NAG Library Function Document

nag_rand_field_2d_generate (g05zsc)

1 Purpose

nag_rand_field_2d_generate (g05zsc) produces realizations of a stationary Gaussian random field in two dimensions, using the circulant embedding method. The square roots of the eigenvalues of the extended covariance matrix (or embedding matrix) need to be input, and can be calculated using nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc).

2 Specification

```
#include <nag.h>
#include <nagg05.h>
void nag_rand_field_2d_generate (const Integer ns[], Integer s,
     const Integer m[], const double lam[], double rho, Integer state[],
     double z[], NagError *fail)
```

3 Description

A two-dimensional random field $Z(\mathbf{x})$ in $\mathbb{R}^2$ is a function which is random at every point $\mathbf{x} \in \mathbb{R}^2$, so $Z(\mathbf{x})$ is a random variable for each $\mathbf{x}$. The random field has a mean function $\mu(\mathbf{x}) = \mathbb{E}[Z(\mathbf{x})]$ and a symmetric positive semidefinite covariance function $C(\mathbf{x},\mathbf{y}) = \mathbb{E}[(Z(\mathbf{x}) - \mu(\mathbf{x}))(Z(\mathbf{y}) - \mu(\mathbf{y}))]$. $Z(\mathbf{x})$ is a Gaussian random field if for any choice of $n \in \mathbb{N}$ and $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^2$, the random vector $[Z(\mathbf{x}_1), \ldots, Z(\mathbf{x}_n)]^{\mathrm{T}}$ follows a multivariate Normal distribution, which would have a mean vector $\tilde{\boldsymbol{\mu}}$ with entries $\tilde{\mu}_i = \mu(\mathbf{x}_i)$ and a covariance matrix $\tilde{C}$ with entries $\tilde{C}_{ij} = C(\mathbf{x}_i, \mathbf{x}_j)$. A Gaussian random field $Z(\mathbf{x})$ is stationary if $\mu(\mathbf{x})$ is constant for all $\mathbf{x} \in \mathbb{R}^2$ and $C(\mathbf{x},\mathbf{y}) = C(\mathbf{x}+\mathbf{a},\mathbf{y}+\mathbf{a})$ for all $\mathbf{x},\mathbf{y},\mathbf{a} \in \mathbb{R}^2$ and hence we can express the covariance function $C(\mathbf{x},\mathbf{y})$ as a function γ of one variable: $C(\mathbf{x},\mathbf{y}) = \gamma(\mathbf{x}-\mathbf{y})$. γ is known as a variogram (or more correctly, a semivariogram) and includes the multiplicative factor σ^2 representing the variance such that $\gamma(0) = \sigma^2$.

The functions nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc) along with nag_rand_field_2d_generate (g05zsc) are used to simulate a two-dimensional stationary Gaussian random field, with mean function zero and variogram $\gamma(\mathbf{x})$, over a domain $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$, using an equally spaced set of $N_1 \times N_2$ points; N_1 points in the x-direction and N_2 points in the y-direction. The problem reduces to sampling a Gaussian random vector $\mathbf{X}$ of size $N_1 \times N_2$, with mean vector zero and a symmetric covariance matrix A, which is an N_2 by N_2 block Toeplitz matrix with Toeplitz blocks of size N_1 by N_1. Since A is in general expensive to factorize, a technique known as the *circulant embedding method* is used. A is embedded into a larger, symmetric matrix B, which is an M_2 by M_2 block circulant matrix with circulant bocks of size M_1 by M_1, where $M_1 \geq 2(N_1 - 1)$ and $M_2 \geq 2(N_2 - 1)$. B can now be factorized as $B = W\Lambda W^* = R^*R$, where W is the two-dimensional Fourier matrix (W^* is the complex conjugate of W), Λ is the diagonal matrix containing the eigenvalues of B and $R = \Lambda^{\frac{1}{2}}W^*$. B is known as the embedding matrix. The eigenvalues can be calculated by performing a discrete Fourier transform of the first row (or column) of B and multiplying by $M_1 \times M_2$, and so only the first row (or column) of B is needed – the whole matrix does not need to be formed.

The symmetry of A as a block matrix, and the symmetry of each block of A, depends on whether the covariance function γ is even or not. γ is even if $\gamma(\mathbf{x}) = \gamma(-\mathbf{x})$ for all $\mathbf{x} \in \mathbb{R}^2$, and uneven otherwise (in higher dimensions, γ can be even in some coordinates and uneven in others, but in two dimensions γ is either even in both coordinates or uneven in both coordinates). If γ is even then A is a symmetric block matrix and has symmetric blocks; if γ is uneven then A is not a symmetric block matrix and has non-symmetric blocks. In the uneven case, M_1 and M_2 are set to be odd in order to guarantee symmetry in B.

As long as all of the values of Λ are non-negative (i.e., B is positive semidefinite), B is a covariance matrix for a random vector $\mathbf{Y}$ which has M_2 'blocks' of size M_1. Two samples of $\mathbf{Y}$ can now be simulated from the real and imaginary parts of $R^*(\mathbf{U}+i\mathbf{V})$, where $\mathbf{U}$ and $\mathbf{V}$ have elements from the standard Normal distribution. Since $R^*(\mathbf{U}+i\mathbf{V}) = W\Lambda^{\frac{1}{2}}(\mathbf{U}+i\mathbf{V})$, this calculation can be done using a discrete Fourier transform of the vector $\Lambda^{\frac{1}{2}}(\mathbf{U}+i\mathbf{V})$. Two samples of the random vector $\mathbf{X}$ can now be recovered by taking the first N_1 elements of the first N_2 blocks of each sample of Y – because the original covariance matrix A is embedded in B, $\mathbf{X}$ will have the correct distribution.

If B is not positive semidefinite, larger embedding matrices B can be tried; however if the size of the matrix would have to be larger than **maxm**, an approximation procedure is used. See the documentation of nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc) for details of the approximation procedure.

nag_rand_field_2d_generate (g05zsc) takes the square roots of the eigenvalues of the embedding matrix B, and its size vector M, as input and outputs S realizations of the random field in Z.

One of the initialization functions nag_rand_init_repeatable (g05kfc) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeatable (g05kgc) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_field_2d_generate (g05zsc).

4 References

Dietrich C R and Newsam G N (1997) Fast and exact simulation of stationary Gaussian processes through circulant embedding of the covariance matrix *SIAM J. Sci. Comput.* **18** 1088–1107

Schlather M (1999) Introduction to positive definite functions and to unconditional simulation of random fields *Technical Report ST 99–10* Lancaster University

Wood A T A and Chan G (1994) Simulation of stationary Gaussian processes in $[0,1]^d$ *Journal of Computational and Graphical Statistics* **3(4)** 409–432

5 Arguments

1: **ns**[**2**] – const Integer *Input*

On entry: the number of sample points to use in each direction, with **ns**[0] sample points in the x-direction and **ns**[1] sample points in the y-direction. The total number of sample points on the grid is therefore **ns**[0] × **ns**[1]. This must be the same value as supplied to nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc) when calculating the eigenvalues of the embedding matrix.

Constraints:

ns[0] ≥ 1;
ns[1] ≥ 1.

2: **s** – Integer *Input*

On entry: S, the number of realizations of the random field to simulate.

Constraint: **s** ≥ 1.

3: **m**[**2**] – const Integer *Input*

On entry: indicates the size, M, of the embedding matrix as returned by nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc). The embedding matrix is a block circulant matrix with circulant blocks. **m**[0] is the size of each block, and **m**[1] is the number of blocks.

Constraints:

m[0] ≥ max(1, 2(**ns**[0] − 1));
m[1] ≥ max(1, 2(**ns**[1] − 1)).

4: **lam**[**m**[0] × **m**[1]] – const double *Input*

On entry: contains the square roots of the eigenvalues of the embedding matrix, as returned by nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc).

Constraint: **lam**$[i-1] \geq 0$, $i = 1, 2, \ldots,$ **m**[0] × **m**[1].

5: **rho** – double *Input*

On entry: indicates the scaling of the covariance matrix, as returned by nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc).

Constraint: $0.0 <$ **rho** ≤ 1.0.

6: **state**[dim] – Integer *Communication Array*

Note: the dimension, dim, of this array is dictated by the requirements of associated functions that must have been previously called. This array MUST be the same array passed as argument **state** in the previous call to nag_rand_init_repeatable (g05kfc) or nag_rand_init_nonrepeatable (g05kgc).

On entry: contains information on the selected base generator and its current state.

On exit: contains updated information on the state of the generator.

7: **z**[dim] – double *Output*

Note: the dimension, *dim*, of the array **z** must be at least **s** × **ns**[0] × **ns**[1].

On exit: contains the realizations of the random field. The kth realization (where $k = 1, 2, \ldots,$ **s**) of the random field on the two-dimensional grid (x_i, y_j) is stored in **z**$[(k-1) \times$ **ns**$[0] \times$ **ns**$[1] + (j-1) \times$ **ns**$[0] + i - 1]$, for $i = 1, 2, \ldots,$ **ns**[0] and for $j = 1, 2, \ldots,$ **ns**[1]. The points are returned in **xx** and **yy** by nag_rand_field_2d_user_setup (g05zqc) or nag_rand_field_2d_predef_setup (g05zrc).

8: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **s** = ⟨*value*⟩.
Constraint: **s** ≥ 1.

NE_INT_ARRAY

On entry, **ns** = [⟨*value*⟩, ⟨*value*⟩].
Constraint: **ns**$[0] \geq 1$, **ns**$[1] \geq 1$.

NE_INT_ARRAY_2

On entry, $\mathbf{m} = [\langle value \rangle, \langle value \rangle]$, and $\mathbf{ns} = [\langle value \rangle, \langle value \rangle]$.
Constraints: $\mathbf{m}[i-1] \geq \max(1, 2(\mathbf{ns}[i-1]) - 1)$, for $i = 1, 2$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_INVALID_STATE

On entry, **state** vector has been corrupted or not initialized.

NE_NEG_ELEMENT

On entry, at least one element of **lam** was negative.
Constraint: all elements of **lam** must be non-negative.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL

On entry, $\mathbf{rho} = \langle value \rangle$.
Constraint: $0.0 < \mathbf{rho} \leq 1.0$.

7 Accuracy

Not applicable.

8 Parallelism and Performance

nag_rand_field_2d_generate (g05zsc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_rand_field_2d_generate (g05zsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Because samples are generated in pairs, calling this routine k times, with $\mathbf{s} = s$, say, will generate a different sequence of numbers than calling the routine once with $\mathbf{s} = ks$, unless s is even.

10 Example

This example calls nag_rand_field_2d_generate (g05zsc) to generate 5 realizations of a two-dimensional random field on a 5 by 5 grid. This uses eigenvalues of the embedding covariance matrix for a symmetric stable variogram as calculated by nag_rand_field_2d_predef_setup (g05zrc) with **cov** = Nag_VgmSymmStab.

10.1 Program Text

```
/* nag_rand_field_2d_generate (g05zsc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg05.h>
#include <nagx04.h>

#define NPMAX 5
#define LENST 17
#define LSEED 1
static void read_input_data(Nag_Variogram *cov, Integer *np, double *params,
                            Nag_NormType *norm, double *var, double *xmin,
                            double *xmax, double *ymin, double *ymax,
                            Integer *ns, Integer *maxm, Nag_EmbedScale *corr,
                            Nag_EmbedPad *pad, Integer *s);
static void display_embedding_results(Integer approx, Integer *m, double rho,
                                      double *eig, Integer icount);
static void initialize_state(Integer *state);
static void display_realizations(Integer *ns, Integer s, double *xx,
                                 double *yy, double *z, Integer *exit_status);

int main(void)
{
  /*  Scalars */
  Integer exit_status = 0;
  double rho, var, xmax, xmin, ymax, ymin;
  Integer approx, icount, np, s;
  /*  Arrays */
  double eig[3], params[NPMAX];
  double *lam = 0, *xx = 0, *yy = 0, *z = 0;
  Integer m[2], maxm[2], ns[2], state[LENST];
  /* Nag types */
  Nag_NormType norm;
  Nag_Variogram cov;
  Nag_EmbedPad pad;
  Nag_EmbedScale corr;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_rand_field_2d_generate (g05zsc) Example Program Results\n\n");
  /* Get problem specifications from data file */
  read_input_data(&cov, &np, params, &norm, &var, &xmin, &xmax, &ymin, &ymax,
                  ns, maxm, &corr, &pad, &s);
  if (!(lam = NAG_ALLOC(maxm[0] * maxm[1], double)) ||
      !(xx = NAG_ALLOC(ns[0], double)) || !(yy = NAG_ALLOC(ns[1], double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }
  /* Get square roots of the eigenvalues of the embedding matrix.
   * nag_rand_field_2d_predef_setup (g05zrc).
   * Setup for simulating two-dimensional random fields, preset variogram,
   * circulant embedding method
   */
  nag_rand_field_2d_predef_setup(ns, xmin, xmax, ymin, ymax, maxm, var, cov,
                                 norm, np, params, pad, corr, lam, xx, yy, m,
                                 &approx, &rho, &icount, eig, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_field_2d_predef_setup (g05zrc).\n%s\n",
```

```
            fail.message);
    exit_status = 1;
    goto END;
  }

  display_embedding_results(approx, m, rho, eig, icount);
  /* Initialize state array */
  initialize_state(state);
  if (!(z = NAG_ALLOC(ns[0] * ns[1] * s, double)))
  {
    printf("Allocation failure\n");
    exit_status = -2;
    goto END;
  }
  /* Compute s random field realizations.
   * nag_rand_field_2d_generate (g05zsc).
   * Generates s realizations of a rwo-dimensional random field by the
   * circulant embedding method.
   */
  nag_rand_field_2d_generate(ns, s, m, lam, rho, state, z, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_field_2d_generate (g05zsc).\n%s\n",
            fail.message);
    exit_status = 2;
    goto END;
  }
  display_realizations(ns, s, xx, yy, z, &exit_status);
END:
  NAG_FREE(lam);
  NAG_FREE(xx);
  NAG_FREE(yy);
  NAG_FREE(z);
  return exit_status;
}

void read_input_data(Nag_Variogram *cov, Integer *np, double *params,
                     Nag_NormType *norm, double *var, double *xmin,
                     double *xmax, double *ymin, double *ymax, Integer *ns,
                     Integer *maxm, Nag_EmbedScale *corr,
                     Nag_EmbedPad *pad, Integer *s)
{
  Integer j;
  char nag_enum_arg[40];

  /* Read in covariance function name and convert to value using
   * nag_enum_name_to_value (x04nac).
   */
#ifdef _WIN32
  scanf_s("%*[^\n] %39s%*[^\n]", nag_enum_arg,
          (unsigned)_countof(nag_enum_arg));
#else
  scanf("%*[^\n] %39s%*[^\n]", nag_enum_arg);
#endif
  *cov = (Nag_Variogram) nag_enum_name_to_value(nag_enum_arg);
  /* Read in parameters */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n]", np);
#else
  scanf("%" NAG_IFMT "%*[^\n]", np);
#endif
  for (j = 0; j < *np; j++)
#ifdef _WIN32
    scanf_s("%lf", &params[j]);
#else
    scanf("%lf", &params[j]);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
```

```
  /* Read in norm type by name and convert to value */
#ifdef _WIN32
  scanf_s(" %39s%*[^\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
  scanf(" %39s%*[^\n]", nag_enum_arg);
#endif
  *norm = (Nag_NormType) nag_enum_name_to_value(nag_enum_arg);
  /* Read in variance of random field. */
#ifdef _WIN32
  scanf_s("%lf%*[^\n]", var);
#else
  scanf("%lf%*[^\n]", var);
#endif
  /* Read in domain endpoints */
#ifdef _WIN32
  scanf_s("%lf %lf%*[^\n]", xmin, xmax);
#else
  scanf("%lf %lf%*[^\n]", xmin, xmax);
#endif
#ifdef _WIN32
  scanf_s("%lf %lf%*[^\n]", ymin, ymax);
#else
  scanf("%lf %lf%*[^\n]", ymin, ymax);
#endif
  /* Read in number of sample points in each direction */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT " %" NAG_IFMT "%*[^\n]", &ns[0], &ns[1]);
#else
  scanf("%" NAG_IFMT " %" NAG_IFMT "%*[^\n]", &ns[0], &ns[1]);
#endif
  /* Read in maximum size of embedding matrix */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT " %" NAG_IFMT "%*[^\n]", &maxm[0], &maxm[1]);
#else
  scanf("%" NAG_IFMT " %" NAG_IFMT "%*[^\n]", &maxm[0], &maxm[1]);
#endif
  /* Read name of scaling in case of approximation and convert to value. */
#ifdef _WIN32
  scanf_s(" %39s%*[^\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
  scanf(" %39s%*[^\n]", nag_enum_arg);
#endif
  *corr = (Nag_EmbedScale) nag_enum_name_to_value(nag_enum_arg);
  /* Read in choice of padding and convert name to value. */
#ifdef _WIN32
  scanf_s(" %39s%*[^\n]", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
  scanf(" %39s%*[^\n]", nag_enum_arg);
#endif
  *pad = (Nag_EmbedPad) nag_enum_name_to_value(nag_enum_arg);
  /* Read in number of realization samples to be generated */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n]", s);
#else
  scanf("%" NAG_IFMT "%*[^\n]", s);
#endif
}

void display_embedding_results(Integer approx, Integer *m, double rho,
                               double *eig, Integer icount)
{
  Integer j;
  /* Display size of embedding matrix */
  printf("\nSize of embedding matrix = %" NAG_IFMT "\n\n", m[0] * m[1]);
  /* Display approximation information if approximation used */
  if (approx == 1) {
    printf("Approximation required\n\n");
    printf("rho = %10.5f\n", rho);
    printf("eig = ");
    for (j = 0; j < 3; j++)
      printf("%10.5f ", eig[j]);
```

```
    printf("\nicount = %" NAG_IFMT "\n", icount);
  }
  else {
    printf("Approximation not required\n");
  }
}

void initialize_state(Integer *state)
{
  /*  Scalars */
  Integer inseed = 14965, lseed = LSEED, subid = 1;
  Integer lstate;
  /*  Arrays */
  Integer seed[LSEED];
  /* Nag types */
  NagError fail;

  INIT_FAIL(fail);
  lstate = LENST;
  seed[0] = inseed;
  /* nag_rand_init_repeatable (g05kfc).
   * Initializes a pseudorandom number generator to give a repeatable sequence.
   */
  nag_rand_init_repeatable(Nag_Basic, subid, seed, lseed, state, &lstate,
                           &fail);
}

void display_realizations(Integer *ns, Integer s, double *xx, double *yy,
                          double *z, Integer *exit_status)
{
  /*  Scalars */
  Integer indent = 0, ncols = 80;
  Integer i, j, nn;
  /*  Arrays */
  char **rlabs = 0;
  /* Nag types */
  NagError fail;

  INIT_FAIL(fail);

  nn = ns[0] * ns[1];
  if (!(rlabs = NAG_ALLOC(nn, char *)))
  {
    printf("Allocation failure\n");
    *exit_status = -3;
    goto END;
  }
  /* Set row labels to mesh points (column label is realization number). */
  for (j = 0; j < ns[1]; j++) {
    for (i = 0; i < ns[0]; i++) {
      if (!(rlabs[j * ns[0] + i] = NAG_ALLOC(13, char)))
      {
        printf("Allocation failure\n");
        *exit_status = -4;
        goto END;
      }
      if (i == 0) {
#ifdef _WIN32
        sprintf_s(rlabs[j * ns[0] + i], 13, "%6.1f%6.1f", xx[i], yy[j]);
#else
        sprintf(rlabs[j * ns[0] + i], "%6.1f%6.1f", xx[i], yy[j]);
#endif
      }
      else {
#ifdef _WIN32
        sprintf_s(rlabs[j * ns[0] + i], 13, "%6.1f%6s", xx[i], ".");
#else
        sprintf(rlabs[j * ns[0] + i], "%6.1f%6s", xx[i], ".");
#endif
      }
    }
```

```
  }
  printf("\n");
  fflush(stdout);
  /* Display random field results, z, using the comprehensive real general
   * matrix print routine nag_gen_real_mat_print_comp (x04cbc).
   */
  nag_gen_real_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                              Nag_NonUnitDiag, nn, s, z, nn, "%10.5f",
                              "Random field "
                              "realizations (x,y coordinates first):",
                              Nag_CharacterLabels, (const char **) rlabs,
                              Nag_IntegerLabels, NULL, ncols, indent, 0,
                              &fail);
END:
  for (i = 0; i < nn; i++) {
    NAG_FREE(rlabs[i]);
  }
  NAG_FREE(rlabs);
}
```

10.2 Program Data

```
nag_rand_field_2d_generate (g05zsc) Example Program Data
 Nag_VgmSymmStab     : cov
  3                  : np (3 parameters for 2D Nag_VgmSymmStab)
  0.1   0.15  1.2    : params (c1, c2 and nu)
  Nag_TwoNorm        : norm
  0.5                : var
 -1.0   1.0          : xmin, xmax
 -0.5   0.5          : ymin, ymax
  5     5            : ns
 64    64            : maxm
 Nag_EmbedScaleOne   : corr
 Nag_EmbedPadValues  : pad
  5                  : s
```

10.3 Program Results

```
nag_rand_field_2d_generate (g05zsc) Example Program Results


Size of embedding matrix = 64

Approximation not required

 Random field realizations (x,y coordinates first):
                     1          2          3          4          5
 -0.8  -0.4   -0.61951   -0.93149   -0.32975   -0.51201    1.38877
 -0.4     .    0.74779    1.33518   -0.51237    0.26595    0.30051
  0.0     .   -0.30579    0.51819    0.50961    0.10379    0.36815
  0.4     .    0.53797   -0.53992   -0.86589   -0.37098    0.21571
  0.8     .   -0.61221   -1.04262    0.00007   -1.22614   -0.06650
 -0.8  -0.2    0.01853    0.64126   -0.42978   -0.79178   -0.55728
 -0.4     .   -0.77912    0.81079   -0.60613    0.07280    1.61511
  0.0     .   -0.23198    1.48744   -0.78145    0.10347    0.07053
  0.4     .    0.32356    0.58676    0.05846    0.34828    1.40522
  0.8     .   -1.24085   -0.92512    0.27247   -0.66965    0.67073
 -0.8   0.0   -1.18183   -0.99775    0.03888    0.01789   -0.65746
 -0.4     .    0.26155   -0.01734   -0.14924    0.28886    0.25940
  0.0     .    1.14960    0.48850   -0.59023    0.22795   -0.60773
  0.4     .   -0.32684   -0.09616   -0.63497   -1.06753   -0.64594
  0.8     .    0.10064    1.06148    0.15020   -0.53168   -0.29251
 -0.8   0.2   -1.30595   -0.03899   -0.35549   -0.20589   -0.35956
 -0.4     .   -0.01776    0.84501    0.20406    0.89039   -0.58338
  0.0     .    0.41898    0.93435   -1.10725    0.76913   -0.74579
  0.4     .   -1.37738    1.72404   -0.20558   -1.41877    1.21816
  0.8     .    0.77866    0.84922   -0.65055    0.83518   -0.26425
```

-0.8	0.4	-0.65163	0.50492	-0.52463	-1.12816	1.12817
-0.4	.	0.15437	0.20739	-0.12675	1.27782	-0.26157
0.0	.	0.20324	0.54670	-1.73909	0.61580	0.17551
0.4	.	-1.09470	0.83967	0.70226	-0.34259	0.29368
0.8	.	1.08452	1.23097	-0.36003	1.06884	0.23594