NAG Library Routine Document

F08QVF (ZTRSYL)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08QVF (ZTRSYL) solves the complex triangular Sylvester matrix equation.

2 Specification

```
SUBROUTINE F08QVF (TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC, SCAL,      &
                   INFO)

INTEGER                  ISGN, M, N, LDA, LDB, LDC, INFO
REAL (KIND=nag_wp)       SCAL
COMPLEX (KIND=nag_wp) A(LDA,*), B(LDB,*), C(LDC,*)
CHARACTER(1)             TRANA, TRANB
```

The routine may be called by its LAPACK name ***ztrsyl***.

3 Description

F08QVF (ZTRSYL) solves the complex Sylvester matrix equation

$$\mathrm{op}(A)X \pm X\,\mathrm{op}(B) = \alpha C,$$

where $\mathrm{op}(A) = A$ or A^{H}, and the matrices A and B are upper triangular; α is a scale factor (≤ 1) determined by the routine to avoid overflow in X; A is m by m and B is n by n while the right-hand side matrix C and the solution matrix X are both m by n. The matrix X is obtained by a straightforward process of back-substitution (see Golub and Van Loan (1996)).

Note that the equation has a unique solution if and only if $\alpha_i \pm \beta_j \neq 0$, where $\{\alpha_i\}$ and $\{\beta_j\}$ are the eigenvalues of A and B respectively and the sign ($+$ or $-$) is the same as that used in the equation to be solved.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1992) Perturbation theory and backward error for $AX - XB = C$ *Numerical Analysis Report* University of Manchester

5 Parameters

1: TRANA – CHARACTER(1) *Input*

On entry: specifies the option $\mathrm{op}(A)$.

TRANA = 'N'
$\mathrm{op}(A) = A$.

TRANA = 'C'
$\mathrm{op}(A) = A^{\mathrm{H}}$.

Constraint: TRANA = 'N' or 'C'.

2: TRANB – CHARACTER(1) *Input*

On entry: specifies the option $\mathrm{op}(B)$.

TRANB = 'N'
$\mathrm{op}(B) = B$.

TRANB = 'C'
$\mathrm{op}(B) = B^{\mathrm{H}}$.

Constraint: TRANB = 'N' or 'C'.

3: ISGN – INTEGER *Input*

On entry: indicates the form of the Sylvester equation.

ISGN $= +1$
The equation is of the form $\mathrm{op}(A)X + X\,\mathrm{op}(B) = \alpha C$.

ISGN $= -1$
The equation is of the form $\mathrm{op}(A)X - X\,\mathrm{op}(B) = \alpha C$.

Constraint: ISGN $= +1$ or -1.

4: M – INTEGER *Input*

On entry: m, the order of the matrix A, and the number of rows in the matrices X and C.

Constraint: M ≥ 0.

5: N – INTEGER *Input*

On entry: n, the order of the matrix B, and the number of columns in the matrices X and C.

Constraint: N ≥ 0.

6: A(LDA,$*$) – COMPLEX (KIND=nag_wp) array *Input*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{M})$.

On entry: the m by m upper triangular matrix A.

7: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08QVF (ZTRSYL) is called.

Constraint: LDA $\geq \max(1, \mathrm{M})$.

8: B(LDB,$*$) – COMPLEX (KIND=nag_wp) array *Input*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{N})$.

On entry: the n by n upper triangular matrix B.

9: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F08QVF (ZTRSYL) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

10: C(LDC,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array C must be at least $\max(1, \mathrm{N})$.

On entry: the m by n right-hand side matrix C.

On exit: C is overwritten by the solution matrix X.

11: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08QVF (ZTRSYL) is called.

Constraint: LDC $\geq \max(1, \mathrm{M})$.

12: SCAL – REAL (KIND=nag_wp) *Output*

On exit: the value of the scale factor α.

13: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO $= 1$

A and B have common or close eigenvalues, perturbed values of which were used to solve the equation.

7 Accuracy

Consider the equation $AX - XB = C$. (To apply the remarks to the equation $AX + XB = C$, simply replace B by $-B$.)

Let $\tilde{X}$ be the computed solution and R the residual matrix:

$$R = C - \left(A\tilde{X} - \tilde{X}B\right).$$

Then the residual is always small:

$$\|R\|_F = O(\epsilon)\left(\|A\|_F + \|B\|_F\right)\left\|\tilde{X}\right\|_F.$$

However, $\tilde{X}$ is **not** necessarily the exact solution of a slightly perturbed equation; in other words, the solution is not backwards stable.

For the forward error, the following bound holds:

$$\left\|\tilde{X} - X\right\|_F \leq \frac{\|R\|_F}{sep(A, B)}$$

but this may be a considerable over estimate. See Golub and Van Loan (1996) for a definition of $sep(A, B)$, and Higham (1992) for further details.

These remarks also apply to the solution of a general Sylvester equation, as described in Section 8.

8 Further Comments

The total number of real floating point operations is approximately $4mn(m + n)$.

To solve the **general** complex Sylvester equation

$$AX \pm XB = C$$

where A and B are general matrices, A and B must first be reduced to Schur form (by calling F08PNF (ZGEES), for example):

$$A = Q_1 \tilde{A} Q_1^{\mathrm{H}} \quad \text{and} \quad B = Q_2 \tilde{B} Q_2^{\mathrm{H}}$$

where $\tilde{A}$ and $\tilde{B}$ are upper triangular and Q_1 and Q_2 are unitary. The original equation may then be transformed to:

$$\tilde{A}\tilde{X} \pm \tilde{X}\tilde{B} = \tilde{C}$$

where $\tilde{X} = Q_1^H X Q_2$ and $\tilde{C} = Q_1^H C Q_2$. $\tilde{C}$ may be computed by matrix multiplication; F08QVF (ZTRSYL) may be used to solve the transformed equation; and the solution to the original equation can be obtained as $X = Q_1 \tilde{X} Q_2^H$.

The real analogue of this routine is F08QHF (DTRSYL).

9 Example

This example solves the Sylvester equation $AX + XB = C$, where

$$A = \begin{pmatrix} -6.00 - 7.00i & 0.36 - 0.36i & -0.19 + 0.48i & 0.88 - 0.25i \\ 0.00 + 0.00i & -5.00 + 2.00i & -0.03 - 0.72i & -0.23 + 0.13i \\ 0.00 + 0.00i & 0.00 + 0.00i & 8.00 - 1.00i & 0.94 + 0.53i \\ 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i & 3.00 - 4.00i \end{pmatrix},$$

$$B = \begin{pmatrix} 0.50 - 0.20i & -0.29 - 0.16i & -0.37 + 0.84i & -0.55 + 0.73i \\ 0.00 + 0.00i & -0.40 + 0.90i & 0.06 + 0.22i & -0.43 + 0.17i \\ 0.00 + 0.00i & 0.00 + 0.00i & -0.90 - 0.10i & -0.89 - 0.42i \\ 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i & 0.30 - 0.70i \end{pmatrix}$$

and

$$C = \begin{pmatrix} 0.63 + 0.35i & 0.45 - 0.56i & 0.08 - 0.14i & -0.17 - 0.23i \\ -0.17 + 0.09i & -0.07 - 0.31i & 0.27 - 0.54i & 0.35 + 1.21i \\ -0.93 - 0.44i & -0.33 - 0.35i & 0.41 - 0.03i & 0.57 + 0.84i \\ 0.54 + 0.25i & -0.62 - 0.05i & -0.52 - 0.13i & 0.11 - 0.08i \end{pmatrix}.$$

9.1 Program Text

```
    Program f08qvfe

!     F08QVF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, ztrsyl
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)                :: scale
      Integer                           :: i, ifail, info, lda, ldb, ldc, m, n
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: a(:,:), b(:,:), c(:,:)
      Character (1)                     :: clabs(1), rlabs(1)
!     .. Executable Statements ..
      Write (nout,*) 'F08QVF Example Program Results'
      Write (nout,*)
      Flush (nout)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) m, n
      lda = m
      ldb = n
      ldc = m
      Allocate (a(lda,m),b(ldb,n),c(ldc,n))

!     Read A, B and C from data file
```

```
      Read (nin,*)(a(i,1:m),i=1,m)
      Read (nin,*)(b(i,1:n),i=1,n)
      Read (nin,*)(c(i,1:n),i=1,m)

!     Solve the Sylvester equation A*X + X*B = C for X
!     The NAG name equivalent of ztrsyl is f08qvf
      Call ztrsyl('No transpose','No transpose',1,m,n,a,lda,b,ldb,c,ldc,scale, &
        info)
      If (info>=1) Then
        Write (nout,99999)
        Write (nout,*)
        Flush (nout)
      End If

!     Print X
!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04dbf('General',' ',m,n,c,ldc,'Bracketed','F8.4', &
        'Solution Matrix','I',rlabs,'I',clabs,80,0,ifail)

99999 Format (/' A and B have common or very close eigenvalues.'/' Pe', &
        'rturbed values were used to solve the equations')
    End Program f08qvfe
```

9.2 Program Data

```
F08QVF Example Program Data
  4  4                                                        :Values of M and N
 (-6.00,-7.00) ( 0.36,-0.36) (-0.19, 0.48) ( 0.88,-0.25)
 ( 0.00, 0.00) (-5.00, 2.00) (-0.03,-0.72) (-0.23, 0.13)
 ( 0.00, 0.00) ( 0.00, 0.00) ( 8.00,-1.00) ( 0.94, 0.53)
 ( 0.00, 0.00) ( 0.00, 0.00) ( 0.00, 0.00) ( 3.00,-4.00)   :End of matrix A
 ( 0.50,-0.20) (-0.29,-0.16) (-0.37, 0.84) (-0.55, 0.73)
 ( 0.00, 0.00) (-0.40, 0.90) ( 0.06, 0.22) (-0.43, 0.17)
 ( 0.00, 0.00) ( 0.00, 0.00) (-0.90,-0.10) (-0.89,-0.42)
 ( 0.00, 0.00) ( 0.00, 0.00) ( 0.00, 0.00) ( 0.30,-0.70)   :End of matrix B
 ( 0.63, 0.35) ( 0.45,-0.56) ( 0.08,-0.14) (-0.17,-0.23)
 (-0.17, 0.09) (-0.07,-0.31) ( 0.27,-0.54) ( 0.35, 1.21)
 (-0.93,-0.44) (-0.33,-0.35) ( 0.41,-0.03) ( 0.57, 0.84)
 ( 0.54, 0.25) (-0.62,-0.05) (-0.52,-0.13) ( 0.11,-0.08)   :End of matrix C
```

9.3 Program Results

```
 F08QVF Example Program Results

 Solution Matrix
                     1                    2                    3
 1  ( -0.0611,  0.0249) ( -0.0031,  0.0798) ( -0.0062,  0.0165)
 2  (  0.0215, -0.0003) ( -0.0155,  0.0570) ( -0.0665,  0.0718)
 3  ( -0.0949, -0.0785) ( -0.0415, -0.0298) (  0.0357,  0.0244)
 4  (  0.0281,  0.1052) ( -0.0970, -0.1214) ( -0.0271, -0.0940)

                     4
 1  (  0.0054, -0.0063)
 2  (  0.0290, -0.2636)
 3  (  0.0284,  0.1108)
 4  (  0.0402,  0.0048)
```