUN and those provided via the constraint Jacobian through CONFUN.

OBJECTIVE
: A more expensive gradient check will be performed on the gradients corresponding to the objective OBJFUN. The gradients of the constraints will not be checked.

CONSTRAINTS
A more expensive check will be performed on the elements of CJAC provided via CONFUN. The objective gradient will not be checked.

FULL
A more expensive check will be performed on both the gradient of the objective and the constraint Jacobian.

Weight Decrease a Default = INTEREST

Determines how particle weights decrease.

OFF
Weights do not decrease.

INTEREST
Weights decrease through compound interest as $w_{IT+1} = w_{IT}(1 - W_{val})$, where W_{val} is the **Weight Value** and IT is the current number of iterations.

LINEAR
Weights decrease linearly following $w_{IT+1} = w_{IT} - IT \times (W_{max} - W_{min})/IT_{max}$, where IT is the iteration number and IT_{max} is the maximum number of iterations as set by **Maximum Iterations Completed**.

Weight Initial r Default = W_{max}

The initial value of any particle's inertial weight, W_{ini}, or the minimum possible initial value if initial weights are randomized. When set, this will override **Weight Initialize** = RANDOMIZED or MAXIMUM, and as such these must be set afterwards if so desired.

Constraint: $W_{min} \leq r \leq W_{max}$.

Weight Initialize a Default = MAXIMUM

Determines how the initial weights are distributed.

INITIAL
All weights are initialized at the initial weight, W_{ini}, if set. If **Weight Initial** has not been set, this will be the maximum weight, W_{max}.

MAXIMUM
All weights are initialized at the maximum weight, W_{max}.

RANDOMIZED
Weights are uniformly distributed in (W_{min}, W_{max}) or (W_{ini}, W_{max}) if **Weight Initial** has been set.

Weight Maximum r Default = 1.0

The maximum particle weight, W_{max}.

Constraint: $1.0 \geq r \geq W_{min}$ (If W_{ini} has been set then $1.0 \geq r \geq W_{ini}$.)

Weight Minimum r Default = 0.1

The minimum achievable weight of any particle, W_{min}. Once achieved, no further weight reduction is possible.

Constraint: $0.0 \leq r \leq W_{max}$ (If W_{ini} has been set then $0.0 \leq r \leq W_{ini}$.)

Weight Reset a Default = MAXIMUM

Determines how particle weights are re-initialized.

INITIAL
Weights are re-initialized at the initial weight if set. If **Weight Initial** has not been set, this will be the maximum weight.

MAXIMUM
Weights are re-initialized at the maximum weight.

RANDOMIZED
Weights are uniformly distributed in (W_{min}, W_{max}) or (W_{ini}, W_{max}) if **Weight Initial** has been set.

Weight Value r Default $= 0.01$

The constant W_{val} used with **Weight Decrease** = INTEREST.

Constraint: $0.0 \leq r \leq \frac{1}{3}$.

11.2 Description of the SMP optional parameters

This section details additional options available to users of the NAG Library for SMP & Multicore. In particular it includes the option **SMP Callback Thread Safe**, which must be set before calling E05SBF with multiple threads.

SMP Callback Thread Safe a Default = WARNING

Declare that the callback routines you provide are or are not thread safe. In particular, this indicates that access to the shared memory arrays IUSER and RUSER from within your provided callbacks is done in a thread-safe manner. If these arrays are just used to pass constant data, then you may assume they are thread safe. If these are also used for workspace, or passing variable data such as random number generator seeds, then you must ensure these are accessed and updated safely. Whilst this can be done using OpenMP critical sections, we suggest their use is minimized to prevent unnecessary bottlenecks, and that instead individual threads have access to independent subsections of the provided arrays where possible.

YES
The callback routines have been programmed in a thread safe way. The algorithm will use `OMP_NUM_THREADS` threads.

NO
The callback routines are not thread safe. Setting this option will force the algorithm to run on a single thread only, and is advisable only for debugging purposes, or if you wish to parallelize your callback functions.

WARNING
This will cause an immediate exit from E05SBF with IFAIL = 51 if multiple threads are detected. This is to inform you that you have not declared the callback functions either to be thread safe, or that they are thread unsafe and you wish the algorithm to run in serial.

SMP Local Minimizer External a Default = ALL

Determines how many threads will attempt to locally minimize the best found solution after the routine has exited the main loop.

MASTER
Only the master thread will attempt to find any improvement. The local minimization will be launched from the best known solution. All other threads will remain effectively idle.

ALL
The master thread will perform a local minimization from the best known solution, while all other threads will perform a local minimization from randomly generated perturbations of the best known solution, increasing the chance of an improvement. Assuming all local minimizations will take approximately the same amount of computation, this will be effectively free in terms of real time. It will however increase the number of function evaluations performed.

SMP Monitor a Default = SINGLE
SMP Monmod a

Determines whether the user-supplied function MONMOD is invoked once every sub-iteration each thread performs, or only once by a single thread after all threads have completed at least one sub-iteration.

SINGLE
: Only one thread will invoke MONMOD, after all threads have performed at least one sub-iteration.

ALL
: Each thread will invoke MONMOD each time it completes a sub-iteration. If you wish to alter X using MONMOD you should use this option, as MONMOD will only receive the arrays X, XBEST, FBEST and CBEST private to the calling thread.

SMP Subswarm i Default = 1

Determines how many threads support a particle subswarm. This is an extra collection of particles constrained to search only within a hypercube of edge length $10.0 \times$ **Distance Tolerance** of the best point known to an individual thread. This may improve the number of iterations required to find a provided target, particularly if no local minimizer is in use.

If $i \leq 0$, then this will be disabled on all the threads.

If $i \geq$ `OMP_NUM_THREADS`, then all the threads will support a particle subswarm.

SMP Thread Overrun i Default = *Imax*

This option provides control over the level of asynchronicity present in a simulation. In particular, a barrier synchronization between all threads is performed if any thread completes i sub-iterations more than the slowest thread, causing all threads to be exposed to the current best solution. Allowing asynchronous behaviour does however allow individual threads to focus on different global optimum candidates some of the time, which can inhibit convergence to unwanted sub-optima. It also allows for threads to continue searching when other threads are completing sub-iterations at a slower rate.

If $i < 1$, then the algorithm will force a synchronization between threads at the end of each iteration.
