NAG Library Function Document

nag_sin_integral (s13adc)

1 Purpose

nag_sin_integral (s13adc) returns the value of the sine integral

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}\,du,$$

.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_sin_integral (double x)
```

3 Description

nag_sin_integral (s13adc) calculates an approximate value for $\mathrm{Si}(x)$.

For $|x| \leq 16.0$ it is based on the Chebyshev expansion

$$\mathrm{Si}(x) = x\sum_{r=0}^{\prime} a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < |x| < x_{\mathrm{hi}}$, where x_{hi} is an implementation-dependent number,

$$\mathrm{Si}(x) = \mathrm{sign}(x)\left\{\frac{\pi}{2} - \frac{f(x)\cos x}{x} - \frac{g(x)\sin x}{x^2}\right\}$$

where $f(x) = \sum_{r=0}^{} f_r T_r(t)$ and $g(x) = \sum_{r=0}^{} g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $|x| \geq x_{\mathrm{hi}}$, $\mathrm{Si}(x) = \frac{1}{2}\pi \,\mathrm{sign}\, x$ to within ***machine precision***.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left|\frac{\delta \sin x}{\mathrm{Si}(x)}\right|.$$

The equality may hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation, then since the factor relating δ to ϵ is always less than one, the accuracy will be limited by ***machine precision***.

8 Parallelism and Performance

nag_sin_integral (s13adc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_sin_integral (s13adc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer exit_status = 0;
  double x, y;

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_sin_integral (s13adc) Example Program Results\n");
  printf("     x           y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
  {
    /* nag_sin_integral (s13adc).
     * Sine integral Si(x)
     */
    y = nag_sin_integral(x);
```

```
    printf("%12.3e%12.3e\n", x, y);
  }

  return exit_status;
}
```

10.2 Program Data

```
nag_sin_integral (s13adc) Example Program Data
             0.0
             0.2
             0.4
             0.6
             0.8
             1.0
```

10.3 Program Results

```
nag_sin_integral (s13adc) Example Program Results
     x           y
   0.000e+00   0.000e+00
   2.000e-01   1.996e-01
   4.000e-01   3.965e-01
   6.000e-01   5.881e-01
   8.000e-01   7.721e-01
   1.000e+00   9.461e-01
```