

## NAG Library Function Document

### nag\_sparse\_nherm\_fac\_sol (f11dq)

#### 1 Purpose

nag\_sparse\_nherm\_fac\_sol (f11dq) solves a complex sparse non-Hermitian system of linear equations, represented in coordinate storage format, using a restarted generalized minimal residual (RGMRES), conjugate gradient squared (CGS), stabilized bi-conjugate gradient (Bi-CGSTAB), or transpose-free quasi-minimal residual (TFQMR) method, with incomplete *LU* preconditioning.

#### 2 Specification

```
#include <nag.h>
#include <nagf11.h>

void nag_sparse_nherm_fac_sol (Nag_SparseNsym_Method method, Integer n,
    Integer nnz, const Complex a[], Integer la, const Integer irow[],
    const Integer icol[], const Integer ipivp[], const Integer ipivq[],
    const Integer istr[], const Integer idiag[], const Complex b[],
    Integer m, double tol, Integer maxitn, Complex x[], double *rnorm,
    Integer *itn, NagError *fail)
```

#### 3 Description

nag\_sparse\_nherm\_fac\_sol (f11dq) solves a complex sparse non-Hermitian linear system of equations

$$Ax = b,$$

using a preconditioned RGMRES (see Saad and Schultz (1986)), CGS (see Sonneveld (1989)), Bi-CGSTAB( $\ell$ ) (see Van der Vorst (1989) and Sleijpen and Fokkema (1993)), or TFQMR (see Freund and Nachtigal (1991) and Freund (1993)) method.

nag\_sparse\_nherm\_fac\_sol (f11dq) uses the incomplete *LU* factorization determined by nag\_sparse\_nherm\_fac (f11dnc) as the preconditioning matrix. A call to nag\_sparse\_nherm\_fac\_sol (f11dq) must always be preceded by a call to nag\_sparse\_nherm\_fac (f11dnc). Alternative preconditioners for the same storage scheme are available by calling nag\_sparse\_nherm\_sol (f11dsc).

The matrix  $A$ , and the preconditioning matrix  $M$ , are represented in coordinate storage (CS) format (see Section 2.1.1 in the f11 Chapter Introduction) in the arrays **a**, **irow** and **icol**, as returned from nag\_sparse\_nherm\_fac (f11dnc). The array **a** holds the nonzero entries in these matrices, while **irow** and **icol** hold the corresponding row and column indices.

#### 4 References

Freund R W (1993) A transpose-free quasi-minimal residual algorithm for non-Hermitian linear systems *SIAM J. Sci. Comput.* **14** 470–482

Freund R W and Nachtigal N (1991) QMR: a Quasi-Minimal Residual Method for Non-Hermitian Linear Systems *Numer. Math.* **60** 315–339

Saad Y and Schultz M (1986) GMRES: a generalized minimal residual algorithm for solving nonsymmetric linear systems *SIAM J. Sci. Statist. Comput.* **7** 856–869

Sleijpen G L G and Fokkema D R (1993) BiCGSTAB( $\ell$ ) for linear equations involving matrices with complex spectrum *ETNA* **1** 11–32

Sonneveld P (1989) CGS, a fast Lanczos-type solver for nonsymmetric linear systems *SIAM J. Sci. Statist. Comput.* **10** 36–52

Van der Vorst H (1989) Bi-CGSTAB, a fast and smoothly converging variant of Bi-CG for the solution of nonsymmetric linear systems *SIAM J. Sci. Statist. Comput.* **13** 631–644

## 5 Arguments

- 1: **method** – Nag\_SparseNsym\_Method *Input*  
*On entry:* specifies the iterative method to be used.  
**method** = Nag\_SparseNsym\_RGMRES  
 Restarted generalized minimum residual method.  
**method** = Nag\_SparseNsym\_CGS  
 Conjugate gradient squared method.  
**method** = Nag\_SparseNsym\_BiCGSTAB  
 Bi-conjugate gradient stabilized ( $\ell$ ) method.  
**method** = Nag\_SparseNsym\_TFQMR  
 Transpose-free quasi-minimal residual method.  
*Constraint:* **method** = Nag\_SparseNsym\_RGMRES, Nag\_SparseNsym\_CGS,  
 Nag\_SparseNsym\_BiCGSTAB or Nag\_SparseNsym\_TFQMR.
- 2: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ . This **must** be the same value as was supplied in the preceding call to nag\_sparse\_nherm\_fac (f11dnc).  
*Constraint:*  $n \geq 1$ .
- 3: **nnz** – Integer *Input*  
*On entry:* the number of nonzero elements in the matrix  $A$ . This **must** be the same value as was supplied in the preceding call to nag\_sparse\_nherm\_fac (f11dnc).  
*Constraint:*  $1 \leq \text{nnz} \leq n^2$ .
- 4: **a[la]** – const Complex *Input*  
*On entry:* the values returned in the array **a** by a previous call to nag\_sparse\_nherm\_fac (f11dnc).
- 5: **la** – Integer *Input*  
*On entry:* the dimension of the arrays **a**, **irow** and **icol**. This **must** be the same value as was supplied in the preceding call to nag\_sparse\_nherm\_fac (f11dnc).  
*Constraint:*  $la \geq 2 \times \text{nnz}$ .
- 6: **irow[la]** – const Integer *Input*  
 7: **icol[la]** – const Integer *Input*  
 8: **ipivp[n]** – const Integer *Input*  
 9: **ipivq[n]** – const Integer *Input*  
 10: **istr[n + 1]** – const Integer *Input*  
 11: **idiag[n]** – const Integer *Input*
- On entry:* the values returned in arrays **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** by a previous call to nag\_sparse\_nherm\_fac (f11dnc).  
**ipivp** and **ipivq** are restored on exit.

- 12: **b[n]** – const Complex *Input*  
*On entry:* the right-hand side vector  $b$ .
- 13: **m** – Integer *Input*  
*On entry:* if **method** = Nag\_SparseNsym\_RGMRES, **m** is the dimension of the restart subspace.  
 If **method** = Nag\_SparseNsym\_BiCGSTAB, **m** is the order  $\ell$  of the polynomial Bi-CGSTAB method.  
 Otherwise, **m** is not referenced.  
*Constraints:*  
 if **method** = Nag\_SparseNsym\_RGMRES,  $0 < \mathbf{m} \leq \min(\mathbf{n}, 50)$ ;  
 if **method** = Nag\_SparseNsym\_BiCGSTAB,  $0 < \mathbf{m} \leq \min(\mathbf{n}, 10)$ .
- 14: **tol** – double *Input*  
*On entry:* the required tolerance. Let  $x_k$  denote the approximate solution at iteration  $k$ , and  $r_k$  the corresponding residual. The algorithm is considered to have converged at iteration  $k$  if
- $$\|r_k\|_\infty \leq \tau \times (\|b\|_\infty + \|A\|_\infty \|x_k\|_\infty).$$
- If **tol**  $\leq 0.0$ ,  $\tau = \max(\sqrt{\epsilon}, 10\epsilon, \sqrt{n}\epsilon)$  is used, where  $\epsilon$  is the *machine precision*. Otherwise  $\tau = \max(\mathbf{tol}, 10\epsilon, \sqrt{n}\epsilon)$  is used.  
*Constraint:* **tol**  $< 1.0$ .
- 15: **maxitn** – Integer *Input*  
*On entry:* the maximum number of iterations allowed.  
*Constraint:* **maxitn**  $\geq 1$ .
- 16: **x[n]** – Complex *Input/Output*  
*On entry:* an initial approximation to the solution vector  $x$ .  
*On exit:* an improved approximation to the solution vector  $x$ .
- 17: **rnorm** – double \* *Output*  
*On exit:* the final value of the residual norm  $\|r_k\|_\infty$ , where  $k$  is the output value of **itn**.
- 18: **itn** – Integer \* *Output*  
*On exit:* the number of iterations carried out.
- 19: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ACCURACY

The required accuracy could not be obtained. However, a reasonable accuracy may have been achieved.

### NE\_ALG\_FAIL

Algorithmic breakdown. A solution is returned, although it is possible that it is completely inaccurate.

**NE\_ALLOC\_FAIL**

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_CONVERGENCE**

The solution has not converged after  $\langle value \rangle$  iterations.

**NE\_INT**

On entry, **maxitn** =  $\langle value \rangle$ .

Constraint: **maxitn**  $\geq 1$ .

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 1$ .

On entry, **nnz** =  $\langle value \rangle$ .

Constraint: **nnz**  $\geq 1$ .

**NE\_INT\_2**

On entry, **la** =  $\langle value \rangle$  and **nnz** =  $\langle value \rangle$ .

Constraint: **la**  $\geq 2 \times \mathbf{nnz}$ .

On entry, **m** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **m**  $\geq 1$  and **m**  $\leq \min(\mathbf{n}, \langle value \rangle)$ .

On entry, **nnz** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .

Constraint: **nnz**  $\leq \mathbf{n}^2$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_INVALID\_CS**

On entry,  $i = \langle value \rangle$ , **icol**[ $i - 1$ ] =  $\langle value \rangle$ , and **n** =  $\langle value \rangle$ .

Constraint: **icol**[ $i - 1$ ]  $\geq 1$  and **icol**[ $i - 1$ ]  $\leq \mathbf{n}$ .

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_fac_sol` (f11dq) and `nag_sparse_nherm_fac` (f11dnc).

On entry,  $i = \langle value \rangle$ , **irow**[ $i - 1$ ] =  $\langle value \rangle$ , **n** =  $\langle value \rangle$ .

Constraint: **irow**[ $i - 1$ ]  $\geq 1$  and **irow**[ $i - 1$ ]  $\leq \mathbf{n}$ .

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_fac_sol` (f11dq) and `nag_sparse_nherm_fac` (f11dnc).

**NE\_INVALID\_CS\_PRECOND**

The CS representation of the preconditioner is invalid.

Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to `nag_sparse_nherm_fac` (f11dnc) and `nag_sparse_nherm_fac_sol` (f11dq).

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_STRICTLY\_INCREASING**

On entry,  $\mathbf{a}[i-1]$  is out of order:  $i = \langle value \rangle$ .

Check that  $\mathbf{a}$ ,  $\mathbf{irow}$ ,  $\mathbf{icol}$ ,  $\mathbf{ipivp}$ ,  $\mathbf{ipivq}$ ,  $\mathbf{istr}$  and  $\mathbf{idiaq}$  have not been corrupted between calls to `nag_sparse_nherm_fac_sol` (f11dq) and `nag_sparse_nherm_fac` (f11dc).

On entry, the location  $(\mathbf{irow}[i-1], \mathbf{icol}[i-1])$  is a duplicate:  $i = \langle value \rangle$ .

Check that  $\mathbf{a}$ ,  $\mathbf{irow}$ ,  $\mathbf{icol}$ ,  $\mathbf{ipivp}$ ,  $\mathbf{ipivq}$ ,  $\mathbf{istr}$  and  $\mathbf{idiaq}$  have not been corrupted between calls to `nag_sparse_nherm_fac_sol` (f11dq) and `nag_sparse_nherm_fac` (f11dc).

**NE\_REAL**

On entry,  $\mathbf{tol} = \langle value \rangle$ .

Constraint:  $\mathbf{tol} < 1.0$ .

**7 Accuracy**

On successful termination, the final residual  $r_k = b - Ax_k$ , where  $k = \mathbf{itn}$ , satisfies the termination criterion

$$\|r_k\|_\infty \leq \tau \times (\|b\|_\infty + \|A\|_\infty \|x_k\|_\infty).$$

The value of the final residual norm is returned in  $\mathbf{rnorm}$ .

**8 Parallelism and Performance**

`nag_sparse_nherm_fac_sol` (f11dq) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_sparse_nherm_fac_sol` (f11dq) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The time taken by `nag_sparse_nherm_fac_sol` (f11dq) for each iteration is roughly proportional to the value of  $\mathbf{nnzc}$  returned from the preceding call to `nag_sparse_nherm_fac` (f11dc).

The number of iterations required to achieve a prescribed accuracy cannot be easily determined *a priori*, as it can depend dramatically on the conditioning and spectrum of the preconditioned coefficient matrix  $\bar{A} = M^{-1}A$ .

**10 Example**

This example solves a complex sparse non-Hermitian linear system of equations using the CGS method, with incomplete *LU* preconditioning.

**10.1 Program Text**

```
/* nag_sparse_nherm_fac_sol (f11dq) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <nag.h>
#include <nag_stdlib.h>
```

```

#include <naga02.h>
#include <nagf11.h>
int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double dtol, rnorm, tol;
    Integer i, itn, la, lfill, m, maxitn, n, nnz, nnzc, npivm;
    /* Arrays */
    Complex *a = 0, *b = 0, *x = 0;
    Integer *icol = 0, *idiag = 0, *ipivp = 0, *ipivq = 0, *irow = 0, *istr = 0;
    char nag_enum_arg[40];
    /* NAG types */
    Nag_SparseNsym_Method method;
    Nag_SparseNsym_Piv pstrat;
    Nag_SparseNsym_Fact milu;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_sparse_nherm_fac_sol (f11dq) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    /* Read algorithmic parameters */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &m);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &n, &m);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n]", &nnz);
#else
    scanf("%" NAG_IFMT "%*[\n]", &nnz);
#endif
    la = 2 * nnz;
    if (!(a = NAG_ALLOC((la), Complex)) ||
        !(b = NAG_ALLOC((n), Complex)) ||
        !(x = NAG_ALLOC((n), Complex)) ||
        !(icol = NAG_ALLOC((la), Integer)) ||
        !(idiag = NAG_ALLOC((n), Integer)) ||
        !(ipivp = NAG_ALLOC((n), Integer)) ||
        !(ipivq = NAG_ALLOC((n), Integer)) ||
        !(irow = NAG_ALLOC((la), Integer)) ||
        !(istr = NAG_ALLOC((n + 1), Integer))
        )
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
#ifdef _WIN32
    scanf_s("%39s%*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s%*[\n]", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    method = (Nag_SparseNsym_Method) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%lf%*[\n]", &lfill, &dtol);
#else
    scanf("%" NAG_IFMT "%lf%*[\n]", &lfill, &dtol);
#endif
#ifdef _WIN32
    scanf_s("%39s%*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));

```

```

#else
    scanf("%39s%*[\n]", nag_enum_arg);
#endif
    pstrat = (Nag_SparseNsym_Piv) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s("%39s%*[\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s%*[\n]", nag_enum_arg);
#endif
    milu = (Nag_SparseNsym_Fact) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s("%lf%" NAG_IFMT "%*[\n]", &tol, &maxitn);
#else
    scanf("%lf%" NAG_IFMT "%*[\n]", &tol, &maxitn);
#endif
    /* Read the matrix a */
#ifdef _WIN32
    for (i = 0; i < nnz; i++)
        scanf_s(" ( %lf , %lf ) %" NAG_IFMT "%" NAG_IFMT "%*[\n]",
#else
    for (i = 0; i < nnz; i++)
        scanf(" ( %lf , %lf ) %" NAG_IFMT "%" NAG_IFMT "%*[\n]",
#endif
                &a[i].re, &a[i].im, &irow[i], &icol[i]);
    /* Read rhs vector b and initial approximate solution x */
#ifdef _WIN32
    for (i = 0; i < n; i++)
        scanf_s(" ( %lf , %lf )", &b[i].re, &b[i].im);
#else
    for (i = 0; i < n; i++)
        scanf(" ( %lf , %lf )", &b[i].re, &b[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    for (i = 0; i < n; i++)
        scanf_s(" ( %lf , %lf )", &x[i].re, &x[i].im);
#else
    for (i = 0; i < n; i++)
        scanf(" ( %lf , %lf )", &x[i].re, &x[i].im);
#endif

    /* Calculate incomplete LU factorization */
    /* nag_sparse_nherm_fac (f11dnc)
     * Complex sparse non-Hermitian linear systems, incomplete LU factorization
     */
    nag_sparse_nherm_fac(n, nnz, a, la, irow, icol, lfill, dtol, pstrat, milu,
                        ipivp, ipivq, istr, iddiag, &nnzc, &npivm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_sparse_nherm_fac (f11dnc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }
    /* solve ax = b */
    /* nag_sparse_nherm_fac_sol (f11dq).
     * Solution of complex sparse non-Hermitian linear system, RGMRES, CGS,
     * Bi-CGSTAB or TFQMR method, preconditioner computed by
     * nag_sparse_nherm_fac (f11dnc) (Black Box).
     */
    nag_sparse_nherm_fac_sol(method, n, nnz, a, la, irow, icol, ipivp, ipivq,
                            istr, iddiag, b, m, tol, maxitn, x, &norm, &itn,
                            &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_sparse_nherm_fac_sol (f11dq).\n%s\n",
                fail.message);
        exit_status = 2;
        goto END;
    }
}

```

```

printf("Converged in%12" NAG_IFMT " iterations\n", itn);
printf("Final residual norm =%11.3e\n\n", rnorm);
/* Output x */
printf("%16s\n", "Solution");
for (i = 0; i < n; i++)
    printf(" (%13.4e, %13.4e) \n", x[i].re, x[i].im);
END:
NAG_FREE(a);
NAG_FREE(b);
NAG_FREE(x);
NAG_FREE(icol);
NAG_FREE(idiag);
NAG_FREE(ipivp);
NAG_FREE(ipivq);
NAG_FREE(irow);
NAG_FREE(istr);
return exit_status;
}

```

## 10.2 Program Data

nag\_sparse\_nherm\_fac\_sol (f11dqc) Example Program Data

```

8           4           : n, m
24          : nnz
Nag_SparseNsym_CGS           : method
0           0.0           : lfill, dtol
Nag_SparseNsym_CompletePiv  : pstrat
Nag_SparseNsym_UnModFact    : milu
1.0E-10    100           : tol, maxitn
( 2., 1.)   1   1
(-1., 1.)   1   4
( 1.,-3.)   1   8
( 4., 7.)   2   1
(-3., 0.)   2   2
( 2., 4.)   2   5
(-7.,-5.)   3   3
( 2., 1.)   3   6
( 3., 2.)   4   1
(-4., 2.)   4   3
( 0., 1.)   4   4
( 5.,-3.)   4   7
(-1., 2.)   5   2
( 8., 6.)   5   5
(-3.,-4.)   5   7
(-6.,-2.)   6   1
( 5.,-2.)   6   3
( 2., 0.)   6   6
( 0.,-5.)   7   3
(-1., 5.)   7   5
( 6., 2.)   7   7
(-1., 4.)   8   2
( 2., 0.)   8   6
( 3., 3.)   8   8           : a[i], irow[i], icol[i], i=0,...,nnz-1
( 7., 11.)
( 1., 24.)
(-13.,-18.)
(-10., 3.)
( 23., 14.)
( 17., -7.)
( 15., -3.)
( -3., 20.)           : b[i], i=0,...,n-1
( 0., 0.)
( 0., 0.)
( 0., 0.)
( 0., 0.)
( 0., 0.)
( 0., 0.)
( 0., 0.)
( 0., 0.)           : x[i], i=0,...,n-1

```

### **10.3 Program Results**

nag\_sparse\_nherm\_fac\_sol (f11dq) Example Program Results

Converged in 4 iterations  
Final residual norm = 1.348e-11

```
      Solution
(  1.0000e+00,  1.0000e+00)
(  2.0000e+00, -1.0000e+00)
(  3.0000e+00,  1.0000e+00)
(  4.0000e+00, -1.0000e+00)
(  3.0000e+00, -1.0000e+00)
(  2.0000e+00,  1.0000e+00)
(  1.0000e+00, -1.0000e+00)
( -1.7424e-12,  3.0000e+00)
```

---