NAG Library Function Document

nag_trans_hessenberg_observer (g13ewc)

1 Purpose

nag_trans_hessenberg_observer (g13ewc) reduces the matrix pair (A, C) to lower or upper observer Hessenberg form using (and optionally accumulating) the unitary state-space transformations.

2 Specification

```
#include <nag.h>
#include <nagg13.h>
void nag_trans_hessenberg_observer (Integer n, Integer p,
     Nag_ObserverForm reduceto, double a[], Integer tda, double c[],
     Integer tdc, double u[], Integer tdu, NagError *fail)
```

3 Description

nag_trans_hessenberg_observer (g13ewc) computes a unitary state-space transformation U, which reduces the matrix pair (A, C) to give a compound matrix in one of the following observer Hessenberg forms:

$$\begin{pmatrix} UAU^{\mathrm{T}} \\ CU^{\mathrm{T}} \end{pmatrix} = \overbrace{\begin{pmatrix} * & . & . & . & . & . & . & * \\ . & & & & & & & . \\ . & & & & & & & . \\ . & & & & & & & . \\ * & & & & & & & . \\ & . & & & & & & . \\ & & . & & & & & . \\ & & & * & . & . & . & * \\ \hline & & & & * & . & . & * \\ \hline & & & & & & . & . \\ \hline & & & & & & . & . \\ \hline & & & & & & & * \end{pmatrix}}^{n} n$$

p

if **reduceto** = Nag_UH_Observer, or

$$\begin{pmatrix} CU^{\mathrm{T}} \\ UAU^{\mathrm{T}} \end{pmatrix} = \underbrace{\begin{pmatrix} * & & & & & & & \\ . & . & & & & & & \\ . & & . & & & & & \\ * & . & . & * & & & & \\ * & . & . & . & * & & & \\ . & & & & & . & & \\ . & & & & & & . & \\ . & & & & & & & * \\ . & & & & & & & . \\ . & & & & & & & . \\ . & & & & & & & . \\ * & . & . & . & . & . & . & * \end{pmatrix}}_{n} p$$

if **reduceto** = Nag_LH_Observer. If $p > n$, then the matrix CU^{T} is trapezoidal and if $p + 1 \geq n$, then the matrix UAU^{T} is full.

4 References

van Dooren P and Verhaegen M (1985) On the use of unitary state-space transformations. *In: Contemporary Mathematics on Linear Algebra and its Role in Systems Theory* **47** AMS, Providence

5 Arguments

1: **n** – Integer *Input*

On entry: the actual state dimension, n, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 1$.

2: **p** – Integer *Input*

On entry: the actual output dimension, p.

Constraint: $\mathbf{p} \geq 1$.

3: **reduceto** – Nag_ObserverForm *Input*

On entry: indicates whether the matrix pair (A, C) is to be reduced to upper or lower observer Hessenberg form

reduceto = Nag_UH_Observer
Upper observer Hessenberg form).

reduceto = Nag_LH_Observer
Lower observer Hessenberg form).

Constraint: **reduceto** = Nag_UH_Observer or Nag_LH_Observer.

4: **a**[**n** × **tda**] – double *Input/Output*

Note: the (i, j)th element of the matrix A is stored in $\mathbf{a}[(i-1) \times \mathbf{tda} + j - 1]$.

On entry: the leading n by n part of this array must contain the state transition matrix A to be transformed.

On exit: the leading n by n part of this array contains the transformed state transition matrix UAU^{T}.

5: **tda** – Integer *Input*

On entry: the stride separating matrix column elements in the array **a**.

Constraint: $\mathbf{tda} \geq \mathbf{n}$.

6: **c**[**p** × **tdc**] – double *Input/Output*

Note: the (i, j)th element of the matrix C is stored in $\mathbf{c}[(i-1) \times \mathbf{tdc} + j - 1]$.

On entry: the leading p by n part of this array must contain the output matrix C to be transformed.

On exit: the leading p by n part of this array contains the transformed output matrix CU^{T}.

7: **tdc** – Integer *Input*

On entry: the stride separating matrix column elements in the array **c**.

Constraint: $\mathbf{tdc} \geq \mathbf{n}$.

8: **u**[**n** × **tdu**] – double *Input/Output*

Note: the (i, j)th element of the matrix U is stored in $\mathbf{u}[(i-1) \times \mathbf{tdu} + j - 1]$.

On entry: if **u** is not **NULL**, then the leading n by n part of this array must contain either a transformation matrix (e.g., from a previous call to this function) or be initialized as the identity matrix. If this information is not to be input then **u** must be set to **NULL**.

On exit: if **u** is not **NULL**, then the leading n by n part of this array contains the product of the input matrix U and the state-space transformation matrix which reduces the given pair to observer Hessenberg form.

9: **tdu** – Integer *Input*

On entry: the stride separating matrix column elements in the array **u**.

Constraint: **tdu** $\geq$ **n** if **u** is defined.

10: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_2_INT_ARG_LT

On entry, **tda** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These arguments must satisfy **tda** $\geq$ **n**.
On entry **tdc** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These arguments must satisfy **tdc** $\geq$ **n**.
On entry **tdu** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These arguments must satisfy **tdu** $\geq$ **n**.

NE_BAD_PARAM

On entry, argument **reduceto** had an illegal value.

NE_INT_ARG_LT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 1.

On entry, **p** = ⟨*value*⟩.
Constraint: **p** ≥ 1.

7 Accuracy

The algorithm is backward stable.

8 Parallelism and Performance

nag_trans_hessenberg_observer (g13ewc) is not threaded in any implementation.

9 Further Comments

The algorithm requires $O\big((n+m)n^2\big)$ operations (see van Dooren and Verhaegen (1985)).

10 Example

To reduce the matrix pair (A, C) to upper observer Hessenberg form.

10.1 Program Text

```
/* nag_trans_hessenberg_observer (g13ewc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group
 *
 * Mark 26, 2016.
 */
```

```
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg13.h>

#define A(I, J) a[(I) *tda + J]
#define C(I, J) c[(I) *tdc + J]
#define U(I, J) u[(I) *tdu + J]
int main(void)
{
  Integer exit_status = 0, i, j, n, p, tda, tdc, tdu;
  double *a = 0, *c = 0, one = 1.0, *u = 0, zero = 0.0;
  Nag_ObserverForm reduceto;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_trans_hessenberg_observer (g13ewc) Example Program Results\n");

  /* Skip the heading in the data file and read the data. */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "", &n, &p);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "", &n, &p);
#endif
  if (n >= 1 || p >= 1) {
    if (!(a = NAG_ALLOC(n * n, double)) ||
        !(c = NAG_ALLOC(p * n, double)) || !(u = NAG_ALLOC(n * n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
    tda = n;
    tdc = n;
    tdu = n;
  }
  else {
    printf("Invalid n or p.\n");
    exit_status = 1;
    return exit_status;
  }

  reduceto = Nag_UH_Observer;

  for (j = 0; j < n; ++j)
    for (i = 0; i < n; ++i)
#ifdef _WIN32
      scanf_s("%lf", &A(i, j));
#else
      scanf("%lf", &A(i, j));
#endif
  for (i = 0; i < p; ++i)
    for (j = 0; j < n; ++j)
#ifdef _WIN32
      scanf_s("%lf", &C(i, j));
#else
      scanf("%lf", &C(i, j));
#endif

  if (u) /* Initialize U as the identity matrix. */
    for (i = 0; i < n; ++i) {
      for (j = 0; j < n; ++j)
        U(i, j) = zero;
      U(i, i) = one;
```

```
    }

  /* Reduce the pair (A,C) to reduceto observer Hessenberg form. */
  /* nag_trans_hessenberg_observer (g13ewc).
   * Unitary state-space transformation to reduce (AC) to
   * lower or upper observer Hessenberg form
   */
  nag_trans_hessenberg_observer(n, p, reduceto, a, tda, c, tdc, u, tdu,
                                &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_trans_hessenberg_observer (g13ewc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  printf("\nThe transformed state transition matrix is \n\n");
  for (i = 0; i < n; ++i) {
    for (j = 0; j < n; ++j)
      printf("%8.4f ", A(i, j));
    printf("\n");
  }
  printf("\nThe transformed input matrix is \n\n");
  for (i = 0; i < p; ++i) {
    for (j = 0; j < n; ++j)
      printf("%8.4f ", C(i, j));
    printf("\n");
  }
  if (u) {
    printf("\nThe transformation matrix that reduces (A,C) "
           "to observer Hessenberg form is \n\n");
    for (i = 0; i < n; ++i) {
      for (j = 0; j < n; ++j)
        printf("%8.4f ", U(i, j));
      printf("\n");
    }
  }
END:
  NAG_FREE(a);
  NAG_FREE(c);
  NAG_FREE(u);
  return exit_status;
}
```

10.2 Program Data

```
nag_trans_hessenberg_observer (g13ewc) Example Program Data
   5     3
  15.0  21.0  -3.0   3.0   9.0
  20.0   1.0   2.0   8.0   9.0
   4.0   1.0   7.0  13.0  14.0
   5.0   6.0  12.0  13.0  -6.0
   5.0  11.0  17.0  -7.0  -1.0
   7.0  -1.0   3.0  -6.0  -3.0
   4.0   5.0   6.0  -2.0  -3.0
   9.0   8.0   5.0   2.0   1.0
```

10.3 Program Results

```
nag_trans_hessenberg_observer (g13ewc) Example Program Results

The transformed state transition matrix is

  7.1637  -0.9691 -16.5046   0.2869   0.9205
 -2.3285  11.5431  -8.7471   3.4122  -3.7118
-10.5440  -7.6032  -0.3215   3.6571  -0.4335
 -3.6845   5.6449   0.5906 -15.6996  17.4267
  0.0000  -6.4260   1.5591  14.4317  32.3143

The transformed input matrix is
```

```
  0.0000   0.0000   7.6585   5.2973  -4.1576
  0.0000   0.0000   0.0000   5.8305  -7.4837
  0.0000   0.0000   0.0000   0.0000 -13.2288

The transformation matrix that reduces (A,C) to observer Hessenberg form is

  0.1863  -0.4823   0.2645   0.6648  -0.4698
 -0.1137  -0.3601   0.6748  -0.0512   0.6320
  0.6742  -0.5151  -0.1897  -0.4940  -0.0097
 -0.1872   0.0813   0.5439  -0.5371  -0.6116
 -0.6803  -0.6047  -0.3780  -0.1512  -0.0756
```