

# NAG Library Function Document

## nag\_deviates\_students\_t (g01fbc)

### 1 Purpose

nag\_deviates\_students\_t (g01fbc) returns the deviate associated with the given tail probability of Student's  $t$ -distribution with real degrees of freedom.

### 2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_students_t (Nag_TailProbability tail, double p,
                                double df, NagError *fail)
```

### 3 Description

The deviate,  $t_p$  associated with the lower tail probability,  $p$ , of the Student's  $t$ -distribution with  $\nu$  degrees of freedom is defined as the solution to

$$P(T < t_p : \nu) = p = \frac{\Gamma((\nu + 1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT, \quad \nu \geq 1; -\infty < t_p < \infty.$$

For  $\nu = 1$  or  $2$  the integral equation is easily solved for  $t_p$ .

For other values of  $\nu < 3$  a transformation to the beta distribution is used and the result obtained from nag\_deviates\_beta (g01fec).

For  $\nu \geq 3$  an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

### 4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's  $t$ -distribution *Comm. ACM* **13(10)** 617–619

### 5 Arguments

1: **tail** – Nag\_TailProbability *Input*

*On entry:* indicates which tail the supplied probability represents.

**tail** = Nag\_UpperTail  
The upper tail probability, i.e.,  $P(T \geq t_p : \nu)$ .

**tail** = Nag\_LowerTail  
The lower tail probability, i.e.,  $P(T \leq t_p : \nu)$ .

**tail** = Nag\_TwoTailSignif  
The two tail (significance level) probability, i.e.,  $P(T \geq |t_p| : \nu) + P(T \leq -|t_p| : \nu)$ .

**tail** = Nag\_TwoTailConfid  
The two tail (confidence interval) probability, i.e.,  $P(T \leq |t_p| : \nu) - P(T \leq -|t_p| : \nu)$ .

*Constraint:* **tail** = Nag\_UpperTail, Nag\_LowerTail, Nag\_TwoTailSignif or Nag\_TwoTailConfid.

- 2: **p** – double *Input*  
*On entry:*  $p$ , the probability from the required Student's  $t$ -distribution as defined by **tail**.  
*Constraint:*  $0.0 < \mathbf{p} < 1.0$ .
- 3: **df** – double *Input*  
*On entry:*  $\nu$ , the degrees of freedom of the Student's  $t$ -distribution.  
*Constraint:*  $\mathbf{df} \geq 1.0$ .
- 4: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

On any of the error conditions listed below except **fail.code** = NE\_SOL\_NOT\_CONV nag\_deviates\_students\_t (g01fbc) returns 0.0.

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.  
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_REAL\_ARG\_GE

On entry,  $\mathbf{p} = \langle value \rangle$ .  
Constraint:  $\mathbf{p} < 1.0$ .

### NE\_REAL\_ARG\_LE

On entry,  $\mathbf{p} = \langle value \rangle$ .  
Constraint:  $\mathbf{p} > 0.0$ .

### NE\_REAL\_ARG\_LT

On entry,  $\mathbf{df} = \langle value \rangle$ .  
Constraint:  $\mathbf{df} \geq 1.0$ .

### NE\_SOL\_NOT\_CONV

The solution has failed to converge. However, the result should be a reasonable approximation.

## 7 Accuracy

The results should be accurate to five significant digits, for most argument values. The error behaviour for various argument values is discussed in Hill (1970).

## 8 Parallelism and Performance

nag\_deviates\_students\_t (g01fbc) is not threaded in any implementation.

## 9 Further Comments

The value  $t_p$  may be calculated by using the transformation described in Section 3 and using nag\_deviates\_beta (g01fec). This function allows you to set the required accuracy.

## 10 Example

This example reads the probability, the tail that probability represents and the degrees of freedom for a number of Student's  $t$ -distributions and computes the corresponding deviates.

### 10.1 Program Text

```

/* nag_deviates_students_t (g01fbc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0;
    double df, p, t;
    int i;
    static Nag_TailProbability tail[] = { Nag_LowerTail, Nag_UpperTail,
        Nag_TwoTailSignif, Nag_TwoTailConfid
    };
    static const char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
        "Nag_TwoTailSignif",
        "Nag_TwoTailConfid"
    };
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_deviates_students_t (g01fbc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("    p          df          tail          t\n\n");
#ifdef _WIN32
    while (scanf_s("%lf %lf %d", &p, &df, &i) != EOF)
#else
    while (scanf("%lf %lf %d", &p, &df, &i) != EOF)
#endif
    {

```

```

/* nag_deviates_students_t (g01fbc).
 * Deviates for Student's t-distribution
 */
t = nag_deviates_students_t(tail[i], p, df, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_deviates_students_t (g01fbc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
}
printf("%8.3f%8.3f   %-19s   %8.3f\n", p, df, tailmess[i], t);
}

END:
return exit_status;
}

```

## 10.2 Program Data

```

nag_deviates_students_t (g01fbc) Example Program Data
0.0100  20.0  2
0.01    7.5  0
0.99   45.0  3

```

## 10.3 Program Results

```

nag_deviates_students_t (g01fbc) Example Program Results

```

p	df	tail	t
0.010	20.000	Nag_TwoTailSignif	2.845
0.010	7.500	Nag_LowerTail	-2.943
0.990	45.000	Nag_TwoTailConfid	2.690

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