NAG Library Routine Document

F08JAF (DSTEV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08JAF (DSTEV) computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric tridiagonal matrix A.

2 Specification

```
SUBROUTINE F08JAF (JOBZ, N, D, E, Z, LDZ, WORK, INFO)

INTEGER             N, LDZ, INFO
REAL (KIND=nag_wp) D(*), E(*), Z(LDZ,*), WORK(*)
CHARACTER(1)        JOBZ
```

The routine may be called by its LAPACK name ***dstev***.

3 Description

F08JAF (DSTEV) computes all the eigenvalues and, optionally, all the eigenvectors of A using a combination of the QR and QL algorithms, with an implicit shift.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: JOBZ – CHARACTER(1) *Input*

On entry: indicates whether eigenvectors are computed.

JOBZ = 'N'
Only eigenvalues are computed.

JOBZ = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: JOBZ = 'N' or 'V'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix.

Constraint: $\mathrm{N} \geq 0$.

3: D($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array D must be at least $\max(1, \mathrm{N})$.

On entry: the n diagonal elements of the tridiagonal matrix A.

On exit: if INFO $= 0$, the eigenvalues in ascending order.

4: E($*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the dimension of the array E must be at least $\max(1, \mathrm{N} - 1)$.

On entry: the $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

On exit: the contents of E are destroyed.

5: Z(LDZ,$*$) – REAL (KIND=nag_wp) array *Output*

Note: the second dimension of the array Z must be at least $\max(1, \mathrm{N})$ if JOBZ = 'V', and at least 1 otherwise.

On exit: if JOBZ = 'V', then if INFO $= 0$, Z contains the orthonormal eigenvectors of the matrix A, with the ith column of Z holding the eigenvector associated with $\mathrm{D}(i)$.

If JOBZ = 'N', Z is not referenced.

6: LDZ – INTEGER *Input*

On entry: the first dimension of the array Z as declared in the (sub)program from which F08JAF (DSTEV) is called.

Constraints:

if JOBZ = 'V', LDZ $\geq \max(1, \mathrm{N})$;
otherwise LDZ ≥ 1.

7: WORK($*$) – REAL (KIND=nag_wp) array *Workspace*

Note: the dimension of the array WORK must be at least $\max(1, 2 \times \mathrm{N} - 2)$.

On exit: if JOBZ = 'N', WORK is not referenced.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the algorithm failed to converge; i off-diagonal elements of E did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating point operations is proportional to n^2 if JOBZ = 'N' and is proportional to n^3 if JOBZ = 'V'.

9 Example

This example finds all the eigenvalues and eigenvectors of the symmetric tridiagonal matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 2 & 0 \\ 0 & 2 & 9 & 3 \\ 0 & 0 & 3 & 16 \end{pmatrix},$$

together with approximate error bounds for the computed eigenvalues and eigenvectors.

9.1 Program Text

```
    Program f08jafe

!     F08JAF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: ddisna, dstev, nag_wp, x02ajf, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter              :: nin = 5, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)              :: eerrbd, eps
      Integer                         :: i, ifail, info, ldz, n
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: d(:), e(:), rcondz(:), work(:),      &
                                         z(:,:), zerrbd(:)
!     .. Intrinsic Procedures ..
      Intrinsic                       :: abs, max
!     .. Executable Statements ..
      Write (nout,*) 'F08JAF Example Program Results'
      Write (nout,*)
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      ldz = n
      Allocate (d(n),e(n-1),rcondz(n),work(2*n-2),z(ldz,n),zerrbd(n))

!     Read the diagonal and off-diagonal elements of the matrix A
!     from data file

      Read (nin,*) d(1:n)
      Read (nin,*) e(1:n-1)

!     Solve the symmetric tridiagonal eigenvalue problem
!     The NAG name equivalent of dstev is f08jaf
      Call dstev('Vectors',n,d,e,z,ldz,work,info)

      If (info==0) Then

!       Print solution

        Write (nout,*) 'Eigenvalues'
        Write (nout,99999) d(1:n)
        Flush (nout)

!       Standardize the eigenvectors so that first elements are non-negative.
        Do i = 1, n
          If (z(1,i)<0.0_nag_wp) z(1:n,i) = -z(1:n,i)
        End Do

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf('General',' ',n,n,z,ldz,'Eigenvectors',ifail)
```

```
!       Get the machine precision, EPS and compute the approximate
!       error bound for the computed eigenvalues.  Note that for
!       the 2-norm, max( abs(D(i)) ) = norm(A), and since the
!       eigenvalues are returned in ascending order
!       max( abs(D(i)) ) = max( abs(D(1)), abs(D(n)))

        eps = x02ajf()
        eerrbd = eps*max(abs(d(1)),abs(d(n)))

!       Call DDISNA (F08FLF) to estimate reciprocal condition
!       numbers for the eigenvectors
        Call ddisna('Eigenvectors',n,n,d,rcondz,info)

!       Compute the error estimates for the eigenvectors

        Do i = 1, n
          zerrbd(i) = eerrbd/rcondz(i)
        End Do

!       Print the approximate error bounds for the eigenvalues
!       and vectors

        Write (nout,*)
        Write (nout,*) 'Error estimate for the eigenvalues'
        Write (nout,99998) eerrbd
        Write (nout,*)
        Write (nout,*) 'Error estimates for the eigenvectors'
        Write (nout,99998) zerrbd(1:n)
      Else
        Write (nout,99997) 'Failure in DSTEV. INFO =', info
      End If

99999 Format (3X,(8F8.4))
99998 Format (4X,1P,6E11.1)
99997 Format (1X,A,I4)
    End Program f08jafe
```

9.2 Program Data

```
F08JAF Example Program Data

  4                         :Value of N

  1.0  4.0  9.0  16.0  :End of diagonal elements
  1.0  2.0  3.0        :End of off-diagonal elements
```

9.3 Program Results

```
 F08JAF Example Program Results

 Eigenvalues
     0.6476  3.5470  8.6578 17.1477
 Eigenvectors
          1       2       3       4
 1   0.9396  0.3388  0.0494  0.0034
 2  -0.3311  0.8628  0.3781  0.0545
 3   0.0853 -0.3648  0.8558  0.3568
 4  -0.0167  0.0879 -0.3497  0.9326

 Error estimate for the eigenvalues
        1.9E-15

 Error estimates for the eigenvectors
        6.6E-16    6.6E-16    3.7E-16    2.2E-16
```