

NAG Library Routine Document

F07CDF (DGTTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07CDF (DGTTRF) computes the LU factorization of a real n by n tridiagonal matrix A .

2 Specification

```
SUBROUTINE F07CDF (N, DL, D, DU, DU2, IPIV, INFO)
```

```
INTEGER          N, IPIV(N), INFO
REAL (KIND=nag_wp) DL(*), D(*), DU(*), DU2(N-2)
```

The routine may be called by its LAPACK name *dgtrf*.

3 Description

F07CDF (DGTTRF) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

5 Parameters

- 1: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 2: DL(*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the dimension of the array DL must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A .
On exit: is overwritten by the $(n - 1)$ multipliers that define the matrix L of the LU factorization of A .
- 3: D(*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the dimension of the array D must be at least $\max(1, N)$.
On entry: must contain the n diagonal elements of the matrix A .
On exit: is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A .

- 4: DU(*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the dimension of the array DU must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ superdiagonal elements of the matrix A .
On exit: is overwritten by the $(n - 1)$ elements of the first superdiagonal of U .
- 5: DU2(N - 2) – REAL (KIND=nag_wp) array *Output*
On exit: contains the $(n - 2)$ elements of the second superdiagonal of U .
- 6: IPIV(N) – INTEGER array *Output*
On exit: contains the n pivot indices that define the permutation matrix P . At the i th step, row i of the matrix was interchanged with row IPIV(i). IPIV(i) will always be either i or $(i + 1)$, IPIV(i) = i indicating that a row interchange was not performed.
- 7: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = PLU,$$

where

$$\|E\|_{\infty} = O(\epsilon)\|A\|_{\infty}$$

and ϵ is the *machine precision*.

Following the use of this routine, F07CEF (DGTTRS) can be used to solve systems of equations $AX = B$ or $A^T X = B$, and F07CGF (DGTCON) can be used to estimate the condition number of A .

8 Further Comments

The total number of floating point operations required to factorize the matrix A is proportional to n .

The complex analogue of this routine is F07CRF (ZGTTRF).

9 Example

This example factorizes the tridiagonal matrix A given by

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix}.$$

9.1 Program Text

```

Program f07cdf

!      F07CDF Example Program Text

!      Mark 24 Release. NAG Copyright 2012.

!      .. Use Statements ..
Use nag_library, Only: dgttrf, nag_wp
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Integer                    :: info, n
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: d(:), dl(:), du(:), du2(:)
Integer, Allocatable       :: ipiv(:)
!      .. Executable Statements ..
Write (nout,*) 'F07CDF Example Program Results'
Write (nout,*)
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n

Allocate (d(n),dl(n-1),du(n-1),du2(n-2),ipiv(n))

!      Read the tridiagonal matrix A from data file

Read (nin,*) du(1:n-1)
Read (nin,*) d(1:n)
Read (nin,*) dl(1:n-1)

!      Factorize the tridiagonal matrix A
!      The NAG name equivalent of dgttrf is f06cdf
Call dgttrf(n,dl,d,du,du2,ipiv,info)

If (info>0) Then
  Write (nout,99999) 'The (' , info, ', ', info, ')', &
    ' element of the factor U is zero'
End If

!      Print details of the factorization

Write (nout,*) 'Details of factorization'
Write (nout,*)
Write (nout,*) ' Second super-diagonal of U'
Write (nout,99998) du2(1:n-2)
Write (nout,*)
Write (nout,*) ' First super-diagonal of U'
Write (nout,99998) du(1:n-1)
Write (nout,*)
Write (nout,*) ' Main diagonal of U'
Write (nout,99998) d(1:n)
Write (nout,*)
Write (nout,*) ' Multipliers'
Write (nout,99998) dl(1:n-1)
Write (nout,*)

```

```

      Write (nout,*) ' Vector of interchanges'
      Write (nout,99997) ipiv(1:n)

99999 Format (1X,A,I3,A,I3,A,A)
99998 Format (1X,8F9.4)
99997 Format (1X,5I9)
      End Program f07cdfe

```

9.2 Program Data

```

F07CDF Example Program Data
  5                               :Value of N
    2.1  -1.0  1.9  8.0
  3.0  2.3  -5.0  -0.9  7.1
  3.4  3.6  7.0  -6.0                               :End of matrix A

```

9.3 Program Results

F07CDF Example Program Results

Details of factorization

```

Second super-diagonal of U
-1.0000  1.9000  8.0000

First super-diagonal of U
 2.3000 -5.0000 -0.9000  7.1000

Main diagonal of U
 3.4000  3.6000  7.0000 -6.0000 -1.0154

Multipliers
 0.8824  0.0196  0.1401 -0.0148

Vector of interchanges
  2      3      4      5      5

```
