

# NAG Library Function Document

## nag\_inteq\_volterra\_weights (d05bwc)

### 1 Purpose

nag\_inteq\_volterra\_weights (d05bwc) computes the quadrature weights associated with the Adams' methods of orders three to six and the Backward Differentiation Formulae (BDF) methods of orders two to five. These rules, which are referred to as reducible quadrature rules, can then be used in the solution of Volterra integral and integro-differential equations.

### 2 Specification

```
#include <nag.h>
#include <nagd05.h>

void nag_inteq_volterra_weights (Nag_ODEMethod method, Integer iorder,
    Integer nomg, double omega[], double sw[], NagError *fail)
```

### 3 Description

nag\_inteq\_volterra\_weights (d05bwc) computes the weights  $W_{i,j}$  and  $\omega_i$  for a family of quadrature rules related to the Adams' methods of orders three to six and the BDF methods of orders two to five, for approximating the integral:

$$\int_0^t \phi(s) ds \simeq h \sum_{j=0}^{p-1} W_{i,j} \phi(j \times h) + h \sum_{j=p}^i \omega_{i-j} \phi(j \times h), \quad 0 \leq t \leq T, \quad (1)$$

with  $t = i \times h$ , for  $i = 0, 1, \dots, n$ , for some given constant  $h$ .

In (1),  $h$  is a uniform mesh,  $p$  is related to the order of the method being used and  $W_{i,j}$ ,  $\omega_i$  are the starting and the convolution weights respectively. The mesh size  $h$  is determined as  $h = \frac{T}{n}$ , where  $n = n_w + p - 1$  and  $n_w$  is the chosen number of convolution weights  $w_j$ , for  $j = 1, 2, \dots, n_w - 1$ . A description of how these weights can be used in the solution of a Volterra integral equation of the second kind is given in Section 9. For a general discussion of these methods, see Wolkenfelt (1982) for more details.

### 4 References

- Lambert J D (1973) *Computational Methods in Ordinary Differential Equations* John Wiley
- Wolkenfelt P H M (1982) The construction of reducible quadrature rules for Volterra integral and integro-differential equations *IMA J. Numer. Anal.* **2** 131–152

### 5 Arguments

- 1: **method** – Nag\_ODEMethod *Input*
- On entry:* the type of method to be used.
- method** = Nag\_Adams  
For Adams' type formulae.
- method** = Nag\_BDF  
For Backward Differentiation Formulae.
- Constraint:* **method** = Nag\_Adams or Nag\_BDF.

- 2: **iorder** – Integer *Input*  
*On entry:* the order of the method to be used. The number of starting weights,  $p$  is determined by **method** and **iorder**.  
 If **method** = Nag\_Adams,  $p = \mathbf{iorder} - 1$ .  
 If **method** = Nag\_BDF,  $p = \mathbf{iorder}$ .  
*Constraints:*  
     if **method** = Nag\_Adams,  $3 \leq \mathbf{iorder} \leq 6$ ;  
     if **method** = Nag\_BDF,  $2 \leq \mathbf{iorder} \leq 5$ .
- 3: **nomg** – Integer *Input*  
*On entry:* the number of convolution weights,  $n_w$ .  
*Constraint:* **nomg**  $\geq 1$ .
- 4: **omega[nomg]** – double *Output*  
*On exit:* contains the first **nomg** convolution weights.
- 5: **sw**[ $n \times p$ ] – double *Output*  
**Note:** the  $(i, j)$ th element of the matrix is stored in **sw**[( $j - 1$ )  $\times n + i - 1$ ].  
*On exit:* **sw**[ $j \times n + i - 1$ ] contains the weights  $W_{i,j}$ , for  $i = 1, 2, \dots, n$  and  $j = 0, 1, \dots, p - 1$ , where  $n$  is as defined in Section 3.
- 6: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_ENUM\_INT

On entry, **method** = Nag\_Adams and **iorder** = 2.

Constraint: if **method** = Nag\_Adams,  $3 \leq \mathbf{iorder} \leq 6$ .

On entry, **method** = Nag\_BDF and **iorder** = 6.

Constraint: if **method** = Nag\_BDF,  $2 \leq \mathbf{iorder} \leq 5$ .

On entry, **method** =  $\langle value \rangle$  and **iorder** =  $\langle value \rangle$ .

Constraint: if **method** = Nag\_Adams,  $3 \leq \mathbf{iorder} \leq 6$ .

On entry, **method** =  $\langle value \rangle$  and **iorder** =  $\langle value \rangle$ .

Constraint: if **method** = Nag\_BDF,  $2 \leq \mathbf{iorder} \leq 5$ .

### NE\_INT

On entry, **iorder** =  $\langle value \rangle$ .

Constraint:  $2 \leq \mathbf{iorder} \leq 6$ .

On entry, **nomg** =  $\langle value \rangle$ .  
 Constraint: **nomg**  $\geq 1$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

Not applicable.

## 8 Parallelism and Performance

nag\_inteq\_volterra\_weights (d05bwc) is not threaded in any implementation.

## 9 Further Comments

Reducible quadrature rules are most appropriate for solving Volterra integral equations (and integro-differential equations). In this section, we propose the following algorithm which you may find useful in solving a linear Volterra integral equation of the form

$$y(t) = f(t) + \int_0^t K(t, s)y(s) ds, \quad 0 \leq t \leq T, \quad (2)$$

using nag\_inteq\_volterra\_weights (d05bwc). In (2),  $K(t, s)$  and  $f(t)$  are given and the solution  $y(t)$  is sought on a uniform mesh of size  $h$  such that  $T = nh$ . Discretization of (2) yields

$$y_i = f(i \times h) + h \sum_{j=0}^{p-1} W_{i,j} K(i, h, j, h) y_j + h \sum_{j=p}^i \omega_{i-j} K(i, h, j, h) y_j, \quad (3)$$

where  $y_i \simeq y(i \times h)$ . We propose the following algorithm for computing  $y_i$  from (3) after a call to nag\_inteq\_volterra\_weights (d05bwc):

- (a) Equation (3) requires starting values,  $y_j$ , for  $j = 1, 2, \dots, p-1$ , with  $y_0 = f(0)$ . These starting values can be computed by solving the linear system

$$y_i = f(i \times h) + h \sum_{j=0}^{p-1} \mathbf{sw}[j \times n + i - 1] K(i, h, j, h) y_j, \quad i = 1, 2, \dots, p-1.$$

- (b) Compute the inhomogeneous terms

$$\sigma_i = f(i \times h) + h \sum_{j=0}^{p-1} \mathbf{sw}[j \times n + i - 1] K(i, h, j, h) y_j, \quad i = p, p+1, \dots, n.$$

- (c) Start the iteration for  $i = p, p+1, \dots, n$  to compute  $y_i$  from:

$$(1 - h \times \mathbf{omega}[0] K(i, h, i, h)) y_i = \sigma_i + h \sum_{j=p}^{i-1} \mathbf{omega}[i-j] K(i, h, j, h) y_j.$$

Note that for a nonlinear integral equation, the solution of a nonlinear algebraic system is required at step (a) and a single nonlinear equation at step (c).

## 10 Example

The following example generates the first ten convolution and thirteen starting weights generated by the fourth-order BDF method.

### 10.1 Program Text

```

/* nag_inteq_volterra_weights (d05bwc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd05.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    Integer i, iorder, j, nomg, p, n;
    char methodstring[10];
    /* Arrays */
    double *omega = 0, *sw = 0;
    /* NAG types */
    NagError fail;
    Nag_ODEMethod method;

    INIT_FAIL(fail);

    printf("nag_inteq_volterra_weights (d05bwc) Example Program Results\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%9s%*[\n] ", methodstring, (unsigned)_countof(methodstring));
#else
    scanf("%9s%*[\n] ", methodstring);
#endif

    /*
     * nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value.
     */
    method = (Nag_ODEMethod) nag_enum_name_to_value(methodstring);

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &iorder);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &iorder);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &nomg);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &nomg);
#endif

    switch (method) {
    case Nag_Adams:
        p = iorder - 1;
        break;
    case Nag_BDF:

```

```

    p = iorder;
    break;
}

n = nomg + p - 1;

if (!(omega = NAG_ALLOC(nomg, double)) || !(sw = NAG_ALLOC(p * n, double))
)
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/*
    nag_inteq_volterra_weights (d05bwc).
    Generate weights for use in solving Volterra equations.
*/
nag_inteq_volterra_weights(method, iorder, nomg, omega, sw, &fail);

if (fail.code != NE_NOERROR) {
    printf("Error from nag_inteq_volterra_weights (d05bwc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

printf("\nThe convolution weights\n\n n-j          omega\n");
for (j = 0; j < nomg; j++)
    printf("%3" NAG_IFMT "          %10.4f\n", j + 1, omega[j]);

printf("\nThe weights W\n");
printf("\n i ");
for (j = 0; j < p; j++)
    printf("%11s" NAG_IFMT " ", "j = ", j);
printf("\n");

#define SW(I, J) sw[J * n + I]

for (i = 0; i < n; i++) {
    printf("%3" NAG_IFMT " ", i + 1);
    for (j = 0; j < p; j++)
        printf("%13.4f", SW(i, j));
    printf("\n");
}

#undef SW

END:

    NAG_FREE(sw);
    NAG_FREE(omega);

    return exit_status;
}

```

## 10.2 Program Data

```

nag_inteq_volterra_weights (d05bwc) Example Program Data
Nag_BDF                               : method
4                                       : iorder
10                                      : nomg

```

### 10.3 Program Results

nag\_inteq\_volterra\_weights (d05bwc) Example Program Results

The convolution weights

n-j	omega
1	0.4800
2	0.9216
3	1.0783
4	1.0504
5	0.9962
6	0.9797
7	0.9894
8	1.0003
9	1.0034
10	1.0017

The weights W

i	j = 0	j = 1	j = 2	j = 3
1	0.3750	0.7917	-0.2083	0.0417
2	0.3333	1.3333	0.3333	0.0000
3	0.3750	1.1250	1.1250	0.3750
4	0.4800	0.7467	1.5467	0.7467
5	0.5499	0.5719	1.5879	0.8886
6	0.5647	0.5829	1.5016	0.8709
7	0.5545	0.6385	1.4514	0.8254
8	0.5458	0.6629	1.4550	0.8098
9	0.5449	0.6578	1.4741	0.8170
10	0.5474	0.6471	1.4837	0.8262
11	0.5491	0.6428	1.4831	0.8292
12	0.5492	0.6438	1.4798	0.8279
13	0.5488	0.6457	1.4783	0.8263

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