NAG Library Routine Document

F07FWF (ZPOTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07FWF (ZPOTRI) computes the inverse of a complex Hermitian positive definite matrix A, where A has been factorized by F07FRF (ZPOTRF).

2 Specification

```
SUBROUTINE F07FWF (UPLO, N, A, LDA, INFO)

INTEGER                 N, LDA, INFO
COMPLEX (KIND=nag_wp) A(LDA,*)
CHARACTER(1)            UPLO
```

The routine may be called by its LAPACK name ***zpotri***.

3 Description

F07FWF (ZPOTRI) is used to compute the inverse of a complex Hermitian positive definite matrix A, the routine must be preceded by a call to F07FRF (ZPOTRF), which computes the Cholesky factorization of A.

If UPLO = 'U', $A = U^{\mathrm{H}}U$ and A^{-1} is computed by first inverting U and then forming $\left(U^{-1}\right)U^{-\mathrm{H}}$.

If UPLO = 'L', $A = LL^{\mathrm{H}}$ and A^{-1} is computed by first inverting L and then forming $L^{-\mathrm{H}}\left(L^{-1}\right)$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'
$A = U^{\mathrm{H}}U$, where U is upper triangular.

UPLO = 'L'
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N $\geq$ 0.

3: A(LDA,$*$) – COMPLEX (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the upper triangular matrix U if UPLO = 'U' or the lower triangular matrix L if UPLO = 'L', as returned by F07FRF (ZPOTRF).

On exit: U is overwritten by the upper triangle of A^{-1} if UPLO = 'U'; L is overwritten by the lower triangle of A^{-1} if UPLO = 'L'.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07FWF (ZPOTRI) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the ith diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \qquad \text{and} \qquad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n, ϵ is the ***machine precision*** and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2\left\|A^{-1}\right\|_2.$$

8 Further Comments

The total number of real floating point operations is approximately $\frac{8}{3}n^3$.

The real analogue of this routine is F07FJF (DPOTRI).

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here A is Hermitian positive definite and must first be factorized by F07FRF (ZPOTRF).

9.1 Program Text

```
    Program f07fwfe

!     F07FWF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: nag_wp, x04dbf, zpotrf, zpotri
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: nin = 5, nout = 6
!     .. Local Scalars ..
      Integer                          :: i, ifail, info, lda, n
      Character (1)                    :: uplo
!     .. Local Arrays ..
      Complex (Kind=nag_wp), Allocatable :: a(:,:)
      Character (1)                    :: clabs(1), rlabs(1)
!     .. Executable Statements ..
      Write (nout,*) 'F07FWF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n
      lda = n
      Allocate (a(lda,n))

!     Read A from data file

      Read (nin,*) uplo
      If (uplo=='U') Then
        Read (nin,*)(a(i,i:n),i=1,n)
      Else If (uplo=='L') Then
        Read (nin,*)(a(i,1:i),i=1,n)
      End If

!     Factorize A
!     The NAG name equivalent of zpotrf is f07frf
      Call zpotrf(uplo,n,a,lda,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute inverse of A
!       The NAG name equivalent of zpotri is f07fwf
        Call zpotri(uplo,n,a,lda,info)

!       Print inverse

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04dbf(uplo,'Nonunit',n,n,a,lda,'Bracketed','F7.4','Inverse', &
          'Integer',rlabs,'Integer',clabs,80,0,ifail)
      Else
        Write (nout,*) 'A is not positive definite'
      End If

    End Program f07fwfe
```

9.2 Program Data

```
F07FWF Example Program Data
  4                                                          :Value of N
  'L'                                                        :Value of UPLO
 (3.23, 0.00)
 (1.51, 1.92) ( 3.58, 0.00)
 (1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
 (0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00)  :End of matrix A
```

9.3 Program Results

```
 F07FWF Example Program Results

 Inverse
                     1                 2                 3                 4
 1  ( 5.4691, 0.0000)
 2  (-1.2624,-1.5491) ( 1.1024, 0.0000)
 3  (-2.9746,-0.9616) ( 0.8989,-0.5672) ( 2.1589, 0.0000)
 4  ( 1.1962, 2.9772) (-0.9826,-0.2566) (-1.3756,-1.4550) ( 2.2934,-0.0000)
```