NAG Library Routine Document

F08QHF (DTRSYL)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08QHF (DTRSYL) solves the real quasi-triangular Sylvester matrix equation.

2 Specification

```
SUBROUTINE F08QHF (TRANA, TRANB, ISGN, M, N, A, LDA, B, LDB, C, LDC, SCALE,     &
                   INFO)

INTEGER            ISGN, M, N, LDA, LDB, LDC, INFO
REAL (KIND=nag_wp) A(LDA,*), B(LDB,*), C(LDC,*), SCALE
CHARACTER(1)       TRANA, TRANB
```

The routine may be called by its LAPACK name ***dtrsyl***.

3 Description

F08QHF (DTRSYL) solves the real Sylvester matrix equation

$$\mathrm{op}(A)X \pm X\,\mathrm{op}(B) = \alpha C,$$

where $\mathrm{op}(A) = A$ or A^{T}, and the matrices A and B are upper quasi-triangular matrices in canonical Schur form (as returned by F08PEF (DHSEQR)); α is a scale factor (≤ 1) determined by the routine to avoid overflow in X; A is m by m and B is n by n while the right-hand side matrix C and the solution matrix X are both m by n. The matrix X is obtained by a straightforward process of back-substitution (see Golub and Van Loan (1996)).

Note that the equation has a unique solution if and only if $\alpha_i \pm \beta_j \neq 0$, where $\{\alpha_i\}$ and $\{\beta_j\}$ are the eigenvalues of A and B respectively and the sign ($+$ or $-$) is the same as that used in the equation to be solved.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1992) Perturbation theory and backward error for $AX - XB = C$ *Numerical Analysis Report* University of Manchester

5 Parameters

1: TRANA – CHARACTER(1) *Input*

On entry: specifies the option $\mathrm{op}(A)$.

TRANA = 'N'
$\mathrm{op}(A) = A$.

TRANA = 'T' or 'C'
$\mathrm{op}(A) = A^{\mathrm{T}}$.

Constraint: TRANA = 'N', 'T' or 'C'.

2: TRANB – CHARACTER(1) *Input*

On entry: specifies the option $\text{op}(B)$.

TRANB = 'N'
$\text{op}(B) = B$.

TRANB = 'T' or 'C'
$\text{op}(B) = B^{\mathrm{T}}$.

Constraint: TRANB = 'N', 'T' or 'C'.

3: ISGN – INTEGER *Input*

On entry: indicates the form of the Sylvester equation.

ISGN $= +1$
The equation is of the form $\text{op}(A)X + X\,\text{op}(B) = \alpha C$.

ISGN $= -1$
The equation is of the form $\text{op}(A)X - X\,\text{op}(B) = \alpha C$.

Constraint: ISGN $= +1$ or -1.

4: M – INTEGER *Input*

On entry: m, the order of the matrix A, and the number of rows in the matrices X and C.

Constraint: M ≥ 0.

5: N – INTEGER *Input*

On entry: n, the order of the matrix B, and the number of columns in the matrices X and C.

Constraint: N ≥ 0.

6: A(LDA,$*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array A must be at least $\max(1,\text{M})$.

On entry: the m by m upper quasi-triangular matrix A in canonical Schur form, as returned by F08PEF (DHSEQR).

7: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08QHF (DTRSYL) is called.

Constraint: LDA $\geq \max(1,\text{M})$.

8: B(LDB,$*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array B must be at least $\max(1,\text{N})$.

On entry: the n by n upper quasi-triangular matrix B in canonical Schur form, as returned by F08PEF (DHSEQR).

9: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F08QHF (DTRSYL) is called.

Constraint: LDB $\geq \max(1,\text{N})$.

10: C(LDC,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array C must be at least $\max(1,\text{N})$.

On entry: the m by n right-hand side matrix C.

On exit: C is overwritten by the solution matrix X.

11: LDC – INTEGER *Input*

On entry: the first dimension of the array C as declared in the (sub)program from which F08QHF (DTRSYL) is called.

Constraint: LDC $\geq \max(1, \mathrm{M})$.

12: SCALE – REAL (KIND=nag_wp) *Output*

On exit: the value of the scale factor α.

13: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO $= 1$

A and B have common or close eigenvalues, perturbed values of which were used to solve the equation.

7 Accuracy

Consider the equation $AX - XB = C$. (To apply the remarks to the equation $AX + XB = C$, simply replace B by $-B$.)

Let $\tilde{X}$ be the computed solution and R the residual matrix:

$$R = C - \left(A\tilde{X} - \tilde{X}B\right).$$

Then the residual is always small:

$$\|R\|_F = O(\epsilon)\left(\|A\|_F + \|B\|_F\right)\left\|\tilde{X}\right\|_F.$$

However, $\tilde{X}$ is **not** necessarily the exact solution of a slightly perturbed equation; in other words, the solution is not backwards stable.

For the forward error, the following bound holds:

$$\left\|\tilde{X} - X\right\|_F \leq \frac{\|R\|_F}{sep(A, B)}$$

but this may be a considerable over estimate. See Golub and Van Loan (1996) for a definition of $sep(A, B)$, and Higham (1992) for further details.

These remarks also apply to the solution of a general Sylvester equation, as described in Section 8.

8 Further Comments

The total number of floating point operations is approximately $mn(m + n)$.

To solve the **general** real Sylvester equation

$$AX \pm XB = C$$

where A and B are general nonsymmetric matrices, A and B must first be reduced to Schur form (by calling F08PAF (DGEES), for example):

$$A = Q_1 \tilde{A} Q_1^{\mathrm{T}} \qquad \text{and} \qquad B = Q_2 \tilde{B} Q_2^{\mathrm{T}}$$

where $\tilde{A}$ and $\tilde{B}$ are upper quasi-triangular and Q_1 and Q_2 are orthogonal. The original equation may then be transformed to:

$$\tilde{A}\tilde{X} \pm \tilde{X}\tilde{B} = \tilde{C}$$

where $\tilde{X} = Q_1^{\mathrm{T}} X Q_2$ and $\tilde{C} = Q_1^{\mathrm{T}} C Q_2$. $\tilde{C}$ may be computed by matrix multiplication; F08QHF (DTRSYL) may be used to solve the transformed equation; and the solution to the original equation can be obtained as $X = Q_1 \tilde{X} Q_2^{\mathrm{T}}$.

The complex analogue of this routine is F08QVF (ZTRSYL).

9 Example

This example solves the Sylvester equation $AX + XB = C$, where

$$A = \begin{pmatrix} 0.10 & 0.50 & 0.68 & -0.21 \\ -0.50 & 0.10 & -0.24 & 0.67 \\ 0.00 & 0.00 & 0.19 & -0.35 \\ 0.00 & 0.00 & 0.00 & -0.72 \end{pmatrix},$$

$$B = \begin{pmatrix} -0.99 & -0.17 & 0.39 & 0.58 \\ 0.00 & 0.48 & -0.84 & -0.15 \\ 0.00 & 0.00 & 0.75 & 0.25 \\ 0.00 & 0.00 & -0.25 & 0.75 \end{pmatrix}$$

and

$$C = \begin{pmatrix} 0.63 & -0.56 & 0.08 & -0.23 \\ -0.45 & -0.31 & 0.27 & 1.21 \\ 0.20 & -0.35 & 0.41 & 0.84 \\ 0.49 & -0.05 & -0.52 & -0.08 \end{pmatrix}.$$

9.1 Program Text

```
    Program f08qhfe

!     F08QHF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dtrsyl, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: scale
      Integer                          :: i, ifail, info, lda, ldb, ldc, m, n, &
                                          sign
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable  :: a(:,:), b(:,:), c(:,:)
!     .. Executable Statements ..
      Write (nout,*) 'F08QHF Example Program Results'
      Write (nout,*)
      Flush (nout)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) m, n
      lda = m
      ldb = n
```

```
      ldc = m
      sign = 1
      Allocate (a(lda,m),b(ldb,n),c(ldc,n))

!     Read A, B and C from data file

      Read (nin,*)(a(i,1:m),i=1,m)
      Read (nin,*)(b(i,1:n),i=1,n)
      Read (nin,*)(c(i,1:n),i=1,m)

!     Solve the Sylvester equation A*X + X*B = C for X
!     The NAG name equivalent of dtrsyl is f08qhf
      Call dtrsyl('No transpose','No transpose',sign,m,n,a,lda,b,ldb,c,ldc, &
        scale,info)
      If (info==1) Then
        Write (nout,99999)
        Write (nout,*)
      End If

      Flush (nout)

!     Print X
!     ifail: behaviour on error exit
!             =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04caf('General',' ',m,n,c,ldc,'Solution Matrix',ifail)

99999 Format (/' A and B have common or very close eigenvalues.'/' Pe', &
        'rturbed values were used to solve the equations')
    End Program f08qhfe
```

9.2 Program Data

```
F08QHF Example Program Data
  4  4                              :Values of M and N
  0.10   0.50   0.68  -0.21
 -0.50   0.10  -0.24   0.67
  0.00   0.00   0.19  -0.35
  0.00   0.00   0.00  -0.72   :End of matrix A
 -0.99  -0.17   0.39   0.58
  0.00   0.48  -0.84  -0.15
  0.00   0.00   0.75   0.25
  0.00   0.00  -0.25   0.75   :End of matrix B
  0.63  -0.56   0.08  -0.23
 -0.45  -0.31   0.27   1.21
  0.20  -0.35   0.41   0.84
  0.49  -0.05  -0.52  -0.08   :End of matrix C
```

9.3 Program Results

```
 F08QHF Example Program Results

 Solution Matrix
          1        2        3        4
 1  -0.4209   0.1764   0.2438  -0.9577
 2   0.5600  -0.8337  -0.7221   0.5386
 3  -0.1246  -0.3392   0.6221   0.8691
 4  -0.2865   0.4113   0.5535   0.3174
```