NAG Library Function Document

nag_prob_von_mises (g01erc)

1   Purpose

nag_prob_von_mises (g01erc) returns the probability associated with the lower tail of the von Mises distribution between $-\pi$ and π .

2   Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_prob_von_mises (double t, double vk, NagError *fail)
```

3   Description

The von Mises distribution is a symmetric distribution used in the analysis of circular data. The lower tail area of this distribution on the circle with mean direction $\mu_0 = 0$ and concentration argument kappa, κ, can be written as

$$\Pr(\Theta \leq \theta : \kappa) = \frac{1}{2\pi I_0(\kappa)} \int_{-\pi}^{\theta} e^{\kappa \cos \Theta} \mathrm{d}\Theta,$$

where θ is reduced modulo 2π so that $-\pi \leq \theta < \pi$ and $\kappa \geq 0$. Note that if $\theta = \pi$ then nag_prob_von_mises (g01erc) returns a probability of 1. For very small κ the distribution is almost the uniform distribution, whereas for $\kappa \to \infty$ all the probability is concentrated at one point.

The method of calculation for small κ involves backwards recursion through a series expansion in terms of modified Bessel functions, while for large κ an asymptotic Normal approximation is used.

In the case of small κ the series expansion of $\Pr(\Theta \leq \theta : \kappa)$ can be expressed as

$$\Pr(\Theta \leq \theta : \kappa) = \tfrac{1}{2} + \frac{\theta}{(2\pi)} + \frac{1}{\pi I_0(\kappa)} \sum_{n=1}^{\infty} n^{-1} I_n(\kappa) \sin n\theta,$$

where $I_n(\kappa)$ is the modified Bessel function. This series expansion can be represented as a nested expression of terms involving the modified Bessel function ratio R_n,

$$R_n(\kappa) = \frac{I_n(\kappa)}{I_{n-1}(\kappa)}, \quad n = 1, 2, 3, \ldots,$$

which is calculated using backwards recursion.

For large values of κ (see Section 7) an asymptotic Normal approximation is used. The angle Θ is transformed to the nearly Normally distributed variate Z,

$$Z = b(\kappa) \sin \tfrac{\Theta}{2},$$

where

$$b(\kappa) = \frac{\sqrt{\tfrac{2}{\pi}} e^{\kappa}}{I_0(\kappa)}$$

and $b(\kappa)$ is computed from a continued fraction approximation. An approximation to order κ^{-4} of the asymptotic normalizing series for z is then used. Finally the Normal probability integral is evaluated.

For a more detailed analysis of the methods used see Hill (1977).

4 References

Hill G W (1977) Algorithm 518: Incomplete Bessel function I_0: The Von Mises distribution *ACM Trans. Math. Software* **3** 279–284

Mardia K V (1972) *Statistics of Directional Data* Academic Press

5 Arguments

1: **t** – double   *Input*

    On entry: θ, the observed von Mises statistic measured in radians.

2: **vk** – double   *Input*

    On entry: the concentration parameter κ, of the von Mises distribution.

    Constraint: **vk** ≥ 0.0.

3: **fail** – NagError *   *Input/Output*

    The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL

On entry, **vk** $= \langle value \rangle$.
Constraint: **vk** ≥ 0.0.

7 Accuracy

nag_prob_von_mises (g01erc) uses one of two sets of constants depending on the value of ***machine precision***. One set gives an accuracy of six digits and uses the Normal approximation when **vk** ≥ 6.5, the other gives an accuracy of 12 digits and uses the Normal approximation when **vk** ≥ 50.0.

8 Parallelism and Performance

nag_prob_von_mises (g01erc) is not threaded in any implementation.

9 Further Comments

Using the series expansion for small κ the time taken by nag_prob_von_mises (g01erc) increases linearly with κ; for larger κ, for which the asymptotic Normal approximation is used, the time taken is much less.

If angles outside the region $-\pi \le \theta < \pi$ are used care has to be taken in evaluating the probability of being in a region $\theta_1 \le \theta \le \theta_2$ if the region contains an odd multiple of π, $(2n+1)\pi$. The value of $F(\theta_2;\kappa) - F(\theta_1;\kappa)$ will be negative and the correct probability should then be obtained by adding one to the value.

10 Example

This example inputs four values from the von Mises distribution along with the values of the argument κ. The probabilities are computed and printed.

10.1 Program Text

```
/* nag_prob_von_mises (g01erc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{

  /* Scalars */
  double p, t, vk;
  Integer exit_status, i__, n;
  NagError fail;

  INIT_FAIL(fail);

  exit_status = 0;

  printf("nag_prob_von_mises (g01erc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n] ", &n);
#else
  scanf("%" NAG_IFMT "%*[^\n] ", &n);
#endif

  for (i__ = 1; i__ <= n; ++i__) {
#ifdef _WIN32
    scanf_s("%lf%lf%*[^\n] ", &t, &vk);
#else
    scanf("%lf%lf%*[^\n] ", &t, &vk);
#endif
    /* nag_prob_von_mises (g01erc).
     * Computes probability for von Mises distribution
     */
    p = nag_prob_von_mises(t, vk, &fail);
```

```
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_prob_von_mises (g01erc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
    printf(" p = %10.4f\n", p);
  }
END:
  return exit_status;
}
```

10.2 Program Data

```
nag_prob_von_mises (g01erc) Example Program Data
 4
 7.0  0.0
 2.8  2.4
 1.0  1.0
-1.4  1.3
```

10.3 Program Results

```
nag_prob_von_mises (g01erc) Example Program Results

 p =     0.6141
 p =     0.9983
 p =     0.7944
 p =     0.1016
```