NAG Library Function Document

nag_regsn_quant_linear_iid (g02qfc)

1 Purpose

nag_regsn_quant_linear_iid (g02qfc) performs a multiple linear quantile regression, returning the parameter estimates and associated confidence limits based on an assumption of Normal, independent, identically distributed errors. nag_regsn_quant_linear_iid (g02qfc) is a simplified version of nag_regsn_quant_linear (g02qgc).

2 Specification

```
#include <nag.h>
#include <nagg02.h>
```

```
void nag_regsn_quant_linear_iid (Integer n, Integer m, const double x[],
     const double y[], Integer ntau, const double tau[], double *df,
     double b[], double bl[], double bu[], Integer info[], NagError *fail)
```

3 Description

Given a vector of n observed values, $y = \{y_i : i = 1, 2, \ldots, n\}$, an $n \times p$ design matrix X, a column vector, x, of length p holding the ith row of X and a quantile $\tau \in (0, 1)$, nag_regsn_quant_linear_iid (g02qfc) estimates the p-element vector β as the solution to

$$\underset{\beta \in \mathbb{R}^p}{\text{minimize}} \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^{\mathrm{T}} \beta \right) \tag{1}$$

where ρ_τ is the piecewise linear loss function $\rho_\tau(z) = z(\tau - I(z < 0))$, and $I(z < 0)$ is an indicator function taking the value 1 if $z < 0$ and 0 otherwise.

nag_regsn_quant_linear_iid (g02qfc) assumes Normal, independent, identically distributed (IID) errors and calculates the asymptotic covariance matrix from

$$\Sigma = \frac{\tau(1 - \tau)}{n} (s\,(\tau))^2 \left(X^{\mathrm{T}} X \right)^{-1}$$

where s is the sparsity function, which is estimated from the residuals, $r_i = y_i - x_i^{\mathrm{T}} \hat{\beta}$ (see Koenker (2005)).

Given an estimate of the covariance matrix, $\hat{\Sigma}$, lower, $\hat{\beta}_L$, and upper, $\hat{\beta}_U$, limits for a 95% confidence interval are calculated for each of the p parameters, via

$$\hat{\beta}_{Li} = \hat{\beta}_i - t_{n-p,0.975} \sqrt{\hat{\Sigma}_{ii}}, \hat{\beta}_{Ui} = \hat{\beta}_i + t_{n-p,0.975} \sqrt{\hat{\Sigma}_{ii}}$$

where $t_{n-p,0.975}$ is the 97.5 percentile of the Student's t distribution with $n - k$ degrees of freedom, where k is the rank of the cross-product matrix $X^{\mathrm{T}} X$.

Further details of the algorithms used by nag_regsn_quant_linear_iid (g02qfc) can be found in the documentation for nag_regsn_quant_linear (g02qgc).

4 References

Koenker R (2005) *Quantile Regression* Econometric Society Monographs, Cambridge University Press, New York

5 Arguments

1: **n** – Integer *Input*

On entry: n, the number of observations in the dataset.

Constraint: $\mathbf{n} \geq 2$.

2: **m** – Integer *Input*

On entry: p, the number of variates in the model.

Constraint: $1 \leq \mathbf{m} < \mathbf{n}$.

3: **x**[**n** × **m**] – const double *Input*

Note: where $\mathbf{X}(i,j)$ appears in this document, it refers to the array element $\mathbf{x}[(i-1) \times \mathbf{m} + j - 1]$.

On entry: X, the design matrix, with the ith value for the jth variate supplied in $\mathbf{X}(i,j)$, for $i = 1, 2, \ldots, \mathbf{n}$ and $j = 1, 2, \ldots, \mathbf{m}$.

4: **y**[**n**] – const double *Input*

On entry: y, the observations on the dependent variable.

5: **ntau** – Integer *Input*

On entry: the number of quantiles of interest.

Constraint: $\mathbf{ntau} \geq 1$.

6: **tau**[**ntau**] – const double *Input*

On entry: the vector of quantiles of interest. A separate model is fitted to each quantile.

Constraint: $\sqrt{\epsilon} < \mathbf{tau}[l-1] < 1 - \sqrt{\epsilon}$ where ϵ is the ***machine precision*** returned by nag_machine_precision (X02AJC), for $l = 1, 2, \ldots, \mathbf{ntau}$.

7: **df** – double * *Output*

On exit: the degrees of freedom given by $n - k$, where n is the number of observations and k is the rank of the cross-product matrix $X^{\mathrm{T}}X$.

8: **b**[**m** × **ntau**] – double *Output*

Note: where $\mathbf{B}(j,l)$ appears in this document, it refers to the array element $\mathbf{b}[(l-1) \times \mathbf{m} + j - 1]$.

On exit: $\hat{\beta}$, the estimates of the parameters of the regression model, with $\mathbf{B}(j,l)$ containing the coefficient for the variable in column j of $\mathbf{X}$, estimated for $\tau = \mathbf{tau}[l-1]$.

9: **bl**[**m** × **ntau**] – double *Output*

Note: where $\mathbf{BL}(j,l)$ appears in this document, it refers to the array element $\mathbf{bl}[(l-1) \times \mathbf{m} + j - 1]$.

On exit: $\hat{\beta}_L$, the lower limit of a 95% confidence interval for $\hat{\beta}$, with $\mathbf{BL}(j,l)$ holding the lower limit associated with $\mathbf{B}(j,l)$.

10: **bu**[**m** × **ntau**] – double *Output*

Note: where $\mathbf{BU}(j,l)$ appears in this document, it refers to the array element $\mathbf{bu}[(l-1) \times \mathbf{m} + j - 1]$.

On exit: $\hat{\beta}_U$, the upper limit of a 95% confidence interval for $\hat{\beta}$, with $\mathbf{BU}(j, l)$ holding the upper limit associated with $\mathbf{B}(j, l)$.

11: **info**[**ntau**] – Integer *Output*

On exit: **info**$[l]$ holds additional information concerning the model fitting and confidence limit calculations when $\tau = \mathbf{tau}[l]$.

Code	Warning
0	Model fitted and confidence limits calculated successfully.
1	The function did not converge whilst calculating the parameter estimates. The returned values are based on the estimate at the last iteration.
2	A singular matrix was encountered during the optimization. The model was not fitted for this value of τ.
8	The function did not converge whilst calculating the confidence limits. The returned limits are based on the estimate at the last iteration.
16	Confidence limits for this value of τ could not be calculated. The returned upper and lower limits are set to a large positive and large negative value respectively.

It is possible for multiple warnings to be applicable to a single model. In these cases the value returned in **info** is the sum of the corresponding individual nonzero warning codes.

12: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 2.

On entry, **ntau** = ⟨*value*⟩.
Constraint: **ntau** ≥ 1.

NE_INT_2

On entry, **m** = ⟨*value*⟩ and **n** = ⟨*value*⟩.
Constraint: $1 \leq \mathbf{m} < \mathbf{n}$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL_ARRAY

On entry, **tau**[⟨*value*⟩] = ⟨*value*⟩ is invalid.

NW_POTENTIAL_PROBLEM

A potential problem occurred whilst fitting the model(s).
Additional information has been returned in **info**.

7 Accuracy

Not applicable.

8 Parallelism and Performance

nag_regsn_quant_linear_iid (g02qfc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_regsn_quant_linear_iid (g02qfc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Calling nag_regsn_quant_linear_iid (g02qfc) is equivalent to calling nag_regsn_quant_linear (g02qgc) with

order = Nag_RowMajor, **intcpt** = Nag_NoIntercept,

no weights supplied, i.e., **wt** set to **NULL**,

pddat = **m**,

setting each element of **isx** to 1,

ip = **m**,

Interval Method = IID, and

Significance Level = 0.95.

10 Example

A quantile regression model is fitted to Engels 1857 study of household expenditure on food. The model regresses the dependent variable, household food expenditure, against household income. An intercept is included in the model by augmenting the dataset with a column of ones.

10.1 Program Text

```
/* nag_regsn_quant_linear_iid (g02qfc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
```

```
 */
/* Pre-processor includes */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg02.h>

#define B(i,j) b[(j) * m + i]
#define BU(i,j) bu[(j) * m + i]
#define BL(i,j) bl[(j) * m + i]
#define X(i,j) x[(i) * m + j]

int main(void)
{
  /* Integer scalar and array declarations */
  Integer i, j, l, m, n, ntau;
  Integer *info = 0;
  Integer exit_status = 0;

  /* NAG structures */
  NagError fail;

  /* Double scalar and array declarations */
  double df;
  double *b = 0, *bl = 0, *bu = 0, *tau = 0, *x = 0, *y = 0;

  /* Initialize the error structure */
  INIT_FAIL(fail);

  printf("nag_regsn_quant_linear_iid (g02qfc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* Read in the problem size */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &n, &m, &ntau);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &n, &m, &ntau);
#endif

  /* Allocate memory for input arrays */
  if (!(y = NAG_ALLOC(n, double)) ||
      !(tau = NAG_ALLOC(ntau, double)) || !(x = NAG_ALLOC(n * m, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Read in the data */
  for (i = 0; i < n; i++) {
    for (j = 0; j < m; j++)
#ifdef _WIN32
      scanf_s("%lf", &X(i, j));
#else
      scanf("%lf", &X(i, j));
#endif
#ifdef _WIN32
    scanf_s("%lf", &y[i]);
#else
    scanf("%lf", &y[i]);
#endif
  }
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
```

```
  scanf("%*[^\n] ");
#endif

  /* Read in the quantiles required */
  for (l = 0; l < ntau; l++) {
#ifdef _WIN32
    scanf_s("%lf", &tau[l]);
#else
    scanf("%lf", &tau[l]);
#endif
  }
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* Allocate memory for output arrays */
  if (!(b = NAG_ALLOC(m * ntau, double)) ||
      !(info = NAG_ALLOC(ntau, Integer)) ||
      !(bl = NAG_ALLOC(m * ntau, double)) ||
      !(bu = NAG_ALLOC(m * ntau, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* nag_regsn_quant_linear_iid (g02qfc).Quantile linear regression, simple
     interface, independent, identically distributed (IID) errors */
  nag_regsn_quant_linear_iid(n, m, x, y, ntau, tau, &df, b, bl, bu, info,
                             &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_regsn_quant_linear_iid (g02qfc).\n%s\n",
           fail.message);
    if (fail.code == NW_POTENTIAL_PROBLEM) {
      printf("Additional error information: ");
      for (i = 0; i < ntau; i++)
        printf("%" NAG_IFMT " ", info[i]);
      printf("\n");
    }
    else {
      exit_status = 1;
      goto END;
    }
  }

  /* Display the parameter estimates */
  for (l = 0; l < ntau; l++) {
    printf(" Quantile: %6.3f\n\n", tau[l]);
    printf("         Lower   Parameter   Upper\n");
    printf("         Limit   Estimate    Limit\n");
    for (j = 0; j < m; j++) {
      printf(" %3" NAG_IFMT "   %7.3f   %7.3f   %7.3f\n", j + 1, BL(j, l),
             B(j, l), BU(j, l));
    }
    printf("\n\n");
  }

END:

  NAG_FREE(info);
  NAG_FREE(b);
  NAG_FREE(bl);
  NAG_FREE(bu);
  NAG_FREE(tau);
  NAG_FREE(x);
  NAG_FREE(y);

  return (exit_status);
}
```

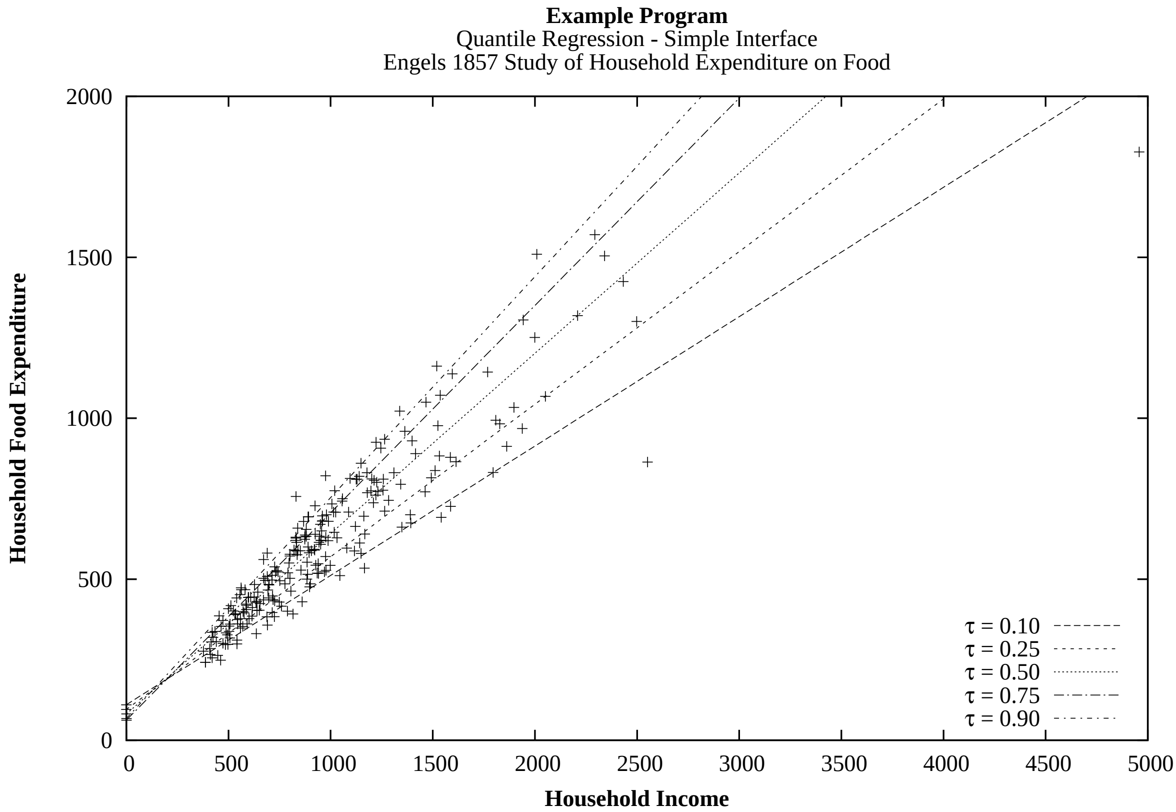
10.2 Program Data

```
nag_regsn_quant_linear_iid (g02qfc) Example Program Data
      235       2          5                                  :: n, m, ntau
1.0    420.1577  255.8394    1.0    800.7990   572.0807   1.0    643.3571   459.8177
1.0    541.4117  310.9587    1.0   1245.6964   907.3969   1.0   2551.6615   863.9199
1.0    901.1575  485.6800    1.0   1201.0002   811.5776   1.0   1795.3226   831.4407
1.0    639.0802  402.9974    1.0    634.4002   427.7975   1.0   1165.7734   534.7610
1.0    750.8756  495.5608    1.0    956.2315   649.9985   1.0    815.6212   392.0502
1.0    945.7989  633.7978    1.0   1148.6010   860.6002   1.0   1264.2066   934.9752
1.0    829.3979  630.7566    1.0   1768.8236  1143.4211   1.0   1095.4056   813.3081
1.0    979.1648  700.4409    1.0   2822.5330  2032.6792   1.0    447.4479   263.7100
1.0   1309.8789  830.9586    1.0    922.3548   590.6183   1.0   1178.9742   769.0838
1.0   1492.3987  815.3602    1.0   2293.1920  1570.3911   1.0    975.8023   630.5863
1.0    502.8390  338.0014    1.0    627.4726   483.4800   1.0   1017.8522   645.9874
1.0    616.7168  412.3613    1.0    889.9809   600.4804   1.0    423.8798   319.5584
1.0    790.9225  520.0006    1.0   1162.2000   696.2021   1.0    558.7767   348.4518
1.0    555.8786  452.4015    1.0   1197.0794   774.7962   1.0    943.2487   614.5068
1.0    713.4412  512.7201    1.0    530.7972   390.5984   1.0   1348.3002   662.0096
1.0    838.7561  658.8395    1.0   1142.1526   612.5619   1.0   2340.6174  1504.3708
1.0    535.0766  392.5995    1.0   1088.0039   708.7622   1.0    587.1792   406.2180
1.0    596.4408  443.5586    1.0    484.6612   296.9192   1.0   1540.9741   692.1689
1.0    924.5619  640.1164    1.0   1536.0201  1071.4627   1.0   1115.8481   588.1371
1.0    487.7583  333.8394    1.0    678.8974   496.5976   1.0   1044.6843   511.2609
1.0    692.6397  466.9583    1.0    671.8802   503.3974   1.0   1389.7929   700.5600
1.0    997.8770  543.3969    1.0    690.4683   357.6411   1.0   2497.7860  1301.1451
1.0    506.9995  317.7198    1.0    860.6948   430.3376   1.0   1585.3809   879.0660
1.0    654.1587  424.3209    1.0    873.3095   624.6990   1.0   1862.0438   912.8851
1.0    933.9193  518.9617    1.0    894.4598   582.5413   1.0   2008.8546  1509.7812
1.0    433.6813  338.0014    1.0   1148.6470   580.2215   1.0    697.3099   484.0605
1.0    587.5962  419.6412    1.0    926.8762   543.8807   1.0    571.2517   399.6703
1.0    896.4746  476.3200    1.0    839.0414   588.6372   1.0    598.3465   444.1001
1.0    454.4782  386.3602    1.0    829.4974   627.9999   1.0    461.0977   248.8101
1.0    584.9989  423.2783    1.0   1264.0043   712.1012   1.0    977.1107   527.8014
1.0    800.7990  503.3572    1.0   1937.9771   968.3949   1.0    883.9849   500.6313
1.0    502.4369  354.6389    1.0    698.8317   482.5816   1.0    718.3594   436.8107
1.0    713.5197  497.3182    1.0    920.4199   593.1694   1.0    543.8971   374.7990
1.0    906.0006  588.5195    1.0   1897.5711  1033.5658   1.0   1587.3480   726.3921
1.0    880.5969  654.5971    1.0    891.6824   693.6795   1.0   4957.8130  1827.2000
1.0    796.8289  550.7274    1.0    889.6784   693.6795   1.0    969.6838   523.4911
1.0    854.8791  528.3770    1.0   1221.4818   761.2791   1.0    419.9980   334.9998
1.0   1167.3716  640.4813    1.0    544.5991   361.3981   1.0    561.9990   473.2009
1.0    523.8000  401.3204    1.0   1031.4491   628.4522   1.0    689.5988   581.2029
1.0    670.7792  435.9990    1.0   1462.9497   771.4486   1.0   1398.5203   929.7540
1.0    377.0584  276.5606    1.0    830.4353   757.1187   1.0    820.8168   591.1974
1.0    851.5430  588.3488    1.0    975.0415   821.5970   1.0    875.1716   637.5483
1.0   1121.0937  664.1978    1.0   1337.9983  1022.3202   1.0   1392.4499   674.9509
1.0    625.5179  444.8602    1.0    867.6427   679.4407   1.0   1256.3174   776.7589
1.0    805.5377  462.8995    1.0    725.7459   538.7491   1.0   1362.8590   959.5170
1.0    558.5812  377.7792    1.0    989.0056   679.9981   1.0   1999.2552  1250.9643
1.0    884.4005  553.1504    1.0   1525.0005   977.0033   1.0   1209.4730   737.8201
1.0   1257.4989  810.8962    1.0    672.1960   561.2015   1.0   1125.0356   810.6772
1.0   2051.1789 1067.9541    1.0    923.3977   728.3997   1.0   1827.4010   983.0009
1.0   1466.3330 1049.8788    1.0    472.3215   372.3186   1.0   1014.1540   708.8968
1.0    730.0989  522.7012    1.0    590.7601   361.5210   1.0    880.3944   633.1200
1.0   2432.3910 1424.8047    1.0    831.7983   620.8006   1.0    873.7375   631.7982
1.0    940.9218  517.9196    1.0   1139.4945   819.9964   1.0    951.4432   608.6419
1.0   1177.8547  830.9586    1.0    507.5169   360.8780   1.0    473.0022   300.9999
1.0   1222.5939  925.5795    1.0    576.1972   395.7608   1.0    601.0030   377.9984
1.0   1519.5811 1162.0024    1.0    696.5991   442.0001   1.0    713.9979   397.0015
1.0    687.6638  383.4580    1.0    650.8180   404.0384   1.0    829.2984   588.5195
1.0    953.1192  621.1173    1.0    949.5802   670.7993   1.0    959.7953   681.7616
1.0    953.1192  621.1173    1.0    497.1193   297.5702   1.0   1212.9613   807.3603
1.0    953.1192  621.1173    1.0    570.1674   353.4882   1.0    958.8743   696.8011
1.0    939.0418  548.6002    1.0    724.7306   383.9376   1.0   1129.4431   811.1962
1.0   1283.4025  745.2353    1.0    408.3399   284.8008   1.0   1943.0419  1305.7201
1.0   1511.5789  837.8005    1.0    638.6713   431.1000   1.0    539.6388   442.0001
1.0   1342.5821  795.3402    1.0   1225.7890   801.3518   1.0    463.5990   353.6013
1.0    511.7980  418.5976    1.0    715.3701   448.4513   1.0    562.6400   468.0008
1.0    689.7988  508.7974    1.0    800.4708   577.9111   1.0    736.7584   526.7573
1.0   1532.3074  883.2780    1.0    975.5974   570.5210   1.0   1415.4461   890.2390
```

```
1.0  1056.0808  742.5276    1.0  1613.7565  865.3205   1.0  2208.7897 1318.8033
1.0   387.3195  242.3202    1.0   608.5019  444.5578   1.0   636.0009  331.0005
1.0   387.3195  242.3202    1.0   958.6634  680.4198   1.0   759.4010  416.4015
1.0   410.9987  266.0010    1.0   835.9426  576.2779   1.0  1078.8382  596.8406
1.0   499.7510  408.4992    1.0  1024.8177  708.4787   1.0   748.6413  429.0399
1.0   832.7554  614.7588    1.0  1006.4353  734.2356   1.0   987.6417  619.6408
1.0   614.9986  385.3184    1.0   726.0000  433.0010   1.0   788.0961  400.7990
1.0   887.4658  515.6200    1.0   494.4174  327.4188   1.0  1020.0225  775.0209
1.0  1595.1611 1138.1620    1.0   776.5958  485.5198   1.0  1230.9235  772.7611
1.0  1807.9520  993.9630    1.0   415.4407  305.4390   1.0   440.5174  306.5191
1.0   541.2006  299.1993    1.0   581.3599  468.0008   1.0   743.0772  522.6019
1.0  1057.6767  750.3202                                   :: (x[1..m],y)[1..n]
0.10    0.25      0.50      0.75    0.90                   :: tau[1..ntau]
```

10.3 Program Results

```
nag_regsn_quant_linear_iid (g02qfc) Example Program Results

 Quantile:  0.100

        Lower   Parameter   Upper
        Limit   Estimate    Limit
   1    74.946   110.142   145.337
   2     0.370     0.402     0.433


 Quantile:  0.250

        Lower   Parameter   Upper
        Limit   Estimate    Limit
   1    64.232    95.483   126.735
   2     0.446     0.474     0.502


 Quantile:  0.500

        Lower   Parameter   Upper
        Limit   Estimate    Limit
   1    55.399    81.482   107.566
   2     0.537     0.560     0.584


 Quantile:  0.750

        Lower   Parameter   Upper
        Limit   Estimate    Limit
   1    41.372    62.396    83.421
   2     0.625     0.644     0.663


 Quantile:  0.900

        Lower   Parameter   Upper
        Limit   Estimate    Limit
   1    26.829    67.351   107.873
   2     0.650     0.686     0.723
```


Example Program
Quantile Regression - Simple Interface
Engels 1857 Study of Household Expenditure on Food
Household Food Expenditure
Household Income
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
τ = 0.10
τ = 0.25
τ = 0.50
τ = 0.75
τ = 0.90