

## NAG Library Chapter Contents

### s – Approximations of Special Functions

s Chapter Introduction – a description of the Chapter and an overview of the algorithms available

Function Name	Mark of Introduction	Purpose
s01bac	7	nag_shifted_log $\ln(1+x)$
s10aac	1	nag_tanh Hyperbolic tangent, $\tanh x$
s10abc	1	nag_sinh Hyperbolic sine, $\sinh x$
s10acc	1	nag_cosh Hyperbolic cosine, $\cosh x$
s11aac	1	nag_arctanh Inverse hyperbolic tangent, $\operatorname{arctanh} x$
s11abc	1	nag_arcsinh Inverse hyperbolic sine, $\operatorname{arcsinh} x$
s11acc	1	nag_arccosh Inverse hyperbolic cosine, $\operatorname{arccosh} x$
s13aac	1	nag_exp_integral Exponential integral $E_1(x)$
s13acc	1	nag_cos_integral Cosine integral $\operatorname{Ci}(x)$
s13adc	1	nag_sin_integral Sine integral $\operatorname{Si}(x)$
s14aac	1	nag_gamma Gamma function $\Gamma(x)$
s14abc	1	nag_log_gamma Log gamma function $\ln(\Gamma(x))$
s14acc	7	nag_polygamma_fun $\psi(x) - \ln x$
s14adc	7	nag_polygamma_deriv Scaled derivatives of $\psi(x)$
s14aec	6	nag_real_polygamma Derivative of the psi function $\psi(x)$
s14afc	6	nag_complex_polygamma Derivative of the psi function $\psi(z)$
s14agc	7	nag_complex_log_gamma Logarithm of the gamma function $\ln \Gamma(z)$ , complex argument
s14ahc	9	nag_scaled_log_gamma Scaled log gamma function $\ln G(x)$ , where $G(x) = \Gamma(x+1)/(x/e)^x$
s14bac	1	nag_incomplete_gamma Incomplete gamma functions $P(a, x)$ and $Q(a, x)$
s14cbc	23	nag_log_beta Logarithm of the beta function $\ln B(a, b)$
s14ccc	23	nag_incomplete_beta Incomplete beta function $I_x(a, b)$ and its complement $1 - I_x$
s15abc	1	nag_cumul_normal Cumulative Normal distribution function $P(x)$
s15acc	1	nag_cumul_normal_complem Complement of cumulative Normal distribution function $Q(x)$
s15adc	1	nag_erfc Complement of error function $\operatorname{erfc}(x)$

s15aec	1	nag_erf Error function $\operatorname{erf}(x)$
s15afc	7	nag_dawson Dawson's integral
s15agc	9	nag_erfcx Scaled complement of error function, $\operatorname{erfcx}(x)$
s15ddc	7	nag_complex_erfc Scaled complex complement of error function, $\exp(-z^2) \operatorname{erfc}(-iz)$
s17acc	1	nag_bessel_y0 Bessel function $Y_0(x)$
s17adc	1	nag_bessel_y1 Bessel function $Y_1(x)$
s17aec	1	nag_bessel_j0 Bessel function $J_0(x)$
s17afc	1	nag_bessel_j1 Bessel function $J_1(x)$
s17agc	1	nag_airy_ai Airy function $\operatorname{Ai}(x)$
s17ahc	1	nag_airy_bi Airy function $\operatorname{Bi}(x)$
s17ajc	1	nag_airy_ai_deriv Airy function $\operatorname{Ai}'(x)$
s17akc	1	nag_airy_bi_deriv Airy function $\operatorname{Bi}'(x)$
s17alc	6	nag_bessel_zeros Zeros of Bessel functions $J_\alpha(x)$ , $J'_\alpha(x)$ , $Y_\alpha(x)$ or $Y'_\alpha(x)$
s17aqc	23	nag_bessel_y0_vector Bessel function vectorized $Y_0(x)$
s17arc	23	nag_bessel_y1_vector Bessel function vectorized $Y_1(x)$
s17asc	23	nag_bessel_j0_vector Bessel function vectorized $J_0(x)$
s17atc	23	nag_bessel_j1_vector Bessel function vectorized $J_1(x)$
s17auc	23	nag_airy_ai_vector Airy function vectorized $\operatorname{Ai}(x)$
s17avc	23	nag_airy_bi_vector Airy function vectorized $\operatorname{Bi}(x)$
s17awc	23	nag_airy_ai_deriv_vector Derivatives of the Airy function, vectorized $\operatorname{Ai}'(x)$
s17axc	23	nag_airy_bi_deriv_vector Derivatives of the Airy function, vectorized $\operatorname{Bi}'(x)$
s17dcc	7	nag_complex_bessel_y Bessel functions $Y_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s17dec	7	nag_complex_bessel_j Bessel functions $J_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s17dgc	7	nag_complex_airy_ai Airy functions $\operatorname{Ai}(z)$ and $\operatorname{Ai}'(z)$ , complex $z$
s17dhc	7	nag_complex_airy_bi Airy functions $\operatorname{Bi}(z)$ and $\operatorname{Bi}'(z)$ , complex $z$
s17dlc	7	nag_complex_hankel Hankel functions $H_{\nu+a}^{(j)}(z)$ , $j = 1, 2$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s17gac	26.1	nag_struve_h0 Struve function of order 0, $H_0(x)$
s17gbc	26.1	nag_struve_h1 Struve function of order 1, $H_1(x)$
s18acc	1	nag_bessel_k0 Modified Bessel function $K_0(x)$

s18adc	1	nag_bessel_k1 Modified Bessel function $K_1(x)$
s18aec	1	nag_bessel_i0 Modified Bessel function $I_0(x)$
s18afc	1	nag_bessel_i1 Modified Bessel function $I_1(x)$
s18aqc	23	nag_bessel_k0_vector Modified Bessel function vectorized $K_0(x)$
s18arc	23	nag_bessel_k1_vector Modified Bessel function vectorized $K_1(x)$
s18asc	23	nag_bessel_i0_vector Modified Bessel function vectorized $I_0(x)$
s18atc	23	nag_bessel_i1_vector Modified Bessel function vectorized $I_1(x)$
s18ccc	2	nag_bessel_k0_scaled Scaled modified Bessel function $e^x K_0(x)$
s18cdc	2	nag_bessel_k1_scaled Scaled modified Bessel function $e^x K_1(x)$
s18cec	2	nag_bessel_i0_scaled Scaled modified Bessel function $e^{- x } I_0(x)$
s18cfc	2	nag_bessel_i1_scaled Scaled modified Bessel function $e^{- x } I_1(x)$
s18cqc	23	nag_bessel_k0_scaled_vector Scaled modified Bessel function vectorized $e^x K_0(x)$
s18crc	23	nag_bessel_k1_scaled_vector Scaled modified Bessel function vectorized $e^x K_1(x)$
s18csc	23	nag_bessel_i0_scaled_vector Scaled modified Bessel function vectorized $e^{- x } I_0(x)$
s18ctc	23	nag_bessel_i1_scaled_vector Scaled modified Bessel function vectorized $e^{- x } I_1(x)$
s18dcc	7	nag_complex_bessel_k Modified Bessel functions $K_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s18dec	7	nag_complex_bessel_i Modified Bessel functions $I_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$
s18ecc	6	nag_bessel_i_nu_scaled Scaled modified Bessel function $e^{-x} I_{\nu/4}(x)$
s18edc	6	nag_bessel_k_nu_scaled Scaled modified Bessel function $e^x K_{\nu/4}(x)$
s18eec	6	nag_bessel_i_nu Modified Bessel function $I_{\nu/4}(x)$
s18efc	6	nag_bessel_k_nu Modified Bessel function $K_{\nu/4}(x)$
s18egc	6	nag_bessel_k_alpha Modified Bessel functions $K_{\alpha+n}(x)$ for real $x > 0$ , selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$
s18ehc	6	nag_bessel_k_alpha_scaled Scaled modified Bessel functions $e^x K_{\alpha+n}(x)$ for real $x > 0$ , selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$
s18ejc	6	nag_bessel_i_alpha Modified Bessel functions $I_{\alpha+n-1}(x)$ or $I_{\alpha-n+1}(x)$ for real $x \neq 0$ , non-negative $\alpha < 1$ and $n = 1, 2, \dots,  N  + 1$
s18ekc	6	nag_bessel_j_alpha Bessel functions $J_{\alpha+n-1}(x)$ or $J_{\alpha-n+1}(x)$ for real $x \neq 0$ , non-negative $\alpha < 1$ and $n = 1, 2, \dots,  N  + 1$
s18gac	26.1	nag_struve_l0 Modified Struve function of order 0, $L_0(x)$
s18gbc	26.1	nag_struve_l1 Modified Struve function of order 1, $L_1(x)$

s18gcc	26.1	nag_struve_i0ml0 The function $I_0(x) - L_0(x)$ , where $I_0(x)$ is a modified Bessel function and $L_0(x)$ is a Struve function
s18gdc	26.1	nag_struve_i1ml1 The function $I_1(x) - L_1(x)$ , where $I_1(x)$ is a modified Bessel function and $L_1(x)$ is a Struve function
s18gkc	7	nag_complex_bessel_j_seq Bessel function of the 1st kind $J_{\alpha \pm n}(z)$
s19aac	1	nag_kelvin_ber Kelvin function ber $x$
s19abc	1	nag_kelvin_bei Kelvin function bei $x$
s19acc	1	nag_kelvin_ker Kelvin function ker $x$
s19adc	1	nag_kelvin_kei Kelvin function kei $x$
s19anc	23	nag_kelvin_ber_vector Kelvin function vectorized ber $x$
s19apc	23	nag_kelvin_bei_vector Kelvin function vectorized bei $x$
s19aqc	23	nag_kelvin_ker_vector Kelvin function vectorized ker $x$
s19arc	23	nag_kelvin_kei_vector Kelvin function vectorized kei $x$
s20acc	1	nag_fresnel_s Fresnel integral $S(x)$
s20adc	1	nag_fresnel_c Fresnel integral $C(x)$
s20aqc	23	nag_fresnel_s_vector Fresnel integral vectorized $S(x)$
s20arc	23	nag_fresnel_c_vector Fresnel integral vectorized $C(x)$
s21bac	1	nag_elliptic_integral_rc Degenerate symmetrised elliptic integral of 1st kind $R_C(x, y)$
s21bbc	1	nag_elliptic_integral_rf Symmetrised elliptic integral of 1st kind $R_F(x, y, z)$
s21bcc	1	nag_elliptic_integral_rd Symmetrised elliptic integral of 2nd kind $R_D(x, y, z)$
s21bdc	1	nag_elliptic_integral_rj Symmetrised elliptic integral of 3rd kind $R_J(x, y, z, r)$
s21bec	9	nag_elliptic_integral_F Elliptic integral of 1st kind, Legendre form, $F(\phi   m)$
s21bfc	9	nag_elliptic_integral_E Elliptic integral of 2nd kind, Legendre form, $E(\phi   m)$
s21bgc	9	nag_elliptic_integral_pi Elliptic integral of 3rd kind, Legendre form, $\Pi(n; \phi   m)$
s21bhc	9	nag_elliptic_integral_complete_K Complete elliptic integral of 1st kind, Legendre form, $K(m)$
s21bjc	9	nag_elliptic_integral_complete_E Complete elliptic integral of 2nd kind, Legendre form, $E(m)$
s21cac	7	nag_real_jacobian_elliptic Jacobian elliptic functions sn, cn and dn of real argument
s21cbc	6	nag_jacobian_elliptic Jacobian elliptic functions sn, cn and dn of complex argument
s21ccc	6	nag_jacobian_theta Jacobian theta functions with real arguments
s21dac	6	nag_general_elliptic_integral_f Elliptic integrals of the second kind with complex arguments

s22aac	6	nag_legendre_p Legendre and associated Legendre functions of the first kind with real arguments
s22bac	24	nag_specfun_1fl_real Real confluent hypergeometric function ${}_1F_1(a; b; x)$
s22bbc	24	nag_specfun_1fl_real_scaled Real confluent hypergeometric function ${}_1F_1(a; b; x)$ in scaled form
s22bec	24	nag_specfun_2fl_real Real Gauss hypergeometric function ${}_2F_1(a, b; c; x)$
s22bfc	24	nag_specfun_2fl_real_scaled Real Gauss hypergeometric function ${}_2F_1(a, b; c; x)$ in scaled form
s30aac	9	nag_bsm_price Black–Scholes–Merton option pricing formula
s30abc	9	nag_bsm_greeks Black–Scholes–Merton option pricing formula with Greeks
s30bac	9	nag_lookback_fls_price Floating-strike lookback option pricing formula in the Black-Scholes-Merton model
s30bbc	9	nag_lookback_fls_greeks Floating-strike lookback option pricing formula with Greeks in the Black-Scholes-Merton model
s30cac	9	nag_binary_con_price Binary option, cash-or-nothing pricing formula
s30cbc	9	nag_binary_con_greeks Binary option, cash-or-nothing pricing formula with Greeks
s30ccc	9	nag_binary_aon_price Binary option, asset-or-nothing pricing formula
s30cdc	9	nag_binary_aon_greeks Binary option, asset-or-nothing pricing formula with Greeks
s30fac	9	nag_barrier_std_price Standard barrier option pricing formula
s30jac	9	nag_jumpdiff_merton_price Jump-diffusion, Merton's model, option pricing formula
s30jbc	9	nag_jumpdiff_merton_greeks Jump-diffusion, Merton's model, option pricing formula with Greeks
s30nac	9	nag_heston_price Heston's model option pricing formula
s30nbc	23	nag_heston_greeks Heston's model option pricing formula with Greeks
s30ncc	24	nag_heston_term Heston's model option pricing with term structure
s30qcc	9	nag_amer_bs_price American option, Bjerksund and Stensland pricing formula
s30sac	9	nag_asian_geom_price Asian option, geometric continuous average rate pricing formula
s30sbc	9	nag_asian_geom_greeks Asian option, geometric continuous average rate pricing formula with Greeks

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