NAG Library Routine Document

E02ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

E02ACF calculates a minimax polynomial fit to a set of data points.

2 Specification

```
SUBROUTINE E02ACF (X, Y, N, A, M1, REF)

INTEGER            N, M1
REAL (KIND=nag_wp) X(N), Y(N), A(M1), REF
```

3 Description

Given a set of data points (x_i, y_i), for $i = 1, 2, \ldots, n$, E02ACF uses the exchange algorithm to compute an mth-order polynomial

$$\mathrm{P}(x) = a_1 + a_2 x + a_3 x^2 + \cdots + a_{m+1} x^m$$

such that $\max_i 2|\mathrm{P}(x_i) - y_i|$ is a minimum.

The routine also returns a number whose absolute value is the final reference deviation (see Section 6). The routine is an adaptation of Boothroyd (1967).

4 References

Boothroyd J B (1967) Algorithm 318 *Comm. ACM* **10** 801

Stieffel E (1959) Numerical methods of Tchebycheff approximation *On Numerical Approximation* (ed R E Langer) 217–232 University of Wisconsin Press

5 Parameters

1: X(N) – REAL (KIND=nag_wp) array *Input*

On entry: the values of the x coordinates, x_i, for $i = 1, 2, \ldots, n$.

Constraint: $x_1 < x_2 < \cdots < x_n$.

2: Y(N) – REAL (KIND=nag_wp) array *Input*

On entry: the values of the y coordinates, y_i, for $i = 1, 2, \ldots, n$.

3: N – INTEGER *Input*

On entry: the number n of data points.

4: A(M1) – REAL (KIND=nag_wp) array *Output*

On exit: the coefficients a_i of the final polynomial, for $i = 1, 2, \ldots, m + 1$.

5: M1 – INTEGER *Input*

On entry: $m+1$, where m is the order of the polynomial to be found.

Constraint: M1 $<$ min(N, 100).

6: REF – REAL (KIND=nag_wp) *Output*

On exit: the final reference deviation (see Section 6).

6 Error Indicators and Warnings

If an error is detected in an input parameter E02ACF will act as if a soft noisy exit has been requested (see Section 3.3.4 in the Essential Introduction).

7 Accuracy

This is wholly dependent on the given data points.

8 Further Comments

The time taken increases with m.

9 Example

This example calculates a minimax fit with a polynomial of degree 5 to the exponential function evaluated at 21 points over the interval $[0, 1]$. It then prints values of the function and the fitted polynomial.

9.1 Program Text

```
    Program e02acfe

!     E02ACF Example Program Text

!     Mark 24 Release. NAG Copyright 2012.

!     .. Use Statements ..
      Use nag_library, Only: e02acf, nag_wp
!     .. Implicit None Statement ..
      Implicit None
!     .. Parameters ..
      Integer, Parameter               :: m1 = 6, n = 21, nout = 6
!     .. Local Scalars ..
      Real (Kind=nag_wp)               :: ref, s, t, z
      Integer                          :: i, j
!     .. Local Arrays ..
      Real (Kind=nag_wp)               :: a(m1), x(n), y(n)
!     .. Intrinsic Procedures ..
      Intrinsic                        :: exp, real
!     .. Executable Statements ..
      Write (nout,*) 'E02ACF Example Program Results'

      x(1:n) = real((/(i-1,i=1,n)/),kind=nag_wp)/real(n-1,kind=nag_wp)
      y(1:n) = exp(x(1:n))

      Call e02acf(x,y,n,a,m1,ref)

      Write (nout,*)
      Write (nout,*) '   Polynomial coefficients'
      Write (nout,99998)(a(i),i=1,m1)
      Write (nout,*)
      Write (nout,99997) '   Reference deviation = ', ref
      Write (nout,*)
      Write (nout,*) '  X     exp(X)    Fit     Residual'
```

```
      Do j = 1, 11
        z = real(j-1,kind=nag_wp)*0.1E0_nag_wp

        s = a(m1)

        Do i = m1 - 1, 1, -1
          s = s*z + a(i)
        End Do

        t = exp(z)
        Write (nout,99999) z, s, t, s - t
      End Do

99999 Format (1X,F5.2,2F9.4,E11.2)
99998 Format (6X,E12.4)
99997 Format (1X,A,E10.2)
    End Program e02acfe
```

9.2 Program Data

None.

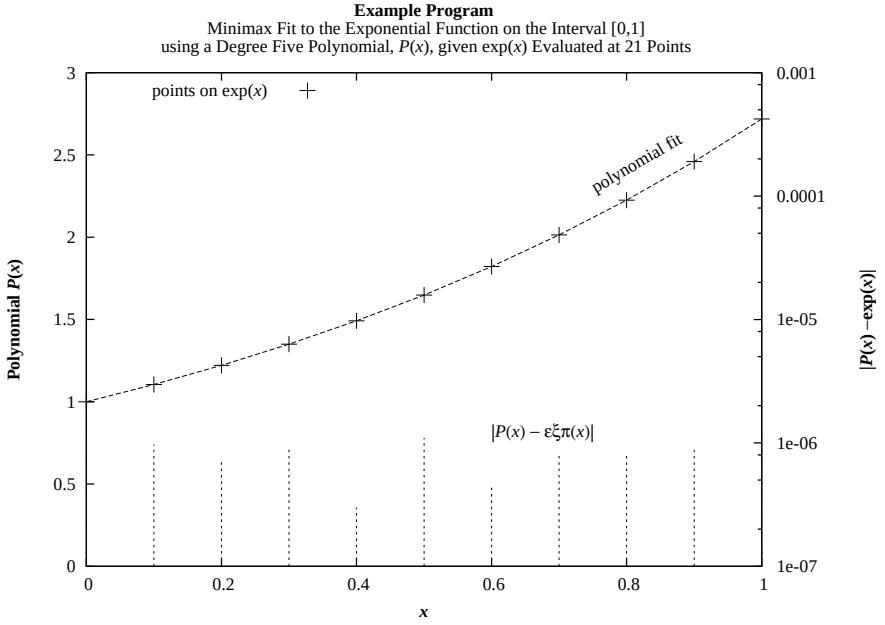
9.3 Program Results

```
 E02ACF Example Program Results

    Polynomial coefficients
        0.1000E+01
        0.1000E+01
        0.4991E+00
        0.1704E+00
        0.3478E-01
        0.1391E-01

    Reference deviation =   0.11E-05

   X     exp(X)    Fit     Residual
  0.00   1.0000   1.0000  -0.11E-05
  0.10   1.1052   1.1052   0.97E-06
  0.20   1.2214   1.2214  -0.74E-06
  0.30   1.3499   1.3499  -0.92E-06
  0.40   1.4918   1.4918   0.30E-06
  0.50   1.6487   1.6487   0.11E-05
  0.60   1.8221   1.8221   0.46E-06
  0.70   2.0138   2.0138  -0.82E-06
  0.80   2.2255   2.2255  -0.84E-06
  0.90   2.4596   2.4596   0.88E-06
  1.00   2.7183   2.7183  -0.11E-05
```


Example Program
Minimax Fit to the Exponential Function on the Interval [0,1]
using a Degree Five Polynomial, P(x), given exp(x) Evaluated at 21 Points
points on exp(x)
polynomial fit
|P(x) − εξπ(x)|
Polynomial P(x)
|P(x) −exp(x)|
x
3
2.5
2
1.5
1
0.5
0
0.001
0.0001
1e-05
1e-06
1e-07
0
0.2
0.4
0.6
0.8
1