NAG Library Routine Document

F07ATF (ZGEEQU)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

F07ATF (ZGEEQU) computes real diagonal scaling matrices D_R and D_C intended to equilibrate a complex m by n matrix A and reduce its condition number.

2 Specification

```
SUBROUTINE F07ATF (M, N, A, LDA, R, C, ROWCND, COLCND, AMAX, INFO)

INTEGER              M, N, LDA, INFO
REAL (KIND=nag_wp)   R(M), C(N), ROWCND, COLCND, AMAX
COMPLEX (KIND=nag_wp) A(LDA,*)
```

The routine may be called by its LAPACK name **_zgeequ_**.

3 Description

F07ATF (ZGEEQU) computes the diagonal scaling matrices. The diagonal scaling matrices are chosen to try to make the elements of largest absolute value in each row and column of the matrix B given by

$$B = D_R A D_C$$

have absolute value 1. The diagonal elements of D_R and D_C are restricted to lie in the safe range $(\delta, 1/\delta)$, where δ is the value returned by routine X02AMF. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

4 References

None.

5 Parameters

1:     M – INTEGER   *Input*

   On entry: m, the number of rows of the matrix A.

   Constraint: $M \geq 0$.

2:     N – INTEGER   *Input*

   On entry: n, the number of columns of the matrix A.

   Constraint: $N \geq 0$.

3:     A(LDA,∗) – COMPLEX (KIND=nag_wp) array   *Input*

   Note: the second dimension of the array A must be at least $\max(1, N)$.

   On entry: the matrix A whose scaling factors are to be computed.

4:     LDA – INTEGER   *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07ATF
(ZGEEQU) is called.

Constraint: $\text{LDA} \geq \max(1, \text{M})$.

5:     R(M) – REAL (KIND=nag_wp) array   *Output*

On exit: if $\text{INFO} = 0$ or $\text{INFO} > \text{M}$, R contains the row scale factors, the diagonal elements of D_R.
The elements of R will be positive.

6:     C(N) – REAL (KIND=nag_wp) array   *Output*

On exit: if $\text{INFO} = 0$, C contains the column scale factors, the diagonal elements of D_C. The
elements of C will be positive.

7:     ROWCND – REAL (KIND=nag_wp)   *Output*

On exit: if $\text{INFO} = 0$ or $\text{INFO} > \text{M}$, ROWCND contains the ratio of the smallest value of $\text{R}(i)$ to
the largest value of $\text{R}(i)$. If $\text{ROWCND} \geq 0.1$ and AMAX is neither too large nor too small, it is
not worth scaling by D_R.

8:     COLCND – REAL (KIND=nag_wp)   *Output*

On exit: if $\text{INFO} = 0$, COLCND contains the ratio of the smallest value of $\text{C}(i)$ to the largest value
of $\text{C}(i)$.

If $\text{COLCND} \geq 0.1$, it is not worth scaling by D_C.

9:     AMAX – REAL (KIND=nag_wp)   *Output*

On exit: $\max \left| a_{ij} \right|$. If AMAX is very close to overflow or underflow, the matrix A should be scaled.

10:    INFO – INTEGER   *Output*

On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6   Error Indicators and Warnings

Errors or warnings detected by the routine:

$\text{INFO} < 0$

If $\text{INFO} = -i$, the ith argument had an illegal value. An explanatory message is output, and
execution of the program is terminated.

$\text{INFO} > 0$ and $\text{INFO} \leq \text{M}$

If $\text{INFO} = i$, the ith row of A is exactly zero.

$\text{INFO} > \text{M}$

If $\text{INFO} = i$, the $(i - \text{M})$th column of A is exactly zero.

7   Accuracy

The computed scale factors will be close to the exact scale factors.

8   Further Comments

The real analogue of this routine is F07AFF (DGEEQU).

9   Example

This example equilibrates the general matrix A given by

$$A = \begin{pmatrix} -1.34 + 2.55i & (0.28 + 3.17i) \times 10^{10} & -6.39 - 2.20i \\ -1.70 - 1.41i & (3.31 - 0.15i) \times 10^{10} & -0.15 + 1.34i \\ (2.41 + 0.39i) \times 10^{-10} & -0.56 + 1.47i & (-0.83 - 0.69i) \times 10^{-10} \end{pmatrix}.$$

Details of the scaling factors, and the scaled matrix are output.

9.1   Program Text

```
    Program f07atfe

!   F07ATF Example Program Text

!   Mark 24 Release. NAG Copyright 2012.

!       .. Use Statements ..
    Use nag_library, Only: f06kcf, nag_wp, x02ajf, x02amf, x02bhf, x04dbf,   &
                           zdscal, zgeequ
!       .. Implicit None Statement ..
    Implicit None
!       .. Parameters ..
    Real (Kind=nag_wp), Parameter    :: one = 1.0_nag_wp
    Real (Kind=nag_wp), Parameter    :: thresh = 0.1_nag_wp
    Integer, Parameter               :: nin = 5, nout = 6
!       .. Local Scalars ..
    Real (Kind=nag_wp)               :: amax, big, colcnd, rowcnd, small
    Integer                          :: i, ifail, info, j, lda, n
!       .. Local Arrays ..
    Complex (Kind=nag_wp), Allocatable :: a(:,:)
    Real (Kind=nag_wp), Allocatable  :: c(:), r(:)
    Character (1)                    :: clabs(1), rlabs(1)
!       .. Intrinsic Procedures ..
    Intrinsic                        :: real
!       .. Executable Statements ..
    Write (nout,*) 'F07ATF Example Program Results'
    Write (nout,*)
    Flush (nout)
!   Skip heading in data file
    Read (nin,*)
    Read (nin,*) n
    lda = n
    Allocate (a(lda,n),c(n),r(n))

!   Read the N by N matrix A from data file

    Read (nin,*)(a(i,1:n),i=1,n)

!   Print the matrix A

!   ifail: behaviour on error exit
!          =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
    ifail = 0
    Call x04dbf('General',' ',n,n,a,lda,'Bracketed','1P,E10.2','Matrix A', &
      'Integer',rlabs,'Integer',clabs,80,0,ifail)

    Write (nout,*)

!   Compute row and column scaling factors

!   The NAG name equivalent of zgeequ is f07atf
    Call zgeequ(n,n,a,lda,r,c,rowcnd,colcnd,amax,info)

    If (info>0) Then
      If (info<=n) Then
        Write (nout,99999) 'Row ', info, ' of A is exactly zero'
      Else
```

```
            Write (nout,99999) 'Column ', info - n, ' of A is exactly zero'
          End If
        Else

!       Print ROWCND, COLCND, AMAX and the scale factors

          Write (nout,99998) 'ROWCND =', rowcnd, ', COLCND =', colcnd, &
            ', AMAX =', amax
          Write (nout,*)
          Write (nout,*) 'Row scale factors'
          Write (nout,99997) r(1:n)
          Write (nout,*)
          Write (nout,*) 'Column scale factors'
          Write (nout,99997) c(1:n)
          Write (nout,*)
          Flush (nout)

!       Compute values close to underflow and overflow

          small = x02amf()/(x02ajf()*real(x02bhf(),kind=nag_wp))
          big = one/small
          If ((rowcnd>=thresh) .And. (amax>=small) .And. (amax<=big)) Then
            If (colcnd<thresh) Then

!           Just column scale A
!           The NAG name equivalent of zdscal is f06jdf
              Do j = 1, n
                Call zdscal(n,c(j),a(1,j),1)
              End Do

            End If
          Else If (colcnd>=thresh) Then

!           Just row scale A
            Do j = 1, n
              Call f06kcf(n,r,1,a(1,j),1)
            End Do

          Else

!           Row and column scale A
            Do j = 1, n
              Call zdscal(n,c(j),a(1,j),1)
              Call f06kcf(n,r,1,a(1,j),1)
            End Do

          End If

!       Print the scaled matrix
          ifail = 0
          Call x04dbf('General',' ',n,n,a,lda,'Bracketed',' ','Scaled matrix', &
            'Integer',rlabs,'Integer',clabs,80,0,ifail)


        End If

99999 Format (1X,A,I4,A)
99998 Format (1X,3(A,1P,E8.1))
99997 Format ((1X,1P,7E11.1))
    End Program f07atfe
```

9.2   Program Data

```
F07ATF Example Program Data

 3                                                              :Value of N

 (-1.34D+00, 2.55D+00) ( 0.28D+10, 3.17D+10) (-6.39D+00,-2.20D+00)
 (-1.70D+00,-1.41D+00) ( 3.31D+10,-0.15D+10) (-0.15D+00, 1.34D+00)
 ( 2.41D-10, 0.39D-10) (-0.56D+00, 1.47D+00) (-0.83D-10,-0.69D-10)

                                                              :End of matrix A
```

9.3   Program Results

```
F07ATF Example Program Results

Matrix A
                           1                    2                    3
1 ( -1.34E+00,  2.55E+00) (  2.80E+09,  3.17E+10) ( -6.39E+00, -2.20E+00)
2 ( -1.70E+00, -1.41E+00) (  3.31E+10, -1.50E+09) ( -1.50E-01,  1.34E+00)
3 (  2.41E-10,  3.90E-11) ( -5.60E-01,  1.47E+00) ( -8.30E-11, -6.90E-11)

ROWCND = 5.9E-11, COLCND = 1.4E-10, AMAX = 3.5E+10

 Row scale factors
    2.9E-11    2.9E-11    4.9E-01

 Column scale factors
    7.3E+09    1.0E+00    4.0E+09

 Scaled matrix
                     1                2                3
1 ( -0.2816,  0.5359) (  0.0812,  0.9188) ( -0.7439, -0.2561)
2 ( -0.3562, -0.2954) (  0.9566, -0.0434) ( -0.0174,  0.1555)
3 (  0.8607,  0.1393) ( -0.2759,  0.7241) ( -0.1642, -0.1365)
```