NAG Library Routine Document

F07BEF (DGBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07BEF (DGBTRS) solves a real band system of linear equations with multiple right-hand sides,

$$AX = B \quad \text{or} \quad A^{\mathrm{T}}X = B,$$

where A has been factorized by F07BDF (DGBTRF).

2 Specification

```
SUBROUTINE F07BEF (TRANS, N, KL, KU, NRHS, AB, LDAB, IPIV, B, LDB, INFO)

INTEGER            N, KL, KU, NRHS, LDAB, IPIV(*), LDB, INFO
REAL (KIND=nag_wp) AB(LDAB,*), B(LDB,*)
CHARACTER(1)       TRANS
```

The routine may be called by its LAPACK name ***dgbtrs***.

3 Description

F07BEF (DGBTRS) is used to solve a real band system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, the routine must be preceded by a call to F07BDF (DGBTRF) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If TRANS = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If TRANS = 'T' or 'C', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: TRANS – CHARACTER(1) *Input*

On entry: indicates the form of the equations.

TRANS = 'N'
$AX = B$ is solved for X.

TRANS = 'T' or 'C'
$A^{\mathrm{T}}X = B$ is solved for X.

Constraint: TRANS = 'N', 'T' or 'C'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N $\geq$ 0.

3: KL – INTEGER *Input*

On entry: k_l, the number of subdiagonals within the band of the matrix A.

Constraint: KL ≥ 0.

4: KU – INTEGER *Input*

On entry: k_u, the number of superdiagonals within the band of the matrix A.

Constraint: KU ≥ 0.

5: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: NRHS ≥ 0.

6: AB(LDAB,$*$) – REAL (KIND=nag_wp) array *Input*

Note: the second dimension of the array AB must be at least $\max(1, \text{N})$.

On entry: the LU factorization of A, as returned by F07BDF (DGBTRF).

7: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07BEF (DGBTRS) is called.

Constraint: LDAB $\geq 2 \times \text{KL} + \text{KU} + 1$.

8: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \text{N})$.

On entry: the pivot indices, as returned by F07BDF (DGBTRF).

9: B(LDB,$*$) – REAL (KIND=nag_wp) array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

10: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07BEF (DGBTRS) is called.

Constraint: LDB $\geq \max(1, \text{N})$.

11: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll n$.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}\left(A^{\mathrm{T}}\right)$ can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07BHF (DGBRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07BGF (DGBCON) with NORM = 'I'.

8 Further Comments

The total number of floating point operations is approximately $2n(2k_l + k_u)r$, assuming $n \gg k_l$ and $n \gg k_u$.

This routine may be followed by a call to F07BHF (DGBRFS) to refine the solution and return an error estimate.

The complex analogue of this routine is F07BSF (ZGBTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4.42 & -36.01 \\ 27.13 & -31.67 \\ -6.14 & -1.16 \\ 10.50 & -25.82 \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by F07BDF (DGBTRF).

9.1 Program Text

```
    Program f07befe

!     F07BEF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: dgbtrf, dgbtrs, nag_wp, x04caf
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter                :: nin = 5, nout = 6
      Character (1), Parameter          :: trans = 'N'
!     .. Local Scalars ..
      Integer                           :: i, ifail, info, j, k, kl, ku, ldab,  &
                                           ldb, n, nrhs
!     .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable   :: ab(:,:), b(:,:)
      Integer, Allocatable              :: ipiv(:)
!     .. Intrinsic Procedures ..
```

```
      Intrinsic                        :: max, min
!     .. Executable Statements ..
      Write (nout,*) 'F07BEF Example Program Results'
!     Skip heading in data file
      Read (nin,*)
      Read (nin,*) n, nrhs, kl, ku
      ldab = 2*kl + ku + 1
      ldb = n
      Allocate (ab(ldab,n),b(ldb,nrhs),ipiv(n))

!     Read A and B from data file

      k = kl + ku + 1
      Read (nin,*)((ab(k+i-j,j),j=max(i-kl,1),min(i+ku,n)),i=1,n)
      Read (nin,*)(b(i,1:nrhs),i=1,n)

!     Factorize A
!     The NAG name equivalent of dgbtrf is f07bdf
      Call dgbtrf(n,n,kl,ku,ab,ldab,ipiv,info)

      Write (nout,*)
      Flush (nout)
      If (info==0) Then

!       Compute solution
!       The NAG name equivalent of dgbtrs is f07bef
        Call dgbtrs(trans,n,kl,ku,nrhs,ab,ldab,ipiv,b,ldb,info)

!       Print solution

!       ifail: behaviour on error exit
!              =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
        ifail = 0
        Call x04caf('General',' ',n,nrhs,b,ldb,'Solution(s)',ifail)

      Else
        Write (nout,*) 'The factor U is singular'
      End If

    End Program f07befe
```

9.2 Program Data

```
F07BEF Example Program Data
  4  2  1  2                 :Values of N, NRHS, KL and KU
 -0.23   2.54  -3.66
 -6.98   2.46  -2.73  -2.13
         2.56   2.46   4.07
               -4.78  -3.82   :End of matrix A
  4.42 -36.01
 27.13 -31.67
 -6.14  -1.16
 10.50 -25.82                 :End of matrix B
```

9.3 Program Results

```
 F07BEF Example Program Results

 Solution(s)
              1          2
 1     -2.0000     1.0000
 2      3.0000    -4.0000
 3      1.0000     7.0000
 4     -4.0000    -2.0000
```