

NAG Library Routine Document

S30FAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

S30FAF computes the price of a standard barrier option.

2 Specification

```

SUBROUTINE S30FAF (CALPUT, TYPE, M, N, X, S, H, K, T, SIGMA, R, Q, P, LDP,      &
                  IFAIL)
INTEGER          M, N, LDP, IFAIL
REAL (KIND=nag_wp) X(M), S, H, K, T(N), SIGMA, R, Q, P(LDP,N)
CHARACTER(1)    CALPUT
CHARACTER(2)    TYPE

```

3 Description

S30FAF computes the price of a standard barrier option, where the exercise, for a given strike price, X , depends on the underlying asset price, S , reaching or crossing a specified barrier level, H . Barrier options of type **In** only become active (are knocked in) if the underlying asset price attains the pre-determined barrier level during the lifetime of the contract. Those of type **Out** start active and are knocked out if the underlying asset price attains the barrier level during the lifetime of the contract. A cash rebate, K , may be paid if the option is inactive at expiration. The option may also be described as **Up** (the underlying price starts below the barrier level) or **Down** (the underlying price starts above the barrier level). This gives the following options which can be specified as put or call contracts.

Down-and-In: the option starts inactive with the underlying asset price above the barrier level. It is knocked in if the underlying price moves down to hit the barrier level before expiration.

Down-and-Out: the option starts active with the underlying asset price above the barrier level. It is knocked out if the underlying price moves down to hit the barrier level before expiration.

Up-and-In: the option starts inactive with the underlying asset price below the barrier level. It is knocked in if the underlying price moves up to hit the barrier level before expiration.

Up-and-Out: the option starts active with the underlying asset price below the barrier level. It is knocked out if the underlying price moves up to hit the barrier level before expiration.

The payoff is $\max(S - X, 0)$ for a call or $\max(X - S, 0)$ for a put, if the option is active at expiration, otherwise it may pay a pre-specified cash rebate, K . Following Haug (2007), the prices of the various standard barrier options can be written as shown below. The volatility, σ , risk-free interest rate, r , and annualised dividend yield, q , are constants. The integer parameters, j and k , take the values ± 1 , depending on the type of barrier.

$$\begin{aligned}
A &= jSe^{-qT}\Phi(jx_1) - jXe^{-rT}\Phi(j[x_1 - \sigma\sqrt{T}]) \\
B &= jSe^{-qT}\Phi(jx_2) - jXe^{-rT}\Phi(j[x_2 - \sigma\sqrt{T}]) \\
C &= jSe^{-qT}\left(\frac{H}{S}\right)^{2(\mu+1)}\Phi(ky_1) - jXe^{-rT}\left(\frac{H}{S}\right)^{2\mu}\Phi(k[y_1 - \sigma\sqrt{T}]) \\
D &= jSe^{-qT}\left(\frac{H}{S}\right)^{2(\mu+1)}\Phi(ky_2) - jXe^{-rT}\left(\frac{H}{S}\right)^{2\mu}\Phi(k[y_2 - \sigma\sqrt{T}]) \\
E &= Ke^{-rT}\left\{\Phi(k[x_2 - \sigma\sqrt{T}]) - \left(\frac{H}{S}\right)^{2\mu}\Phi(k[y_2 - \sigma\sqrt{T}])\right\} \\
F &= K\left\{\left(\frac{H}{S}\right)^{\mu+\lambda}\Phi(kz) + \left(\frac{H}{S}\right)^{\mu-\lambda}\Phi(k[z - \sigma\sqrt{T}])\right\}
\end{aligned}$$

with

$$\begin{aligned}
x_1 &= \frac{\ln(S/X)}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
x_2 &= \frac{\ln(S/H)}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
y_1 &= \frac{\ln(H^2/(SX))}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
y_2 &= \frac{\ln(H/S)}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
z &= \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \\
\mu &= \frac{r-q-\sigma^2/2}{\sigma^2} \\
\lambda &= \sqrt{\mu^2 + \frac{2r}{\sigma^2}}
\end{aligned}$$

and where Φ denotes the cumulative Normal distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp(-y^2/2) dy.$$

Down-and-In ($S > H$):

When $X \geq H$, with $j = k = 1$,

$$P_{\text{call}} = C + E$$

and with $j = -1$, $k = 1$

$$P_{\text{put}} = B - C + D + E$$

When $X < H$, with $j = k = 1$

$$P_{\text{call}} = A - B + D + E$$

and with $j = -1$, $k = 1$

$$P_{\text{put}} = A + E.$$

Down-and-Out ($S > H$):

When $X \geq H$, with $j = k = 1$,

$$P_{\text{call}} = A - C + F$$

and with $j = -1$, $k = 1$

$$P_{\text{put}} = A - B + C - D + F$$

When $X < H$, with $j = k = 1$,

$$P_{\text{call}} = B - D + F$$

and with $j = -1$, $k = 1$

$$P_{\text{put}} = F.$$

Up-and-In ($S < H$):

When $X \geq H$, with $j = 1, k = -1$,

$$P_{\text{call}} = A + E$$

and with $j = k = -1$,

$$P_{\text{put}} = A - B + D + E$$

When $X < H$, with $j = 1, k = -1$,

$$P_{\text{call}} = B - C + D + E$$

and with $j = k = -1$,

$$P_{\text{put}} = C + E.$$

Up-and-Out ($S < H$):

When $X \geq H$, with $j = 1, k = -1$,

$$P_{\text{call}} = F$$

and with $j = k = -1$,

$$P_{\text{put}} = B - D + F$$

When $X < H$, with $j = 1, k = -1$,

$$P_{\text{call}} = A - B + C - D + F$$

and with $j = k = -1$,

$$P_{\text{put}} = A - C + F.$$

4 References

Haug E G (2007) *The Complete Guide to Option Pricing Formulas* (2nd Edition) McGraw-Hill

5 Parameters

- 1: CALPUT – CHARACTER(1) *Input*
On entry: determines whether the option is a call or a put.
 CALPUT = 'C'
 A call. The holder has a right to buy.
 CALPUT = 'P'
 A put. The holder has a right to sell.
Constraint: CALPUT = 'C' or 'P'.
- 2: TYPE – CHARACTER(2) *Input*
On entry: indicates the barrier type as **In** or **Out** and its relation to the price of the underlying asset as **Up** or **Down**.
 TYPE = 'DI'
 Down-and-In.
 TYPE = 'DO'
 Down-and-Out.
 TYPE = 'UI'
 Up-and-In.

- TYPE = 'UO'
Up-and-Out.
Constraint: TYPE = 'DI', 'DO', 'UI' or 'UO'.
- 3: M – INTEGER *Input*
On entry: the number of strike prices to be used.
Constraint: $M \geq 1$.
- 4: N – INTEGER *Input*
On entry: the number of times to expiry to be used.
Constraint: $N \geq 1$.
- 5: X(M) – REAL (KIND=nag_wp) array *Input*
On entry: X(i) must contain X_i , the i th strike price, for $i = 1, 2, \dots, M$.
Constraint: $X(i) \geq z$ and $X(i) \leq 1/z$, where $z = X02AMF()$, the safe range parameter, for $i = 1, 2, \dots, M$.
- 6: S – REAL (KIND=nag_wp) *Input*
On entry: S, the price of the underlying asset.
Constraint: $S \geq z$ and $S \leq 1.0/z$, where $z = X02AMF()$, the safe range parameter.
- 7: H – REAL (KIND=nag_wp) *Input*
On entry: the barrier price.
Constraint: $H \geq z$ and $H \leq 1/z$, where $z = X02AMF()$, the safe range parameter.
- 8: K – REAL (KIND=nag_wp) *Input*
On entry: the value of a possible cash rebate to be paid if the option has not been knocked in (or out) before expiration.
Constraint: $K \geq 0.0$.
- 9: T(N) – REAL (KIND=nag_wp) array *Input*
On entry: T(i) must contain T_i , the i th time, in years, to expiry, for $i = 1, 2, \dots, N$.
Constraint: $T(i) \geq z$, where $z = X02AMF()$, the safe range parameter, for $i = 1, 2, \dots, N$.
- 10: SIGMA – REAL (KIND=nag_wp) *Input*
On entry: σ , the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
Constraint: SIGMA > 0.0.
- 11: R – REAL (KIND=nag_wp) *Input*
On entry: r , the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
Constraint: $R \geq 0.0$.
- 12: Q – REAL (KIND=nag_wp) *Input*
On entry: q , the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.
Constraint: $Q \geq 0.0$.

- 13: P(LDP,N) – REAL (KIND=nag_wp) array Output
On exit: the leading $M \times N$ part of the array P contains the computed option prices.
- 14: LDP – INTEGER Input
On entry: the first dimension of the array P as declared in the (sub)program from which S30FAF is called.
Constraint: $LDP \geq M$.
- 15: IFAIL – INTEGER Input/Output
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**
On exit: IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, CALPUT \neq 'C' or 'P'.

IFAIL = 2

On entry, TYPE \neq 'DI', 'DO', 'UI' or 'UO'.

IFAIL = 3

On entry, $M \leq 0$.

IFAIL = 4

On entry, $N \leq 0$.

IFAIL = 5

On entry, $X(i) < z$ or $X(i) > 1/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 6

On entry, $S < z$ or $S > 1.0/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 7

On entry, $H < z$ or $H > 1.0/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 8

On entry, $K < 0.0$.

IFAIL = 9

On entry, $T(i) < z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 10

On entry, $SIGMA \leq 0.0$.

IFAIL = 11

On entry, $R < 0.0$.

IFAIL = 12

On entry, $Q < 0.0$.

IFAIL = 15

S and H are not consistent with TYPE.

IFAIL = 14

On entry, $LDP < M$.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ . This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the *machine precision* (see S15ABF and S15ADF). An accuracy close to *machine precision* can generally be expected.

8 Further Comments

None.

9 Example

This example computes the price of a Down-and-In put with a time to expiry of 6 months, a stock price of 100 and a strike price of 100. The barrier value is 95 and there is a cash rebate of 3, payable on expiry if the option has not been knocked in. The risk-free interest rate is 8% per year, there is an annual dividend return of 4% and the volatility is 30% per year.

9.1 Program Text

```

Program s30fafa
!
!   S30FAF Example Program Text
!
!   Mark 24 Release. NAG Copyright 2012.
!
!   .. Use Statements ..
Use nag_library, Only: nag_wp, s30faf
!   .. Implicit None Statement ..
Implicit None
!   .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!   .. Local Scalars ..
Real (Kind=nag_wp)         :: h, k, q, r, s, sigma
Integer                    :: i, ifail, j, ldp, m, n
Character (1)              :: calput
Character (2)              :: type
!   .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: p(:,,:), t(:,), x(:)
!   .. Executable Statements ..
Write (nout,*) 'S30FAF Example Program Results'
```

```

!      Skip heading in data file
      Read (nin,*)

      Read (nin,*) calput, type
      Read (nin,*) s, h, k, sigma, r, q
      Read (nin,*) m, n

      ldp = m
      Allocate (p(ldp,n),t(n),x(m))

      Read (nin,*)(x(i),i=1,m)
      Read (nin,*)(t(i),i=1,n)

      ifail = 0
      Call s30faf(calput,type,m,n,x,s,h,k,t,sigma,r,q,p,ldp,ifail)

      Write (nout,*)
      Write (nout,*) 'Standard Barrier Option'

      Select Case (calput)
      Case ('C','c')
        Write (nout,*) 'Call :'
      Case ('P','p')
        Write (nout,*) 'Put :'
      End Select

      Select Case (type)
      Case ('DI','di','Di','dI')
        Write (nout,*) 'Down-and-In'
      Case ('DO','do','Do','dO')
        Write (nout,*) 'Down-and-Out'
      Case ('UI','ui','Ui','uI')
        Write (nout,*) 'Up-and-In'
      Case ('UO','uo','Uo','uO')
        Write (nout,*) 'Up-and-Out'
      End Select

      Write (nout,99998) ' Spot      = ', s
      Write (nout,99998) ' Barrier   = ', h
      Write (nout,99998) ' Rebate    = ', k
      Write (nout,99998) ' Volatility = ', sigma
      Write (nout,99998) ' Rate      = ', r
      Write (nout,99998) ' Dividend  = ', q

      Write (nout,*)
      Write (nout,*) '   Strike    Expiry    Option Price'

      Do i = 1, m

        Do j = 1, n
          Write (nout,99999) x(i), t(j), p(i,j)
        End Do

      End Do

99999 Format (1X,2(F9.4,1X),6X,F9.4)
99998 Format (A,1X,F8.4)
      End Program s30faf

```

9.2 Program Data

```

S30FAF Example Program Data
'P' 'DI' : Call = 'C', Put = 'P', Type
100.0 95.0 3.0 0.3 0.08 0.04 : S, H, K, SIGMA, R, Q
1 1 : M, N
100.0 : X(I), I = 1,2,...M
0.5 : T(I), I = 1,2,...N

```

9.3 Program Results

S30FAF Example Program Results

Standard Barrier Option

Put :

Down-and-In

Spot = 100.0000
Barrier = 95.0000
Rebate = 3.0000
Volatility = 0.3000
Rate = 0.0800
Dividend = 0.0400

Strike	Expiry	Option Price
100.0000	0.5000	7.7988
