NAG Library Function Document

nag_kalman_sqrt_filt_cov_var (g13eac)

1 Purpose

nag_kalman_sqrt_filt_cov_var (g13eac) performs a combined measurement and time update of one iteration of the time-varying Kalman filter. The method employed for this update is the square root covariance filter with the system matrices in their original form.

2 Specification

```
#include <nag.h>
#include <nagg13.h>
```

```
void nag_kalman_sqrt_filt_cov_var (Integer n, Integer m, Integer p,
     double s[], Integer tds, const double a[], Integer tda,
     const double b[], Integer tdb, const double q[], Integer tdq,
     const double c[], Integer tdc, const double r[], Integer tdr,
     double k[], Integer tdk, double h[], Integer tdh, double tol,
     NagError *fail)
```

3 Description

For the state space system defined by

$$\begin{array}{rcll} X_{i+1} & = & A_iX_i + B_iW_i & \mathrm{var}\left(W_i\right) = Q_i \\ Y_i & = & C_iX_i + V_i & \mathrm{var}\left(V_i\right) = R_i \end{array}$$

the estimate of X_i given observations Y_1 to Y_{i-1} is denoted by $\hat{X}_{i|i-1}$ with $\mathrm{var}\left(\hat{X}_{i|i-1}\right) = P_{i|i-1} = S_iS_i^{\mathrm{T}}$. nag_kalman_sqrt_filt_cov_var (g13eac) performs one recursion of the square root covariance filter algorithm, summarised as follows:

$$\underset{\text{(Pre-array)}}{\begin{pmatrix} R_i^{1/2} & C_iS_i & 0 \\ 0 & A_iS_i & B_iQ_i^{1/2} \end{pmatrix}} U = \underset{\text{(Post-array)}}{\begin{pmatrix} H_i^{1/2} & 0 & 0 \\ G_i & S_{i+1} & 0 \end{pmatrix}}$$

where U is an orthogonal transformation triangularizing the pre-array. The triangularisation is carried out via Householder transformations exploiting the zero pattern in the pre-array. The measurement-update for the estimated state vector X is

$$\hat{X}_{i|i} = \hat{X}_{i|i-1} - K_i\left[C_i\hat{X}_{i|i-1} - Y_i\right] \tag{1}$$

where K_i is the Kalman gain matrix, whilst the time-update for X is

$$\hat{X}_{i+1|i} = A_i\hat{X}_{i|i} + D_iU_i \tag{2}$$

where D_iU_i represents any deterministic control used. The relationship between the Kalman gain matrix K_i and G_i is given by

$$A_iK_i = G_i\left(H_i^{1/2}\right)^{-1}$$

The function returns the product of the matrices A_i and K_i represented as AK_i, and the state covariance matrix $P_{i|i-1}$ factorized as $P_{i|i-1} = S_iS_i^{\mathrm{T}}$ (see the g13 Chapter Introduction for more information concerning the covariance filter).

4 References

Anderson B D O and Moore J B (1979) *Optimal Filtering* Prentice–Hall

Harvey A C and Phillips G D A (1979) Maximum likelihood estimation of regression models with autoregressive — moving average disturbances *Biometrika* **66** 49–58

Vanbegin M, van Dooren P and Verhaegen M H G (1989) Algorithm 675: FORTRAN subroutines for computing the square root covariance filter and square root information filter in dense or Hessenberg forms *ACM Trans. Math. Software* **15** 243–256

Verhaegen M H G and van Dooren P (1986) Numerical aspects of different Kalman filter implementations *IEEE Trans. Auto. Contr.* **AC-31** 907–917

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Arguments

1: **n** – Integer *Input*

On entry: the actual state dimension, n, i.e., the order of the matrices S_i and A_i.

Constraint: **n** ≥ 1.

2: **m** – Integer *Input*

On entry: the actual input dimension, m, i.e., the order of the matrix $Q_i^{1/2}$.

Constraint: **m** ≥ 1.

3: **p** – Integer *Input*

On entry: the actual output dimension, p, i.e., the order of the matrix $R_i^{1/2}$.

Constraint: **p** ≥ 1.

4: **s**[**n** $\times$ **tds**] – double *Input/Output*

Note: the (i,j)th element of the matrix S is stored in **s**$[(i-1) \times$ **tds** $+ j - 1]$.

On entry: the leading n by n lower triangular part of this array must contain S_i, the left Cholesky factor of the state covariance matrix $P_{i|i-1}$.

On exit: the leading n by n lower triangular part of this array contains S_{i+1}, the left Cholesky factor of the state covariance matrix $P_{i+1|i}$.

5: **tds** – Integer *Input*

On entry: the stride separating matrix column elements in the array **s**.

Constraint: **tds** $\geq$ **n**.

6: **a**[**n** $\times$ **tda**] – const double *Input*

Note: the (i,j)th element of the matrix A is stored in **a**$[(i-1) \times$ **tda** $+ j - 1]$.

On entry: the leading n by n part of this array must contain A_i, the state transition matrix of the discrete system.

7: **tda** – Integer *Input*

On entry: the stride separating matrix column elements in the array **a**.

Constraint: **tda** $\geq$ **n**.

8: **b**[**n** × **tdb**] – const double *Input*

Note: the (i, j)th element of the matrix B is stored in **b**$[(i-1) \times$ **tdb** $+ j - 1]$.

On entry: if **q** is not **NULL** then the leading n by m part of this array must contain the matrix B_i, otherwise if **q** is **NULL** then the leading n by m part of the array must contain the matrix $B_i Q_i^{1/2}$. B_i is the input weight matrix and Q_i is the noise covariance matrix.

9: **tdb** – Integer *Input*

On entry: the stride separating matrix column elements in the array **b**.

Constraint: **tdb** $\geq$ **m**.

10: **q**[**m** × **tdq**] – const double *Input*

Note: the (i, j)th element of the matrix Q is stored in **q**$[(i-1) \times$ **tdq** $+ j - 1]$.

On entry: if the noise covariance matrix is to be supplied separately from the input weight matrix then the leading m by m lower triangular part of this array must contain $Q_i^{1/2}$, the left Cholesky factor of the input process noise covariance matrix. If the noise covariance matrix is to be input with the weight matrix as $B_i Q_i^{1/2}$ then the array **q** must be set to **NULL**.

11: **tdq** – Integer *Input*

On entry: the stride separating matrix column elements in the array **q**.

Constraint: **tdq** $\geq$ **m** if **q** is defined.

12: **c**[**p** × **tdc**] – const double *Input*

Note: the (i, j)th element of the matrix C is stored in **c**$[(i-1) \times$ **tdc** $+ j - 1]$.

On entry: the leading p by n part of this array must contain C_i, the output weight matrix of the discrete system.

13: **tdc** – Integer *Input*

On entry: the stride separating matrix column elements in the array **c**.

Constraint: **tdc** $\geq$ **n**.

14: **r**[**p** × **tdr**] – const double *Input*

Note: the (i, j)th element of the matrix R is stored in **r**$[(i-1) \times$ **tdr** $+ j - 1]$.

On entry: the leading p by p lower triangular part of this array must contain $R_i^{1/2}$, the left Cholesky factor of the measurement noise covariance matrix.

15: **tdr** – Integer *Input*

On entry: the stride separating matrix column elements in the array **r**.

Constraint: **tdr** $\geq$ **p**.

16: **k**[**n** × **tdk**] – double *Output*

Note: the (i, j)th element of the matrix K is stored in **k**$[(i-1) \times$ **tdk** $+ j - 1]$.

On exit: if **k** is not **NULL** then the leading n by p part of **k** contains AK_i, the product of the Kalman filter gain matrix K_i with the state transition matrix A_i. If AK_i is not required then **k** must be set to **NULL**.

17: **tdk** – Integer *Input*

On entry: the stride separating matrix column elements in the array **k**.

Constraint: if **k** is not **NULL**, **tdk** $\geq$ **p**

18: **h**[**p** $\times$ **tdh**] – double *Output*

Note: the (i, j)th element of the matrix H is stored in **h**$[(i-1) \times$ **tdh** $+ j - 1]$.

On exit: if **k** is not **NULL** then the leading p by p lower triangular part of this array contains $H_i^{1/2}$. If **k** is **NULL** then **h** is not referenced and may be set to **NULL**.

19: **tdh** – Integer *Input*

On entry: the stride separating matrix column elements in the array **h**.

Constraint: if both **k** and **h** are not **NULL**, **tdh** $\geq$ **p**

20: **tol** – double *Input*

On entry: if both **k** and **h** are not **NULL** then **tol** is used to test for near singularity of the matrix $H_i^{1/2}$. If you set **tol** to be less than $p^2\epsilon$ then the tolerance is taken as $p^2\epsilon$, where ϵ is the ***machine precision***. Otherwise, **tol** need not be set by you.

21: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_2_INT_ARG_LT

On entry **tda** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These arguments must satisfy **tda** $\geq$ **n**.

On entry **tdb** = ⟨*value*⟩ while **m** = ⟨*value*⟩. These arguments must satisfy **tdb** $\geq$ **m**.

On entry **tdc** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These arguments must satisfy **tdc** $\geq$ **n**.

On entry **tdh** = ⟨*value*⟩ while **p** = ⟨*value*⟩. These arguments must satisfy **tdh** $\geq$ **p**.

On entry **tdk** = ⟨*value*⟩ while **p** = ⟨*value*⟩. These arguments must satisfy **tdk** $\geq$ **p**.

On entry **tdq** = ⟨*value*⟩ while **m** = ⟨*value*⟩. These arguments must satisfy **tdq** $\geq$ **m**.

On entry **tdr** = ⟨*value*⟩ while **p** = ⟨*value*⟩. These arguments must satisfy **tdr** $\geq$ **p**.

On entry, **tds** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These arguments must satisfy **tds** $\geq$ **n**.

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_INT_ARG_LT

On entry, **m** = ⟨*value*⟩.
Constraint: **m** ≥ 1.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 1.

On entry, **p** = ⟨*value*⟩.
Constraint: **p** ≥ 1.

NE_MAT_SINGULAR

The matrix sqrt(H) is singular.

NE_NULL_ARRAY

Array **h** has null address.

7 Accuracy

The use of the square root algorithm improves the stability of the computations.

8 Parallelism and Performance

nag_kalman_sqrt_filt_cov_var (g13eac) is not threaded in any implementation.

9 Further Comments

The algorithm requires $\frac{7}{6}n^3 + n^2\left(\frac{5}{2}p + m\right) + n\left(\frac{1}{2}m^2 + p^2\right)$ operations and is backward stable (see Vanbegin *et al.* (1989)).

10 Example

For this function two examples are presented. There is a single example program for nag_kalman_sqrt_filt_cov_var (g13eac), with a main program and the code to solve the two example problems is given in the functions ex1 and ex2.

Example 1 (ex1)

To apply three iterations of the Kalman filter (in square root covariance form) to the system (A_i, B_i, C_i). The same data is used for all three iterative steps.

Example 2 (ex2)

In the second example 2000 terms of an ARMA(1,1) time series (with $\sigma^2 = 1.0, \theta = 0.9$ and $\phi = 0.4$) are generated using the function nag_rand_arma (g05phc). The Kalman filter and optimization function nag_opt_nlp (e04ucc) are then used to find the maximum likelihood estimate for the time series arguments θ and ϕ. The ARMA(1,1) time series is defined by

$$y_k = \phi y_{k-1} + \epsilon_k - \theta\epsilon_{k-1}$$

This has the following state space representation (Harvey and Phillips (1979))

$$\begin{aligned} x_k &= \begin{pmatrix} \phi & 1 \\ 0 & 0 \end{pmatrix} x_{k-1} + \begin{pmatrix} 1 \\ -\theta \end{pmatrix} \epsilon_k \\ y_k &= \begin{pmatrix} 1 & 0 \end{pmatrix} x_k \end{aligned}$$

where the state vector $x_k = \begin{pmatrix} y_k \\ -\theta\epsilon_k \end{pmatrix}$ and ϵ_k is uncorrelated white noise with zero mean and variance σ^2, i.e.,

$$E[\epsilon_k] = 0, E[\epsilon_k\epsilon_k] = \sigma^2, E[y_k\epsilon_k] = \sigma^2 \text{ and } E[\epsilon_k\epsilon_{k-1}] = 0.$$

Since $\sigma^2 = 1$ we arrive at the following Kalman Filter matrices

$$\begin{aligned} A_k &= \begin{pmatrix} \phi & 1 \\ 0 & 0 \end{pmatrix}, B_k = \begin{pmatrix} 1 \\ -\theta \end{pmatrix} \\ C_k &= \begin{pmatrix} 1 & 0 \end{pmatrix}, Q_k = 0 \text{ and } R_k = 1. \end{aligned}$$

The initial estimates for the state vector, $x_{1|0}$, and state covariance matrix, $P_{1|0}$, are:

$$x_{1|0} = E[x_k] = 0 \text{ and } P_{1|0} = E\left[x_k x_k^{\mathrm{T}}\right] = \begin{pmatrix} E[y_k y_k] & -\theta E[y_k\epsilon_k] \\ -\theta E[y_k\epsilon_k] & \theta^2 E[\epsilon_k\epsilon_k] \end{pmatrix}.$$

Since $E[y_k y_k] = \gamma_\circ = \frac{(1+\theta^2-2\phi\theta)\sigma^2}{(1-\phi^2)}$ (Wei (1990))

$$P_{1|0} = \begin{pmatrix} \gamma_\circ & -\theta \\ -\theta & \theta^2 \end{pmatrix}.$$

Using $P_{1|0} = S_{1|0} S_{1|0}^{\mathrm{T}}$ gives an initial Cholesky 'square root' of

$$S_{1|0} = \begin{pmatrix} \sqrt{\gamma_\circ} & 0 \\ \frac{-\theta}{\sqrt{\gamma_\circ}} & \theta\sqrt{\frac{\gamma_\circ-1}{\gamma_\circ}} \end{pmatrix}.$$

10.1 Program Text

```
/* nag_kalman_sqrt_filt_cov_var (g13eac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group
 *
 * Mark 26, 2016.
 *
 */

#include <nag.h>
#include <math.h>
#include <stdio.h>
#include <string.h>
#include <nag_stdlib.h>
#include <nage04.h>
#include <nagf06.h>
#include <nagg05.h>
#include <nagg13.h>
#include <nagx02.h>

#ifdef __cplusplus
extern "C"
{
#endif
  static void NAG_CALL objfun(Integer n, const double theta_phi[],
                              double *objf, double g[], Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

static int ex1(void);
static int ex2(void);

int main(void)
{
  /* Integer scalar and array declarations */
  Integer exit_status_ex1 = 0;
  Integer exit_status_ex2 = 0;

  printf("nag_kalman_sqrt_filt_cov_var (g13eac) Example Program Results\n\n");

  /* Run example 1 */
  exit_status_ex1 = ex1();

  /* Run example 2 */
  exit_status_ex2 = ex2();

  return (exit_status_ex1 == 0 && exit_status_ex2 == 0) ? 0 : 1;
}

/* Start of the first example ... */
#define A(I, J) a[(I) *tda + J]
#define B(I, J) b[(I) *tdb + J]
#define C(I, J) c[(I) *tdc + J]
```

```
#define K(I, J) k[(I) *tdk + J]
#define Q(I, J) q[(I) *tdq + J]
#define R(I, J) r[(I) *tdr + J]
#define S(I, J) s[(I) *tds + J]

static int ex1()
{
  /* Integer scalar and array declarations */
  Integer exit_status = 0;
  Integer i, j, m, n, p, istep, tda, tdb, tdc, tdk, tdq, tdr, tds, tdh;

  /* Double scalar and array declarations */
  double tol;
  double *a = 0, *b = 0, *c = 0, *k = 0, *q = 0, *r = 0, *s = 0, *h = 0;

  /* NAG structures */
  NagError fail;

  /* Initialize the error structure */
  INIT_FAIL(fail);

  printf("Example 1\n");

  /* Skip the heading in the data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  /* Get the problem size */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%lf", &n, &m, &p, &tol);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%lf", &n, &m, &p, &tol);
#endif
  if (n < 1 || m < 1 || p < 1) {
    printf("Invalid n or m or p.\n");
    exit_status = 1;
    goto END;
  }

  tda = tdc = tds = n;
  tdb = tdq = m;
  tdk = tdr = tdh = p;

  /* Allocate arrays */
  if (!(a = NAG_ALLOC(n * tda, double)) ||
      !(b = NAG_ALLOC(n * tdb, double)) ||
      !(c = NAG_ALLOC(p * tdc, double)) ||
      !(k = NAG_ALLOC(n * tdk, double)) ||
      !(q = NAG_ALLOC(m * tdq, double)) ||
      !(r = NAG_ALLOC(p * tdr, double)) ||
      !(s = NAG_ALLOC(n * tds, double)) || !(h = NAG_ALLOC(n * tdh, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Read data */
  for (i = 0; i < n; ++i)
    for (j = 0; j < n; ++j)
#ifdef _WIN32
      scanf_s("%lf", &S(i, j));
#else
      scanf("%lf", &S(i, j));
#endif
  for (i = 0; i < n; ++i)
    for (j = 0; j < n; ++j)
#ifdef _WIN32
```

```
      scanf_s("%lf", &A(i, j));
#else
      scanf("%lf", &A(i, j));
#endif
  for (i = 0; i < n; ++i)
    for (j = 0; j < m; ++j)
#ifdef _WIN32
      scanf_s("%lf", &B(i, j));
#else
      scanf("%lf", &B(i, j));
#endif
  if (q) {
    for (i = 0; i < m; ++i)
      for (j = 0; j < m; ++j)
#ifdef _WIN32
        scanf_s("%lf", &Q(i, j));
#else
        scanf("%lf", &Q(i, j));
#endif
  }
  for (i = 0; i < p; ++i)
    for (j = 0; j < n; ++j)
#ifdef _WIN32
      scanf_s("%lf", &C(i, j));
#else
      scanf("%lf", &C(i, j));
#endif
  for (i = 0; i < p; ++i)
    for (j = 0; j < p; ++j)
#ifdef _WIN32
      scanf_s("%lf", &R(i, j));
#else
      scanf("%lf", &R(i, j));
#endif

  /* Perform three iterations of the Kalman filter recursion */
  for (istep = 1; istep <= 3; ++istep) {
    /* Run the kalman filter */
    nag_kalman_sqrt_filt_cov_var(n, m, p, s, tds, a, tda, b, tdb, q, tdq,
                                 c, tdc, r, tdr, k, tdk, h, tdh, tol, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_kalman_sqrt_filt_cov_var (g13eac).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }
  }

  /* Print the results */
  printf("\nThe square root of the state covariance matrix is\n\n");
  for (i = 0; i < n; ++i) {
    for (j = 0; j < n; ++j)
      printf("%8.4f ", S(i, j));
    printf("\n");
  }
  if (k) {
    printf("\nThe matrix AK (the product of the Kalman gain\n");
    printf("matrix with the state transition matrix) is\n\n");
    for (i = 0; i < n; ++i) {
      for (j = 0; j < p; ++j)
        printf("%8.4f ", K(i, j));
      printf("\n");
    }
  }

END:
  NAG_FREE(a);
  NAG_FREE(b);
  NAG_FREE(c);
  NAG_FREE(k);
  NAG_FREE(q);
```

```
  NAG_FREE(r);
  NAG_FREE(s);
  NAG_FREE(h);

  return exit_status;
}

/* ... end of the first example */

/* Start of the second example ... */
#define NY 2000

/* This illustrates the use of the kalman filter to estimate the
   parameters of an ARMA(1,1) time series model.
   Note : theta_phi[0] contains theta (moving average coefficient), and
   theta_phi[1] contains phi (autoregressive coefficient)
 */
static int ex2(void)
{
  /* Integer scalar and array declarations */
  Integer exit_status = 0;
  Integer n, lr;
  Integer lstate;
  Integer *state = 0;

  /* Double scalar and array declarations */
  double a[1], sy[NY];
  double *theta_phi = 0, *g = 0, *bl = 0, *bu = 0, *r = 0;
  double objf, var;

  /* NAG structures and data types */
  Nag_Comm comm;
  Nag_E04_Opt options;
  NagError fail;
  Nag_ModeRNG mode;

  /* Choose the base generator */
  Nag_BaseRNG genid = Nag_Basic;
  Integer subid = 0;

  /* Set the seed */
  Integer seed[] = { 1762543 };
  Integer lseed = 1;

  /* Set the autoregressive coefficients for the (randomly generated)
     time series */
  Integer ip = 1;
  double sphi[] = { 0.4 };

  /* Set the moving average coefficients for the (randomly generated)
     time series */
  Integer iq = 1;
  double stheta[] = { 0.9 };

  /* Number of terms in the (randomly generated) time series */
  Integer nterms = NY;

  /* Mean and variance of the (randomly generated) time series */
  double mean = 0.0, vara = 1.0;

  /* Initialize the error structure */
  INIT_FAIL(fail);

  printf("\n\nExample 2\n\n");

  /* Get the length of the state array */
  lstate = -1;
  nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n",
           fail.message);
```

```
    exit_status = 1;
    goto END;
  }

  /* Calculate the size of the reference vector */
  lr = (ip > iq + 1) ? ip : iq + 1;
  lr += ip + iq + 6;

  /* Allocate arrays */
  if (!(state = NAG_ALLOC(lstate, Integer)) || !(r = NAG_ALLOC(lr, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Initialize the generator to a repeatable sequence */
  nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  /* Generate a time series with nterms terms */
  mode = Nag_InitializeAndGenerate;
  nag_rand_arma(mode, nterms, mean, ip, sphi, iq, stheta, vara, r, lr, state,
                &var, sy, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_arma (g05phc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* Number of parameters to estimate */
  n = ip + iq;

  /* Allocate arrays */
  if (!(theta_phi = NAG_ALLOC(n, double)) ||
      !(g = NAG_ALLOC(n, double)) ||
      !(bl = NAG_ALLOC(n, double)) || !(bu = NAG_ALLOC(n, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Make an initial guess of the parameters */
  theta_phi[0] = 0.6;
  theta_phi[1] = 0.4;

  /* Set the bounds */
  bl[0] = -1.0;
  bu[0] = 1.0;
  bl[1] = -1.0;
  bu[1] = 1.0;
  comm.user = &sy[0];

  /* nag_opt_init (e04xxc).
   * Initialization function for option setting
   */
  nag_opt_init(&options);
#ifdef _WIN32
  strcpy_s(options.outfile, (unsigned)_countof(options.outfile), "stdout");
#else
  strcpy(options.outfile, "stdout");
#endif
  options.print_level = Nag_NoPrint;
  options.list = Nag_FALSE;
```

```
  options.obj_deriv = Nag_FALSE;
  options.con_deriv = Nag_FALSE;
  options.print_deriv = Nag_D_NoPrint;

  /* nag_opt_nlp (e04ucc).
   * Minimization with nonlinear constraints using a sequential QP method.
   */
  nag_opt_nlp(n, 0, 0, a, 1, bl, bu, objfun, NULLFN, theta_phi, &objf,
              g, &options, &comm, &fail);
  if (fail.code != NE_NOERROR && fail.code != NW_KT_CONDITIONS) {
    printf("Error from nag_opt_nlp (e04ucc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* nag_opt_free (e04xzc).
   * Memory freeing function for use with option setting
   */
  nag_opt_free(&options, "all", &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_opt_free (e04xzc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* Display the results */
  printf("The estimates are :  theta = %7.3f, phi = %7.3f \n",
         theta_phi[0], theta_phi[1]);

END:
  NAG_FREE(state);
  NAG_FREE(theta_phi);
  NAG_FREE(g);
  NAG_FREE(bl);
  NAG_FREE(bu);
  NAG_FREE(r);

  return exit_status;
}

/* Define  objective function used in the non-linear optimization routine ... */

static void NAG_CALL objfun(Integer n, const double theta_phi[], double *objf,
                            double g[], Nag_Comm *comm)
{
  /* Routine to evaluate objective function. */

  Integer ione = 1, itwo = 2, k, m1 = 1, n1 = 2, nsteps = NY, nsum = 0, p1 =
         1;
  double a[2][2], ak[2][1], b[2][1], c[1][2], h[1][1], hs, k11, logdet = 0.0;
  double phi, q[1][1], r[1][1];
  double s[2][2], ss = 0.0, temp1, temp2, theta, tol = 0.0;
  double v, xp[2], *y;
  NagError fail;

  /* Initialize the error structure */
  INIT_FAIL(fail);

  y = comm->user;
  /* The expectation of the mean of an ARMA(1,1) is 0.0 */
  xp[0] = 0.0;
  xp[1] = 0.0;
  q[0][0] = 1.0;

  c[0][0] = 1.0;
  c[0][1] = 0.0;

  /* There is no measurement noise */
  r[0][0] = 0.0;
  theta = theta_phi[0];
  phi = theta_phi[1];
```

```
  b[0][0] = 1.0;
  b[1][0] = -theta;

  a[0][0] = phi;
  a[1][0] = 0.0;
  a[0][1] = 1.0;
  a[1][1] = 0.0;

  /* set value for Cholesky factor of state covariance matrix  */
  temp1 = 1.0 + (theta * theta) - (2.0 * theta * phi);
  temp2 = 1.0 - (phi * phi);
  k11 = temp1 / temp2;
  s[0][0] = sqrt(k11);
  s[0][1] = 0.0;
  s[1][0] = -theta / s[0][0];
  s[1][1] = theta * sqrt(1.0 - (1.0 / k11));

  /* iterate kalman filter for number of observations  */
  for (k = 1; k <= nsteps; ++k) {
    /* nag_kalman_sqrt_filt_cov_var (g13eac).
     * One iteration step of the time-varying Kalman filter
     * recursion using the square root covariance implementation
     */
    nag_kalman_sqrt_filt_cov_var(n1, m1, p1, &s[0][0], itwo, &a[0][0], itwo,
                                 &b[0][0], ione, &q[0][0], ione, &c[0][0],
                                 itwo, &r[0][0], ione, &ak[0][0], ione,
                                 &h[0][0], ione, tol, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from  nag_kalman_sqrt_filt_cov_var (g13eac).\n%s\n",
             fail.message);
      *objf = 0.0;
      return;
    }

    v = y[k - 1] - c[0][0] * xp[0];
    hs = h[0][0] * h[0][0];
    logdet = logdet + log(hs);
    ss = ss + (v * v / hs);
    nsum = nsum + 1;

    xp[0] = a[0][0] * xp[0] + a[0][1] * xp[1] + ak[0][0] * v;
    xp[1] = ak[1][0] * v;
  }
  *objf = nsum * log(ss / nsum) + logdet;
}

/* ... end of the objective function definition */
/* ... end of the second example */
```

10.2 Program Data

None.

10.3 Program Results

```
nag_kalman_sqrt_filt_cov_var (g13eac) Example Program Results

Example 1

The square root of the state covariance matrix is

 -1.2936   0.0000   0.0000   0.0000
 -1.1382  -0.2579   0.0000   0.0000
 -0.9622  -0.1529   0.2974   0.0000
 -1.3076   0.0936   0.4508  -0.4897

The matrix AK (the product of the Kalman gain
matrix with the state transition matrix) is
```

```
  0.3638   0.9469
  0.3532   0.8179
  0.2471   0.5542
  0.1982   0.6471

Example 2

The estimates are :  theta =   0.898, phi =   0.406
```