

NAG Library Function Document

nag_nearest_correlation_bounded (g02abc)

1 Purpose

nag_nearest_correlation_bounded (g02abc) computes the nearest correlation matrix, in the Frobenius norm or weighted Frobenius norm, and optionally with bounds on the eigenvalues, to a given square, input matrix.

2 Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_nearest_correlation_bounded (Nag_OrderType order, double g[],
    Integer pdg, Integer n, Nag_NearCorr_ProbType opt, double alpha,
    double w[], double errtol, Integer maxits, Integer maxit, double x[],
    Integer pdx, Integer *iter, Integer *feval, double *nrmgrd,
    NagError *fail)
```

3 Description

Finds the nearest correlation matrix X by minimizing $\frac{1}{2}\|G - X\|^2$ where G is an approximate correlation matrix.

The norm can either be the Frobenius norm or the weighted Frobenius norm $\frac{1}{2}\|W^{\frac{1}{2}}(G - X)W^{\frac{1}{2}}\|_F^2$.

You can optionally specify a lower bound on the eigenvalues, α , of the computed correlation matrix, forcing the matrix to be positive definite, $0 < \alpha < 1$.

Note that if the weights vary by several orders of magnitude from one another the algorithm may fail to converge.

4 References

Borsdorf R and Higham N J (2010) A preconditioned (Newton) algorithm for the nearest correlation matrix *IMA Journal of Numerical Analysis* **30(1)** 94–107

Qi H and Sun D (2006) A quadratically convergent Newton method for computing the nearest correlation matrix *SIAM J. Matrix AnalAppl* **29(2)** 360–385

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **g[*pdg* × *n*]** – double *Input/Output*

Note: the (i, j)th element of the matrix G is stored in

g[($j - 1$) × **pdg** + $i - 1$] when **order** = Nag_ColMajor;
g[($i - 1$) × **pdg** + $j - 1$] when **order** = Nag_RowMajor.

- On entry:* G , the initial matrix.
On exit: G is overwritten.
- 3: **pdg** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) in the array **g**.
Constraint: **pdg** \geq **n**.
- 4: **n** – Integer *Input*
On entry: the order of the matrix G .
Constraint: **n** $>$ 0.
- 5: **opt** – Nag_NearCorr_ProbType *Input*
On entry: indicates the problem to be solved.
opt = Nag_LowerBound
 The lower bound problem is solved.
opt = Nag_WeightedNorm
 The weighted norm problem is solved.
opt = Nag_Both
 Both problems are solved.
Constraint: **opt** = Nag_LowerBound, Nag_WeightedNorm or Nag_Both.
- 6: **alpha** – double *Input*
On entry: the value of α .
 If **opt** = Nag_WeightedNorm, **alpha** need not be set.
Constraint: $0.0 < \mathbf{alpha} < 1.0$.
- 7: **w[n]** – double *Input/Output*
On entry: the square roots of the diagonal elements of W , that is the diagonal of $W^{\frac{1}{2}}$.
 If **opt** = Nag_LowerBound, **w** is not referenced and may be **NULL**.
On exit: if **opt** = Nag_WeightedNorm or Nag_Both, the array is scaled so $0 < \mathbf{w}[i-1] \leq 1$, for $i = 1, 2, \dots, n$.
Constraint: $\mathbf{w}[i-1] > 0.0$, for $i = 1, 2, \dots, n$.
- 8: **errtol** – double *Input*
On entry: the termination tolerance for the Newton iteration. If **errtol** \leq 0.0 then $\mathbf{n} \times \sqrt{\mathit{machine\ precision}}$ is used.
- 9: **maxits** – Integer *Input*
On entry: specifies the maximum number of iterations to be used by the iterative scheme to solve the linear algebraic equations at each Newton step.
 If **maxits** \leq 0, $2 \times \mathbf{n}$ is used.
- 10: **maxit** – Integer *Input*
On entry: specifies the maximum number of Newton iterations.
 If **maxit** \leq 0, 200 is used.

- 11: **x**[**pdx** × **n**] – double *Output*
Note: the (i, j) th element of the matrix X is stored in
 $\mathbf{x}[(j - 1) \times \mathbf{pdx} + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{x}[(i - 1) \times \mathbf{pdx} + j - 1]$ when **order** = Nag_RowMajor.
On exit: contains the nearest correlation matrix.
- 12: **pdx** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) in the array **x**.
Constraint: **pdx** ≥ **n**.
- 13: **iter** – Integer * *Output*
On exit: the number of Newton steps taken.
- 14: **feval** – Integer * *Output*
On exit: the number of function evaluations of the dual problem.
- 15: **nrmgrd** – double * *Output*
On exit: the norm of the gradient of the last Newton step.
- 16: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_CONVERGENCE

Newton iteration fails to converge in $\langle value \rangle$ iterations. Increase **maxit** or check the call to the function.

The *machine precision* is limiting convergence. In this instance the returned value of **x** may be useful.

NE_EIGENPROBLEM

An intermediate eigenproblem could not be solved. This should not occur. Please contact NAG with details of your call.

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: **n** > 0.

NE_INT_2

On entry, **pdg** = $\langle value \rangle$ and **n** = $\langle value \rangle$.

Constraint: **pdg** \geq **n**.

On entry, **pdx** = $\langle value \rangle$ and **n** = $\langle value \rangle$.

Constraint: **pdx** \geq **n**.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL

On entry, **alpha** = $\langle value \rangle$.

Constraint: $0.0 < \mathbf{alpha} < 1.0$.

NE_WEIGHTS_NOT_POSITIVE

On entry, all elements of **w** were not positive.

7 Accuracy

The returned accuracy is controlled by **errtol** and limited by *machine precision*.

8 Parallelism and Performance

nag_nearest_correlation_bounded (g02abc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_nearest_correlation_bounded (g02abc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

Arrays are internally allocated by nag_nearest_correlation_bounded (g02abc). The total size of these arrays is $12 \times \mathbf{n} + 3 \times \mathbf{n} \times \mathbf{n} + \max(2 \times \mathbf{n} \times \mathbf{n} + 6 \times \mathbf{n} + 1, 120 + 9 \times \mathbf{n})$ double elements and $5 \times \mathbf{n} + 3$ Integer elements. All allocated memory is freed before return of nag_nearest_correlation_bounded (g02abc).

10 Example

This example finds the nearest correlation matrix to:

$$G = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

weighted by $W^{\frac{1}{2}} = \text{diag}(100, 20, 20, 20)$ with minimum eigenvalue 0.02.

10.1 Program Text

```

/* nag_nearest_correlation_bounded (g02abc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagg02.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double alpha, errtol, nrmgrd;
    Integer feval, i, iter, j, maxit, maxits, n, pdeig, pdg, pdx;

    /* Arrays */
    char nag_enum_arg[100];
    double *eig = 0, *g = 0, *w = 0, *x = 0;

    /* Nag Types */
    Nag_OrderType order;
    Nag_NearCorr_ProbType opt;
    NagError fail;

    INIT_FAIL(fail);

#ifdef NAG_COLUMN_MAJOR
#define G(I, J) g[(J-1)*pdg + I-1]
    order = Nag_ColMajor;
#else
#define G(I, J) g[(I-1)*pdg + J-1]
    order = Nag_RowMajor;
#endif

    /* Output preamble */
    printf("nag_nearest_correlation_bounded (g02abc)");
    printf(" Example Program Results\n\n");
    fflush(stdout);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    /* Read in the problem size, opt and alpha */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT " ", &n);

```

```

#else
    scanf("%" NAG_IFMT " ", &n);
#endif
#ifdef _WIN32
    scanf_s("%39s", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf("%39s", nag_enum_arg);
#endif
/*
 * nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
opt = (Nag_NearCorr_ProbType) nag_enum_name_to_value(nag_enum_arg);
#ifdef _WIN32
    scanf_s("%lf%*[\n]", &alpha);
#else
    scanf("%lf%*[\n]", &alpha);
#endif

    pdg = n;
    pdx = n;
    if (order == Nag_ColMajor)
        pdeig = 1;
    else
        pdeig = n;

    if (!(g = NAG_ALLOC((pdg) * (n), double)) ||
        !(w = NAG_ALLOC((n), double)) ||
        !(x = NAG_ALLOC((pdx) * (n), double)) || !(eig = NAG_ALLOC((n), double))
        )
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read in the matrix g */
    for (i = 1; i <= n; i++)
        for (j = 1; j <= n; j++)
#ifdef _WIN32
            scanf_s("%lf", &G(i, j));
#else
            scanf("%lf", &G(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Read in the vector w */
    for (i = 0; i < n; i++)
#ifdef _WIN32
        scanf_s("%lf", &w[i]);
#else
        scanf("%lf", &w[i]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Use the defaults for errtol, maxits and maxit */
    errtol = 0.0;
    maxits = 0;
    maxit = 0;

    /*
     * nag_nearest_correlation_bounded (g02abc).
     * Computes the nearest correlation matrix incorporating weights

```

```

    * and/or bounds
    */
nag_nearest_correlation_bounded(order, g, pdg, n, opt, alpha, w, errtol,
                                maxits, maxit, x, pdx, &iter, &feval,
                                &nrmsgrd, &fail);

if (fail.code != NE_NOERROR) {
    printf("%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/*
 * nag_gen_real_mat_print (x04cac).
 * Print real general matrix (easy-to-use)
 */
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n, x,
                        pdx, "Nearest Correlation Matrix x", NULL, &fail);
if (fail.code != NE_NOERROR) {
    printf("%s\n", fail.message);
    exit_status = 1;
    goto END;
}
printf("\nNumber of Newton steps taken: %11" NAG_IFMT "\n", iter);
printf("Number of function evaluations: %9" NAG_IFMT "\n\n", feval);
printf("alpha: %37.3f \n\n", alpha);
fflush(stdout);

/* nag_dsyev (f08fac).
 * Computes all eigenvalues and, optionally, eigenvectors of a real
 * symmetric matrix
 */
nag_dsyev(order, Nag_EigVals, Nag_Upper, n, x, pdx, eig, &fail);
if (fail.code != NE_NOERROR) {
    printf("%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* nag_gen_real_mat_print (x04cac).
 * Print real general matrix (easy-to-use)
 */
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, 1, n,
                        eig, pdeig, "Eigenvalues of x", NULL, &fail);
if (fail.code != NE_NOERROR) {
    printf("%s\n", fail.message);
    exit_status = 1;
}

END:
    NAG_FREE(eig);
    NAG_FREE(g);
    NAG_FREE(w);
    NAG_FREE(x);
    return exit_status;
}

```

10.2 Program Data

```

nag_nearest_correlation_bounded (g02abc) Example Program Data
4  Nag_Both  0.02      :: n, opt, alpha
   2.0   -1.0   0.0     0.0
  -1.0   2.0   -1.0     0.0
   0.0  -1.0   2.0    -1.0
   0.0   0.0  -1.0     2.0  :: End of g
100.0  20.0  20.0    20.0  :: w

```

10.3 Program Results

nag_nearest_correlation_bounded (g02abc) Example Program Results

```
Nearest Correlation Matrix x
      1      2      3      4
1  1.0000 -0.9187  0.0257  0.0086
2 -0.9187  1.0000 -0.3008  0.2270
3  0.0257 -0.3008  1.0000 -0.8859
4  0.0086  0.2270 -0.8859  1.0000
```

```
Number of Newton steps taken:      5
Number of function evaluations:    6
```

```
alpha:                             0.020
```

```
Eigenvalues of x
      1      2      3      4
1  0.0392  0.1183  1.6515  2.1910
```
