NAG Library Function Document

nag_erfc (s15adc)

1 Purpose

nag_erfc (s15adc) returns the value of the complementary error function, $\mathrm{erfc}(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_erfc (double x)
```

3 Description

nag_erfc (s15adc) calculates an approximate value for the complement of the error function

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt = 1 - \mathrm{erf}(x).$$

Let $\hat{x}$ be the root of the equation $\mathrm{erfc}(x) - \mathrm{erf}(x) = 0$ (then $\hat{x} \approx 0.46875$). For $|x| \leq \hat{x}$ the value of $\mathrm{erfc}(x)$ is based on the following rational Chebyshev expansion for $\mathrm{erf}(x)$:

$$\mathrm{erf}(x) \approx x R_{\ell,m}\left(x^2\right),$$

where $R_{\ell,m}$ denotes a rational function of degree ℓ in the numerator and m in the denominator.

For $|x| > \hat{x}$ the value of $\mathrm{erfc}(x)$ is based on a rational Chebyshev expansion for $\mathrm{erfc}(x)$: for $\hat{x} < |x| \leq 4$ the value is based on the expansion

$$\mathrm{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);$$

and for $|x| > 4$ it is based on the expansion

$$\mathrm{erfc}(x) \approx \frac{e^{x^2}}{x}\left(\frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}\left(1/x^2\right)\right).$$

For each expansion, the specific values of ℓ and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d}, where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag_decimal_digits (X02BEC)).

For $|x| \geq x_{\mathrm{hi}}$ there is a danger of setting underflow in $\mathrm{erfc}(x)$ (the value of x_{hi} is given in the Users' Note for your implementation). For $x \geq x_{\mathrm{hi}}$, nag_erfc (s15adc) returns $\mathrm{erfc}(x) = 0$; for $x \leq -x_{\mathrm{hi}}$ it returns $\mathrm{erfc}(x) = 2$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J (1969) Rational Chebyshev approximations for the error function *Math.Comp.* **23** 631–637

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{2xe^{-x^2}}{\sqrt{\pi}\,\mathrm{erfc}(x)} \delta \right|.$$

That is, the relative error in the argument, x, is amplified by a factor $\frac{2xe^{-x^2}}{\sqrt{\pi}\,\mathrm{erfc}(x)}$ in the result.

The behaviour of this factor is shown in Figure 1.

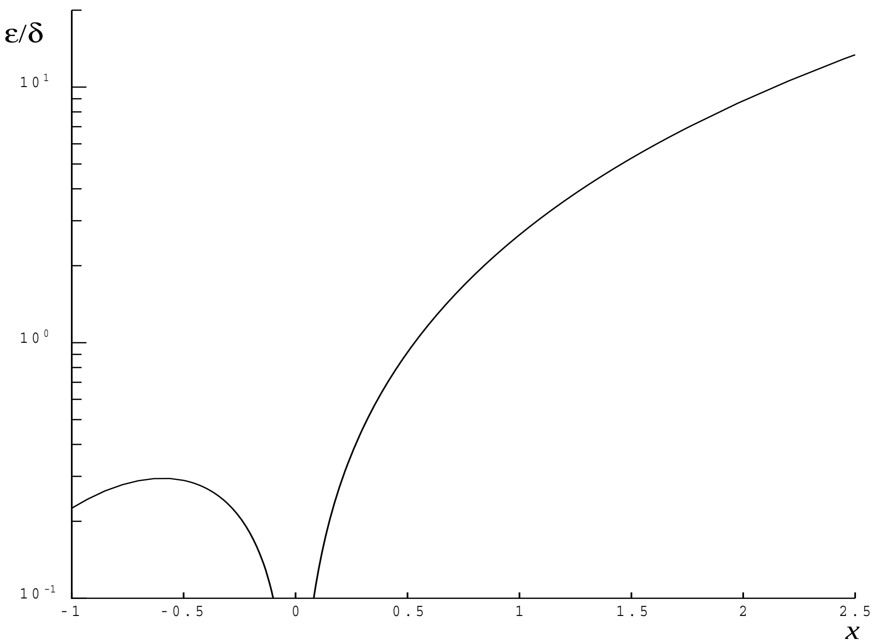

Figure 1

It should be noted that near $x = 0$ this factor behaves as $\frac{2x}{\sqrt{\pi}}$ and hence the accuracy is largely determined by the ***machine precision***. Also for large negative x, where the factor is $\sim \frac{xe^{-x^2}}{\sqrt{\pi}}$, accuracy is mainly limited by ***machine precision***. However, for large positive x, the factor becomes $\sim 2x^2$ and to an extent relative accuracy is necessarily lost. The absolute accuracy E is given by

$$E \simeq \frac{2xe^{-x^2}}{\sqrt{\pi}} \delta$$

so absolute accuracy is guaranteed for all x.

8 Parallelism and Performance

nag_erfc (s15adc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_erfc (s15adc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer exit_status = 0;
  double x, y;

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_erfc (s15adc) Example Program Results\n");
  printf("     x           y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
  {
    /* nag_erfc (s15adc).
     * Complement of error function erfc(x)
     */
    y = nag_erfc(x);
    printf("%12.3e%12.3e\n", x, y);
  }

  return exit_status;
}
```

10.2 Program Data

```
nag_erfc (s15adc) Example Program Data
           -10.0
            -1.0
             0.0
             1.0
            10.0
```

10.3 Program Results

```
nag_erfc (s15adc) Example Program Results
     x          y
  -1.000e+01   2.000e+00
  -1.000e+00   1.843e+00
   0.000e+00   1.000e-00
   1.000e+00   1.573e-01
   1.000e+01   2.088e-45
```