

NAG Library Routine Document

F08WEF (DGGHRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08WEF (DGGHRD) reduces a pair of real matrices (A, B) , where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations.

2 Specification

```
SUBROUTINE F08WEF (COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,      &
                  LDZ, INFO)
INTEGER          N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
REAL (KIND=nag_wp) A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
CHARACTER(1)     COMPQ, COMPZ
```

The routine may be called by its LAPACK name *dgghrd*.

3 Description

F08WEF (DGGHRD) is the third step in the solution of the real generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using F08WHF (DGGBAL). In the second step, matrix B is reduced to upper triangular form using the QR factorization routine F08AEF (DGEQRF) and this orthogonal transformation Q is applied to matrix A by calling F08AGF (DORMQR).

F08WEF (DGGHRD) reduces a pair of real matrices (A, B) , where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations. This two-sided transformation is of the form

$$\begin{aligned} Q^T A Z &= H \\ Q^T B Z &= T \end{aligned}$$

where H is an upper Hessenberg matrix, T is an upper triangular matrix and Q and Z are orthogonal matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1 , so that

$$\begin{aligned} Q_1 A Z_1^T &= (Q_1 Q) H (Z_1 Z)^T, \\ Q_1 B Z_1^T &= (Q_1 Q) T (Z_1 Z)^T. \end{aligned}$$

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

5 Parameters

- 1: COMPQ – CHARACTER(1) *Input*
On entry: specifies the form of the computed orthogonal matrix Q .
 COMPQ = 'N'
 Do not compute Q .
 COMPQ = 'I'
 The orthogonal matrix Q is returned.
 COMPQ = 'V'
 Q must contain an orthogonal matrix Q_1 , and the product Q_1Q is returned.
Constraint: COMPQ = 'N', 'I' or 'V'.
- 2: COMPZ – CHARACTER(1) *Input*
On entry: specifies the form of the computed orthogonal matrix Z .
 COMPZ = 'N'
 Do not compute Z .
 COMPZ = 'I'
 The orthogonal matrix Z is returned.
 COMPZ = 'V'
 Z must contain an orthogonal matrix Z_1 , and the product Z_1Z is returned.
Constraint: COMPZ = 'N', 'I' or 'V'.
- 3: N – INTEGER *Input*
On entry: n , the order of the matrices A and B .
Constraint: $N \geq 0$.
- 4: ILO – INTEGER *Input*
 5: IHI – INTEGER *Input*
On entry: i_{lo} and i_{hi} as determined by a previous call to F08WHF (DGGBAL). Otherwise, they should be set to 1 and n , respectively.
Constraints:
 if $N > 0$, $1 \leq ILO \leq IHI \leq N$;
 if $N = 0$, $ILO = 1$ and $IHI = 0$.
- 6: A(LDA,*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the matrix A of the matrix pair (A, B) . Usually, this is the matrix A returned by F08AGF (DORMQR).
On exit: A is overwritten by the upper Hessenberg matrix H .
- 7: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F08WEF (DGGHRD) is called.
Constraint: $LDA \geq \max(1, N)$.

- 8: B(LDB,*) – REAL (KIND=nag_wp) array Input/Output
Note: the second dimension of the array B must be at least $\max(1, N)$.
On entry: the upper triangular matrix B of the matrix pair (A, B) . Usually, this is the matrix B returned by the QR factorization routine F08AEF (DGEQRF).
On exit: B is overwritten by the upper triangular matrix T .
- 9: LDB – INTEGER Input
On entry: the first dimension of the array B as declared in the (sub)program from which F08WEF (DGGHRD) is called.
Constraint: $LDB \geq \max(1, N)$.
- 10: Q(LDQ,*) – REAL (KIND=nag_wp) array Input/Output
Note: the second dimension of the array Q must be at least $\max(1, N)$ if COMPQ = 'I' or 'V' and at least 1 if COMPQ = 'N'.
On entry: if COMPQ = 'V', Q must contain an orthogonal matrix Q_1 .
If COMPQ = 'N', Q is not referenced.
On exit: if COMPQ = 'I', Q contains the orthogonal matrix Q .
If COMPQ = 'V', Q is overwritten by Q_1Q .
- 11: LDQ – INTEGER Input
On entry: the first dimension of the array Q as declared in the (sub)program from which F08WEF (DGGHRD) is called.
Constraints:
if COMPQ = 'I' or 'V', $LDQ \geq \max(1, N)$;
if COMPQ = 'N', $LDQ \geq 1$.
- 12: Z(LDZ,*) – REAL (KIND=nag_wp) array Input/Output
Note: the second dimension of the array Z must be at least $\max(1, N)$ if COMPZ = 'I' or 'V' and at least 1 if COMPZ = 'N'.
On entry: if COMPZ = 'V', Z must contain an orthogonal matrix Z_1 .
If COMPZ = 'N', Z is not referenced.
On exit: if COMPZ = 'I', Z contains the orthogonal matrix Z .
If COMPZ = 'V', Z is overwritten by Z_1Z .
- 13: LDZ – INTEGER Input
On entry: the first dimension of the array Z as declared in the (sub)program from which F08WEF (DGGHRD) is called.
Constraints:
if COMPZ = 'I' or 'V', $LDZ \geq \max(1, N)$;
if COMPZ = 'N', $LDZ \geq 1$.
- 14: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The reduction to the generalized Hessenberg form is implemented using orthogonal transformations which are backward stable.

8 Further Comments

This routine is usually followed by F08XEF (DHGEQZ) which implements the QZ algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The complex analogue of this routine is F08WSF (ZGGHRD).

9 Example

See Section 9 in F08XEF (DHGEQZ) and F08YKF (DTGEVC).
