

# NAG Library Function Document

## nag\_complex\_polygamma (s14afc)

### 1 Purpose

nag\_complex\_polygamma (s14afc) returns the value of the  $k$ th derivative of the psi function  $\psi(z)$  for complex  $z$  and  $k = 0, 1, \dots, 4$ .

### 2 Specification

```
#include <nag.h>
#include <nags.h>
```

```
Complex nag_complex_polygamma (Complex z, Integer k, NagError *fail)
```

### 3 Description

nag\_complex\_polygamma (s14afc) evaluates an approximation to the  $k$ th derivative of the psi function  $\psi(z)$  given by

$$\psi^{(k)}(z) = \frac{d^k}{dz^k} \psi(z) = \frac{d^k}{dz^k} \left( \frac{d}{dz} \log_e \Gamma(z) \right),$$

where  $z = x + iy$  is complex provided  $y \neq 0$  and  $k = 0, 1, \dots, 4$ . If  $y = 0$ ,  $z$  is real and thus  $\psi^{(k)}(z)$  is singular when  $z = 0, -1, -2, \dots$ .

Note that  $\psi^{(k)}(z)$  is also known as the *polygamma* function. Specifically,  $\psi^{(0)}(z)$  is often referred to as the *digamma* function and  $\psi^{(1)}(z)$  as the *trigamma* function in the literature. Further details can be found in Abramowitz and Stegun (1972).

nag\_complex\_polygamma (s14afc) is based on a modification of the method proposed by K lbig (1972).

To obtain the value of  $\psi^{(k)}(z)$  when  $z$  is real, nag\_real\_polygamma (s14aec) can be used.

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

K lbig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

### 5 Arguments

1: **z** – Complex *Input*

*On entry:* the argument  $z$  of the function.

*Constraint:* **z.re** must not be ‘too close’ (see Section 6) to a non-positive integer when **z.im** = 0.0.

2: **k** – Integer *Input*

*On entry:* the function  $\psi^{(k)}(z)$  to be evaluated.

*Constraint:*  $0 \leq \mathbf{k} \leq 4$ .

- 3: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.  
 See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_COMPLEX

On entry,  $z.re$  is ‘too close’ to a non-positive integer when  $z.im = 0.0$ :  $z.re = \langle value \rangle$ ,  $nint(z.re) = \langle value \rangle$ .

### NE\_INT

On entry,  $k = \langle value \rangle$ .  
 Constraint:  $k \leq 4$ .

On entry,  $k = \langle value \rangle$ .  
 Constraint:  $k \geq 0$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_OVERFLOW\_LIKELY

Evaluation abandoned due to likelihood of overflow.

## 7 Accuracy

Empirical tests have shown that the maximum relative error is a loss of approximately two decimal places of precision.

## 8 Parallelism and Performance

nag\_complex\_polygamma (s14afc) is not threaded in any implementation.

## 9 Further Comments

None.

## 10 Example

This example evaluates the psi (trigamma) function  $\psi^{(1)}(z)$  at  $z = -1.5 + 2.5i$ , and prints the results.

## 10.1 Program Text

```

/* nag_complex_polygamma (s14afc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Complex y, z;
    Integer exit_status = 0, k;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    printf("nag_complex_polygamma (s14afc) Example Program Results\n");
    printf("      z          k          (d^k/dz^k)psi(z)\n");
#ifdef _WIN32
    while (scanf_s(" (%lf,%lf)%" NAG_IFMT "%*[\n] ", &z.re, &z.im, &k) != EOF)
#else
    while (scanf(" (%lf,%lf)%" NAG_IFMT "%*[\n] ", &z.re, &z.im, &k) != EOF)
#endif
    {
        /* nag_complex_polygamma (s14afc).
         * Derivative of the psi function psi(z)
         */
        y = nag_complex_polygamma(z, k, &fail);
        if (fail.code == NE_NOERROR)
            printf("(%5.1f, %5.1f) %6" NAG_IFMT " (%13.4e, %13.4e)\n",
                z.re, z.im, k, y.re, y.im);
        else {
            printf("Error from nag_complex_polygamma (s14afc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
    }
END:
    return exit_status;
}

```

## 10.2 Program Data

```

nag_complex_polygamma (s14afc) Example Program Data
(1.2,5.0) 0
(0.5,-0.2) 1
(-1.5,2.5) 1
(8.0,3.3) 3
(2.9,7.5) 4 : Values of z and k

```

### 10.3 Program Results

```
nag_complex_polygamma (s14afc) Example Program Results
      z          k      (d^k/dz^k)psi(z)
(  1.2,   5.0)   0 (  1.6176e+00,   1.4312e+00)
(  0.5,  -0.2)   1 (  3.4044e+00,   2.5394e+00)
( -1.5,   2.5)   1 ( -1.9737e-01,  -2.4271e-01)
(  8.0,   3.3)   3 (  1.1814e-03,  -3.4188e-03)
(  2.9,   7.5)   4 ( -5.0227e-04,  -1.4955e-03)
```

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