NAG Library Routine Document

S30BBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S30BBF computes the price of a floating-strike lookback option together with its sensitivities (Greeks).

2 Specification

```
SUBROUTINE S30BBF (CALPUT, M, N, SM, S, T, SIGMA, R, Q, P, LDP, DELTA,          &
                   GAMMA, VEGA, THETA, RHO, CRHO, VANNA, CHARM, SPEED,          &
                   COLOUR, ZOMMA, VOMMA, IFAIL)

INTEGER            M, N, LDP, IFAIL
REAL (KIND=nag_wp) SM(M), S, T(N), SIGMA, R, Q, P(LDP,N), DELTA(LDP,N),         &
                   GAMMA(LDP,N), VEGA(LDP,N), THETA(LDP,N), RHO(LDP,N),         &
                   CRHO(LDP,N), VANNA(LDP,N), CHARM(LDP,N), SPEED(LDP,N),       &
                   COLOUR(LDP,N), ZOMMA(LDP,N), VOMMA(LDP,N)
CHARACTER(1)       CALPUT
```

3 Description

S30BBF computes the price of a floating-strike lookback call or put option, together with the Greeks or sensitivities, which are the partial derivatives of the option price with respect to certain of the other input parameters. A call option of this type confers the right to buy the underlying asset at the lowest price, S_{min}, observed during the lifetime of the contract. A put option gives the holder the right to sell the underlying asset at the maximum price, S_{max}, observed during the lifetime of the contract. Thus, at expiry, the payoff for a call option is $S - S_{\mathrm{min}}$, and for a put, $S_{\mathrm{max}} - S$.

For a given minimum value the price of a floating-strike lookback call with underlying asset price, S, and time to expiry, T, is

$$P_{\mathrm{call}} = Se^{-qT}\Phi(a_1) - S_{\mathrm{min}}e^{-rT}\Phi(a_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{\mathrm{min}}}\right)^{-2b/\sigma^2}\Phi\left(-a_1 + \frac{2b}{\sigma}\sqrt{T}\right) - e^{bT}\Phi(-a_1)\right],$$

where $b = r - q \neq 0$. The volatility, σ, risk-free interest rate, r, and annualised dividend yield, q, are constants.

The corresponding put price is

$$P_{\mathrm{put}} = S_{\mathrm{max}}e^{-rT}\Phi(-a_2) - Se^{-qT}\Phi(-a_1) + Se^{-rT}\frac{\sigma^2}{2b}\left[-\left(\frac{S}{S_{\mathrm{max}}}\right)^{-2b/\sigma^2}\Phi\left(a_1 - \frac{2b}{\sigma}\sqrt{T}\right) + e^{bT}\Phi(a_1)\right].$$

In the above, Φ denotes the cumulative Normal distribution function,

$$\Phi(x) = \int_{-\infty}^{x}\phi(y)dy$$

where ϕ denotes the standard Normal probability density function

$$\phi(y) = \frac{1}{\sqrt{2\pi}}\exp\left(-y^2/2\right)$$

and

$$a_1 = \frac{\ln(S/S_{\mathrm{m}}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$a_2 = a_1 - \sigma\sqrt{T}$$

where S_m is taken to be the minimum price attained by the underlying asset, S_{min}, for a call and the maximum price, S_{max}, for a put.

4 References

Goldman B M, Sosin H B and Gatto M A (1979) Path dependent options: buy at the low, sell at the high *Journal of Finance* **34** 1111–1127

5 Parameters

1: CALPUT – CHARACTER(1) *Input*

On entry: determines whether the option is a call or a put.

CALPUT = 'C'
A call. The holder has a right to buy.

CALPUT = 'P'
A put. The holder has a right to sell.

Constraint: CALPUT = 'C' or 'P'.

2: M – INTEGER *Input*

On entry: the number of minimum or maximum prices to be used.

Constraint: M ≥ 1.

3: N – INTEGER *Input*

On entry: the number of times to expiry to be used.

Constraint: N ≥ 1.

4: SM(M) – REAL (KIND=nag_wp) array *Input*

On entry: SM(i) must contain $S_{\mathrm{min}}(i)$, the ith minimum observed price of the underlying asset when CALPUT = 'C', or $S_{\mathrm{max}}(i)$, the maximum observed price when CALPUT = 'P', for $i = 1, 2, \ldots,$ M.

Constraints:

SM(i) $\geq z$ and SM(i) $\leq 1/z$, where $z =$ X02AMF(), the safe range parameter, for $i = 1, 2, \ldots,$ M;
if CALPUT = 'C', SM(i) $\leq S$, for $i = 1, 2, \ldots,$ M;
if CALPUT = 'P', SM(i) $\geq S$, for $i = 1, 2, \ldots,$ M.

5: S – REAL (KIND=nag_wp) *Input*

On entry: S, the price of the underlying asset.

Constraint: S $\geq z$ and S $\leq 1.0/z$, where $z =$ X02AMF(), the safe range parameter.

6: T(N) – REAL (KIND=nag_wp) array *Input*

On entry: T(i) must contain T_i, the ith time, in years, to expiry, for $i = 1, 2, \ldots,$ N.

Constraint: T(i) $\geq z$, where $z =$ X02AMF(), the safe range parameter, for $i = 1, 2, \ldots,$ N.

7: SIGMA – REAL (KIND=nag_wp) *Input*

On entry: σ, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.

Constraint: SIGMA > 0.0.

8: R – REAL (KIND=nag_wp) *Input*

On entry: the annual risk-free interest rate, r, continuously compounded. Note that a rate of 5% should be entered as 0.05.

Constraint: R ≥ 0.0 and $\text{abs}(\text{R} - \text{Q}) > 10 \times \text{eps} \times \max(\text{abs}(\text{R}), 1)$, where eps = X02AJF(), the ***machine precision***.

9: Q – REAL (KIND=nag_wp) *Input*

On entry: the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.

Constraint: Q ≥ 0.0 and $\text{abs}(\text{R} - \text{Q}) > 10 \times \text{eps} \times \max(\text{abs}(\text{R}), 1)$, where eps = X02AJF(), the ***machine precision***.

10: P(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array P contains the computed option prices.

11: LDP – INTEGER *Input*

On entry: the first dimension of the arrays P, DELTA, GAMMA, VEGA, THETA, RHO, CRHO, VANNA, CHARM, SPEED, COLOUR, ZOMMA and VOMMA as declared in the (sub)program from which S30BBF is called.

Constraint: LDP $\geq$ M.

12: DELTA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array DELTA contains the sensitivity, $\frac{\partial P}{\partial S}$, of the option price to change in the price of the underlying asset.

13: GAMMA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array GAMMA contains the sensitivity, $\frac{\partial^2 P}{\partial S^2}$, of DELTA to change in the price of the underlying asset.

14: VEGA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array VEGA contains the sensitivity, $\frac{\partial P}{\partial \sigma}$, of the option price to change in the volatility of the underlying asset.

15: THETA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array THETA contains the sensitivity, $-\frac{\partial P}{\partial T}$, of the option price to change in the time to expiry of the option.

16: RHO(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array RHO contains the sensitivity, $\frac{\partial P}{\partial r}$, of the option price to change in the annual risk-free interest rate.

17: CRHO(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\text{M} \times \text{N}$ part of the array CRHO containing the sensitivity, $\frac{\partial P}{\partial b}$, of the option price to change in the annual cost of carry rate, b, where $b = r - q$.

18: VANNA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\mathrm{M} \times \mathrm{N}$ part of the array VANNA contains the sensitivity, $\frac{\partial^2 P}{\partial S \partial \sigma}$, of VEGA to change in the price of the underlying asset or, equivalently, the sensitivity of DELTA to change in the volatility of the asset price.

19: CHARM(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\mathrm{M} \times \mathrm{N}$ part of the array CHARM contains the sensitivity, $-\frac{\partial^2 P}{\partial S \partial T}$, of DELTA to change in the time to expiry of the option.

20: SPEED(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\mathrm{M} \times \mathrm{N}$ part of the array SPEED contains the sensitivity, $\frac{\partial^3 P}{\partial S^3}$, of GAMMA to change in the price of the underlying asset.

21: COLOUR(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\mathrm{M} \times \mathrm{N}$ part of the array COLOUR contains the sensitivity, $-\frac{\partial^3 P}{\partial S^2 \partial T}$, of GAMMA to change in the time to expiry of the option.

22: ZOMMA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\mathrm{M} \times \mathrm{N}$ part of the array ZOMMA contains the sensitivity, $\frac{\partial^3 P}{\partial S^2 \partial \sigma}$, of GAMMA to change in the volatility of the underlying asset.

23: VOMMA(LDP,N) – REAL (KIND=nag_wp) array *Output*

On exit: the leading $\mathrm{M} \times \mathrm{N}$ part of the array VOMMA contains the sensitivity, $\frac{\partial^2 P}{\partial \sigma^2}$, of VEGA to change in the volatility of the underlying asset.

24: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

On exit: $\mathrm{IFAIL} = 0$ unless the routine detects an error or a warning has been flagged (see Section 6).

6 Error Indicators and Warnings

If on entry $\mathrm{IFAIL} = 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$\mathrm{IFAIL} = 1$

On entry, $\mathrm{CALPUT} \neq$ 'C' or 'P'.

$\mathrm{IFAIL} = 2$

On entry, $\mathrm{M} \leq 0$.

IFAIL = 3

On entry, $N \leq 0$.

IFAIL = 4

On entry, $SM(i) < z$ or $SM(i) > 1/z$, where $z = X02AMF()$, the safe range parameter,
or CALPUT = 'C' and $SM(i) > S$,
or CALPUT = 'P' and $SM(i) < S$.

IFAIL = 5

On entry, $S < z$ or $S > 1.0/z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 6

On entry, $T(i) < z$, where $z = X02AMF()$, the safe range parameter.

IFAIL = 7

On entry, $SIGMA \leq 0.0$.

IFAIL = 8

On entry, $R < 0.0$.

IFAIL = 9

On entry, $Q < 0.0$.

IFAIL = 11

On entry, LDP < M.

IFAIL = 12

On entry, $abs(R - Q) \leq 10 \times eps \times \max(abs(R), 1)$, where $eps = X02AJF()$, the ***machine precision*** .

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the ***machine precision*** (see S15ABF and S15ADF). An accuracy close to ***machine precision*** can generally be expected.

8 Further Comments

None.

9 Example

This example computes the price of a floating-strike lookback put with a time to expiry of 6 months and a stock price of 87. The maximum price observed so far is 100. The risk-free interest rate is 6% per year and the volatility is 30% per year with an annual dividend return of 4%.

9.1 Program Text

```
    Program s30bbfe

!     S30BBF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, s30bbf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: q, r, s, sigma
      Integer                          :: i, ifail, j, ldp, m, n
      Character (1)                    :: calput
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: charm(:,:), colour(:,:), crho(:,:),  &
                                          delta(:,:), gamma(:,:), p(:,:),      &
                                          rho(:,:), sm(:), speed(:,:), t(:),   &
                                          theta(:,:), vanna(:,:), vega(:,:),   &
                                          vomma(:,:), zomma(:,:)
!     .. Executable Statements ..
      Write (nout,*) 'S30BBF Example Program Results'

!     Skip heading in data file
      Read (nin,*)

      Read (nin,*) calput
      Read (nin,*) s, sigma, r, q
      Read (nin,*) m, n

      ldp = m
      Allocate (charm(ldp,n),colour(ldp,n),crho(ldp,n),delta(ldp,n), &
        gamma(ldp,n),p(ldp,n),rho(ldp,n),sm(m),speed(ldp,n),t(n),theta(ldp,n), &
        vanna(ldp,n),vega(ldp,n),vomma(ldp,n),zomma(ldp,n))

      Read (nin,*)(sm(i),i=1,m)
      Read (nin,*)(t(i),i=1,n)

      ifail = 0
      Call s30bbf(calput,m,n,sm,s,t,sigma,r,q,p,ldp,delta,gamma,vega,theta, &
        rho,crho,vanna,charm,speed,colour,zomma,vomma,ifail)

      Write (nout,*)
      Write (nout,*) 'Floating-Strike Lookback'

      Select Case (calput)
      Case ('C','c')
        Write (nout,*) 'European Call :'
      Case ('P','p')
        Write (nout,*) 'European Put :'
      End Select

      Write (nout,99997) '  Spot       = ', s
      Write (nout,99997) '  Volatility = ', sigma
      Write (nout,99997) '  Rate       = ', r
      Write (nout,99997) '  Dividend   = ', q

      Write (nout,*)

      Do j = 1, n
        Write (nout,*)
        Write (nout,99999) t(j)
        Write (nout,*) 'S-Max/Min     Price     Delta     Gamma' // &
          '      Vega      Theta      Rho      CRho'

        Do i = 1, m
          Write (nout,99998) sm(i), p(i,j), delta(i,j), gamma(i,j), vega(i,j), &
```

```
            theta(i,j), rho(i,j), crho(i,j)
        End Do

        Write (nout,*) 'S-Max/Min     Price     Vanna     Charm' // &
          '     Speed     Colour    Zomma     Vomma'

        Do i = 1, m
          Write (nout,99998) sm(i), p(i,j), vanna(i,j), charm(i,j), &
            speed(i,j), colour(i,j), zomma(i,j), vomma(i,j)
        End Do

      End Do

99999 Format (1X,'Time to Expiry : ',1X,F8.4)
99998 Format (8(1X,F9.4))
99997 Format (A,1X,F8.4)
    End Program s30bbfe
```

9.2 Program Data

```
S30BBF Example Program Data
 'P'                    : Call = 'C', Put = 'P'
 87.0 0.3 0.06 0.04     : S, SIGMA, R, Q
 1 1                    : M, N
 100.0                  : SM(I), I = 1,2,...M
 0.5                    : T(I), I = 1,2,...N
```

9.3 Program Results

```
 S30BBF Example Program Results

 Floating-Strike Lookback
 European Put :
  Spot       =   87.0000
  Volatility =    0.3000
  Rate       =    0.0600
  Dividend   =    0.0400


 Time to Expiry :    0.5000
 S-Max/Min     Price     Delta     Gamma      Vega     Theta       Rho      CRho
  100.0000   18.3530   -0.3560    0.0391   45.5353  -11.6139  -32.8139  -23.6374
 S-Max/Min     Price     Vanna     Charm     Speed    Colour     Zomma     Vomma
  100.0000   18.3530    1.9141   -0.6199    0.0007    0.0221   -0.0648   76.1292
```