NAG Library Routine Document

F07MSF (ZHETRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07MSF (ZHETRS) solves a complex Hermitian indefinite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07MRF (ZHETRF).

2 Specification

```
SUBROUTINE F07MSF (UPLO, N, NRHS, A, LDA, IPIV, B, LDB, INFO)

INTEGER               N, NRHS, LDA, IPIV(*), LDB, INFO
COMPLEX (KIND=nag_wp) A(LDA,*), B(LDB,*)
CHARACTER(1)          UPLO
```

The routine may be called by its LAPACK name ***zhetrs***.

3 Description

F07MSF (ZHETRS) is used to solve a complex Hermitian indefinite system of linear equations $AX = B$, this routine must be preceded by a call to F07MRF (ZHETRF) which computes the Bunch–Kaufman factorization of A.

If UPLO = 'U', $A = PUDU^{\mathrm{H}}P^{\mathrm{T}}$, where P is a permutation matrix, U is an upper triangular matrix and D is an Hermitian block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^{\mathrm{H}}P^{\mathrm{T}}X = Y$.

If UPLO = 'L', $A = PLDL^{\mathrm{H}}P^{\mathrm{T}}$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^{\mathrm{H}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'
$A = PUDU^{\mathrm{H}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
$A = PLDL^{\mathrm{H}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $N \geq 0$.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: $NRHS \geq 0$.

4: A(LDA,$*$) – COMPLEX (KIND=nag_wp) array *Input*

Note: the second dimension of the array A must be at least $\max(1, N)$.

On entry: details of the factorization of A, as returned by F07MRF (ZHETRF).

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07MSF (ZHETRS) is called.

Constraint: $LDA \geq \max(1, N)$.

6: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, N)$.

On entry: details of the interchanges and the block structure of D, as returned by F07MRF (ZHETRF).

7: B(LDB,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07MSF (ZHETRS) is called.

Constraint: $LDB \geq \max(1, N)$.

9: INFO – INTEGER *Output*

On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If $INFO = -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if UPLO = 'U', $|E| \leq c(n)\epsilon P|U||D||U^{\mathrm{H}}|P^{\mathrm{T}}$;

if UPLO = 'L', $|E| \leq c(n)\epsilon P|L||D||L^{\mathrm{H}}|P^{\mathrm{T}}$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07MVF (ZHERFS), and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling F07MUF (ZHECON).

8 Further Comments

The total number of real floating point operations is approximately $8n^2r$.

This routine may be followed by a call to F07MVF (ZHERFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07MEF (DSYTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.36+0.00i & 1.58+0.90i & 2.21-0.21i & 3.91+1.50i \\ 1.58-0.90i & -8.87+0.00i & -1.84-0.03i & -1.78+1.18i \\ 2.21+0.21i & -1.84+0.03i & -4.63+0.00i & 0.11+0.11i \\ 3.91-1.50i & -1.78-1.18i & 0.11-0.11i & -1.84+0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 7.79+\ 5.48i & -35.39+18.01i \\ -0.77-16.05i & 4.23-70.02i \\ -9.58+\ 3.88i & -24.79-\ 8.40i \\ 2.98-10.18i & 28.68-39.89i \end{pmatrix}.$$

Here A is Hermitian indefinite and must first be factorized by F07MRF (ZHETRF).

9.1 Program Text

```
    Program f07msfe

!     F07MSF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zhetrf, zhetrs
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, ldb, lwork, n,  &
                                          nrhs
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: a(:,:), b(:,:), work(:)
      Integer, Allocatable             :: ipiv(:)
      Character (1)                    :: clabs(1), rlabs(1)
!     .. Executable Statements ..
```

```
      Write (nout,*) 'F07MSF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs
      lda = n
      ldb = n
      lwork = 64*n
      Allocate (a(lda,n),b(ldb,nrhs),work(lwork),ipiv(n))

!     Read A and B from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)(a(i,i:n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)(a(i,1:i),i=1,n)
      End If
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Factorize A
!     The NAG name equivalent of zhetrf is f07mrf
      Call zhetrf(uplo,n,a,lda,ipiv,work,lwork,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute solution
!       The NAG name equivalent of zhetrs is f07msf
        Call zhetrs(uplo,n,nrhs,a,lda,ipiv,b,ldb,info)

!       Print solution

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04dbf('General',' ',n,nrhs,b,ldb,'Bracketed','F7.4', &
          'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

      Else
        Write (nout,*) 'The factor D is singular'
      End If

    End Program f07msfe
```

9.2 Program Data

```
F07MSF Example Program Data
  4  2                                                     :Values of N and NRHS
  'L'                                                      :Value of UPLO
 (-1.36, 0.00)
 ( 1.58,-0.90) (-8.87, 0.00)
 ( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
 ( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00)  :End of matrix A
 ( 7.79,  5.48) (-35.39, 18.01)
 (-0.77,-16.05) (  4.23,-70.02)
 (-9.58,  3.88) (-24.79, -8.40)
 ( 2.98,-10.18) ( 28.68,-39.89)                            :End of matrix B
```

9.3 Program Results

```
 F07MSF Example Program Results

 Solution(s)
                    1                  2
 1  ( 1.0000,-1.0000) ( 3.0000,-4.0000)
 2  (-1.0000, 2.0000) (-1.0000, 5.0000)
 3  ( 3.0000,-2.0000) ( 7.0000,-2.0000)
 4  ( 2.0000, 1.0000) (-8.0000, 6.0000)
```