## **NAG Library Routine Document**

#### F11MMF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

### 1 Purpose

F11MMF computes the reciprocal pivot growth factor of an LU factorization of a real sparse matrix in compressed column (Harwell–Boeing) format.

## 2 Specification

```
SUBROUTINE F11MMF (N, ICOLZP, A, IPRM, IL, LVAL, IU, UVAL, RPG, IFAIL)

INTEGER

N, ICOLZP(*), IPRM(7*N), IL(*), IU(*), IFAIL

REAL (KIND=nag_wp) A(*), LVAL(*), UVAL(*), RPG
```

## 3 Description

F11MMF computes the reciprocal pivot growth factor  $\max_j \left( \|A_j\|_{\infty} / \|U_j\|_{\infty} \right)$  from the columns  $A_j$  and  $U_j$  of an LU factorization of the matrix A,  $P_rAP_c = LU$  where  $P_r$  is a row permutation matrix,  $P_c$  is a column permutation matrix, L is unit lower triangular and U is upper triangular as computed by F11MEF.

#### 4 References

None.

### 5 Parameters

1: N – INTEGER Input

On entry: n, the order of the matrix A.

Constraint:  $N \geq 0$ .

2: ICOLZP(\*) – INTEGER array

Input

Note: the dimension of the array ICOLZP must be at least N+1.

On entry: ICOLZP(i) contains the index in A of the start of a new column. See Section 2.1.3 in the F11 Chapter Introduction.

3: A(\*) – REAL (KIND=nag wp) array

Input

**Note**: the dimension of the array A must be at least ICOLZP(N + 1) - 1, the number of nonzeros of the sparse matrix A.

On entry: the array of nonzero values in the sparse matrix A.

4:  $IPRM(7 \times N) - INTEGER$  array

Input

On entry: the column permutation which defines  $P_c$ , the row permutation which defines  $P_r$ , plus associated data structures as computed by F11MEF.

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### 5: IL(\*) – INTEGER array

Input

**Note**: the dimension of the array IL must be at least as large as the dimension of the array of the same name in F11MEF.

On entry: records the sparsity pattern of matrix L as computed by F11MEF.

#### 6: LVAL(\*) – REAL (KIND=nag wp) array

Input

**Note**: the dimension of the array LVAL must be at least as large as the dimension of the array of the same name in F11MEF.

On entry: records the nonzero values of matrix L and some nonzero values of matrix U as computed by F11MEF.

#### 7: IU(\*) - INTEGER array

Input

**Note**: the dimension of the array IU must be at least as large as the dimension of the array of the same name in F11MEF.

On entry: records the sparsity pattern of matrix U as computed by F11MEF.

#### 8: UVAL(\*) – REAL (KIND=nag\_wp) array

Input

**Note**: the dimension of the array UVAL must be at least as large as the dimension of the array of the same name in F11MEF.

On entry: records some nonzero values of matrix U as computed by F11MEF.

#### 9: RPG - REAL (KIND=nag wp)

Output

On exit: the reciprocal pivot growth factor  $\max_j (\|A_j\|_{\infty}/\|U_j\|_{\infty})$ . If the reciprocal pivot growth factor is much less than 1, the stability of the LU factorization may be poor.

#### 10: IFAIL - INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.

On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

### 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, N < 0.

IFAIL = 2

Ill-defined column permutations in array IPRM. Internal checks have revealed that the IPRM array is corrupted.

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IFAIL = 301

Unable to allocate required internal workspace.

## 7 Accuracy

Not applicable.

#### **8 Further Comments**

If the reciprocal pivot growth factor, RPG, is much less than 1, then the factorization of the matrix A could be poor. This means that using the factorization to obtain solutions to a linear system, forward error bounds and estimates of the condition number could be unreliable. Consider increasing the THRESH parameter in the call to F11MEF.

### 9 Example

To compute the reciprocal pivot growth for the factorization of the matrix A, where

$$A = \begin{pmatrix} 2.00 & 1.00 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & -1.00 & 0 \\ 4.00 & 0 & 1.00 & 0 & 1.00 \\ 0 & 0 & 0 & 1.00 & 2.00 \\ 0 & -2.00 & 0 & 0 & 3.00 \end{pmatrix}.$$

In this case, it should be equal to 1.0.

#### 9.1 Program Text

```
Program f11mmfe
     F11MMF Example Program Text
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     .. Use Statements ..
!
     Use nag_library, Only: f11mdf, f11mef, f11mmf, nag_wp
     .. Implicit None Statement ..
!
     Implicit None
     .. Parameters ..
     Real (Kind=nag_wp), Parameter :: one = 1.E0_nag_wp
                                    :: nin = 5, nout = 6
     Integer, Parameter
     .. Local Scalars ..
!
     Real (Kind=nag_wp)
                                    :: flop, rpg, thresh
     Integer
                                    :: i, ifail, n, nnz, nnzl, nnzu, nzlmx, &
                                       nzlumx, nzumx
     Character (1)
                                    :: spec
     .. Local Arrays ..
!
     Real (Kind=nag_wp), Allocatable :: a(:), lval(:), uval(:)
     .. Executable Statements ..
!
     Write (nout,*) 'F11MMF Example Program Results'
!
     Skip heading in data file
     Read (nin,*)
!
     Read order of matrix
     Read (nin,*) n
     Allocate (icolzp(n+1), iprm(7*n))
     Read the matrix A
     Read (nin,*) icolzp(1:n+1)
```

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```
nnz = icolzp(n+1) - 1
 Allocate (a(nnz),lval(8*nnz),uval(8*nnz),il(7*n+8*nnz+4),irowix(nnz), &
   iu(2*n+8*nnz+1))
 Do i = 1, nnz
    Read (nin,*) a(i), irowix(i)
 End Do
 Calculate COLAMD permutation
 spec = 'M'
 ifail: behaviour on error exit
          =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
  ifail = 0
 Call f11mdf(spec,n,icolzp,irowix,iprm,ifail)
 Factorise
 thresh = one
 ifail = 0
 nzlmx = 8*nnz
 nzlumx = 8*nnz
 nzumx = 8*nnz
 Call f11mef(n,irowix,a,iprm,thresh,nzlmx,nzlmx,nzumx,i1,lval,iu,uval, &
   nnzl, nnzu, flop, ifail)
 Calculate reciprocal pivot growth
 Call f11mmf(n,icolzp,a,iprm,il,lval,iu,uval,rpg,ifail)
 Output result
 Write (nout,*)
 Write (nout,*) 'Reciprocal pivot growth'
Write (nout,'(F7.3)') rpg
End Program fllmmfe
```

### 9.2 Program Data

```
F11MMF Example Program Data
 5 N
3
 5
 7
 9
 12
      ICOLZP(I) I=1,...,N+1
 2.
      1
  4.
       3
 1.
       1
 -2.
       5
 1.
       2
 1.
       3
       2
 -1.
  1.
       4
       3
  1.
  2.
       4
  3.
       5
            A(I), IROWIX(I) I=1, NNZ
```

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# 9.3 Program Results

F11MMF Example Program Results

Reciprocal pivot growth 1.000

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