NAG Library Routine Document

G13AAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13AAF carries out non-seasonal and seasonal differencing on a time series. Information which allows the original series to be reconstituted from the differenced series is also produced. This information is required in time series forecasting.

2 Specification

```
SUBROUTINE G13AAF (X, NX, ND, NDS, NS, XD, NXD, IFAIL)

INTEGER            NX, ND, NDS, NS, NXD, IFAIL
REAL (KIND=nag_wp) X(NX), XD(NX)
```

3 Description

Let $\nabla^d \nabla_s^D x_i$ be the ith value of a time series x_i, for $i = 1, 2, \ldots, n$ after non-seasonal differencing of order d and seasonal differencing of order D (with period or seasonality s). In general,

$$\begin{aligned} \nabla^d \nabla_s^D x_i &= \nabla^{d-1} \nabla_s^D x_{i+1} - \nabla^{d-1} \nabla_s^D x_i && d > 0 \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i && D > 0 \end{aligned}$$

Non-seasonal differencing up to the required order d is obtained using

$$\begin{aligned} \nabla^1 x_i &= x_{i+1} - x_i && \text{for } i = 1, 2, \ldots, (n-1) \\ \nabla^2 x_i &= \nabla^1 x_{i+1} - \nabla^1 x_i && \text{for } i = 1, 2, \ldots, (n-2) \\ &\vdots \\ \nabla^d x_i &= \nabla^{d-1} x_{i+1} - \nabla^{d-1} x_i && \text{for } i = 1, 2, \ldots, (n-d) \end{aligned}$$

Seasonal differencing up to the required order D is then obtained using

$$\begin{aligned} \nabla^d \nabla_s^1 x_i &= \nabla^d x_{i+s} - \nabla^d x_i && \text{for } i = 1, 2, \ldots, (n-d-s) \\ \nabla^d \nabla_s^2 x_i &= \nabla^d \nabla_s^1 x_{i+s} - \nabla^d \nabla_s^1 x_i && \text{for } i = 1, 2, \ldots, (n-d-2s) \\ &\vdots \\ \nabla^d \nabla_s^D x_i &= \nabla^d \nabla_s^{D-1} x_{i+s} - \nabla^d \nabla_s^{D-1} x_i && \text{for } i = 1, 2, \ldots, (n-d-D \times s) \end{aligned}$$

Mathematically, the sequence in which the differencing operations are performed does not affect the final resulting series of $m = n - d - D \times s$ values.

4 References

None.

5 Parameters

1: X(NX) – REAL (KIND=nag_wp) array *Input*

On entry: the undifferenced time series, x_i, for $i = 1, 2, \ldots, n$.

2: NX – INTEGER *Input*

On entry: n, the number of values in the undifferenced time series.

Constraint: NX $>$ ND $+$ (NDS $\times$ NS).

3: ND – INTEGER *Input*

On entry: d, the order of non-seasonal differencing.

Constraint: ND ≥ 0.

4: NDS – INTEGER *Input*

On entry: D, the order of seasonal differencing.

Constraint: NDS ≥ 0.

5: NS – INTEGER *Input*

On entry: s, the seasonality.

Constraints:

if NDS > 0, NS > 0;
if NDS $= 0$, NS ≥ 0.

6: XD(NX) – REAL (KIND=nag_wp) array *Output*

On exit: the differenced values in elements 1 to NXD, and reconstitution data in the remainder of the array.

7: NXD – INTEGER *Output*

On exit: the number of differenced values in the array XD.

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: IFAIL $= 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, ND < 0,
or NDS < 0,
or NS < 0,
or NS $= 0$ when NDS > 0.

IFAIL $= 2$

On entry, NX $\leq$ ND $+$ (NDS $\times$ NS).

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by G13AAF is approximately proportional to $(\mathrm{ND} + \mathrm{NDS}) \times \mathrm{NX}$.

9 Example

This example reads in a set of data consisting of 20 observations from a time series. Non-seasonal differencing of order 2 and seasonal differencing of order 1 (with seasonality of 4) are applied to the input data, giving an output array holding 14 differenced values and 6 values which can be used to reconstitute the output array.

9.1 Program Text

```
    Program g13aafe

!     G13AAF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: g13aaf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: ifail, nd, nds, ns, nx, nxd
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: x(:), xd(:)
!     .. Executable Statements ..
      Write (nout,*) 'G13AAF Example Program Results'
      Write (nout,*)

!     Skip heading in data file
      Read (nin,*)

!     Read in the problem size
      Read (nin,*) nx, nd, nds, ns

      Allocate (x(nx),xd(nx))

!     Read in data
      Read (nin,*) x(1:nx)

!     Perform differencing
      ifail = 0
      Call g13aaf(x,nx,nd,nds,ns,xd,nxd,ifail)

!     Display results
      Write (nout,99999) 'Non-seasonal differencing of order ', nd, &
        ' and seasonal differencing'
      Write (nout,99999) 'of order ', nds, ' with seasonality ', ns, &
        ' are applied'
      Write (nout,*)
      Write (nout,99998) 'The output array holds ', nx, &
        ' values, of which the first ', nxd, ' are differenced values'
      Write (nout,*)
      Write (nout,99997) xd(1:nx)

99999 Format (1X,A,I1,A,I1,A)
99998 Format (1X,A,I2,A,I2,A)
99997 Format (1X,5F9.1)
    End Program g13aafe
```

9.2 Program Data

```
G13AAF Example Program Data
 20  2  1  4
 120.0 108.0  98.0 118.0 135.0
 131.0 118.0 125.0 121.0 100.0
  82.0  82.0  89.0  88.0  86.0
  96.0 108.0 110.0  99.0 105.0
```

9.3 Program Results

```
 G13AAF Example Program Results

 Non-seasonal differencing of order 2 and seasonal differencing
 of order 1 with seasonality 4 are applied

 The output array holds 20 values, of which the first 14 are differenced values

     -11.0    -10.0     -8.0      4.0     12.0
      -2.0     18.0      9.0     -4.0     -6.0
      -5.0     -2.0    -12.0      5.0      2.0
     -10.0    -13.0     17.0      6.0    105.0
```