NAG Library Function Document

nag_deviates_f_dist (g01fdc)

1 Purpose

nag_deviates_f_dist (g01fdc) returns the deviate associated with the given lower tail probability of the F or variance-ratio distribution with real degrees of freedom.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_f_dist (double p, double df1, double df2, NagError *fail)
```

3 Description

The deviate, f_p, associated with the lower tail probability, p, of the F-distribution with degrees of freedom ν_1 and ν_2 is defined as the solution to

$$P\left(F \leq f_p : \nu_1, \nu_2\right) = p = \frac{\nu_1^{\frac{1}{2}\nu_1} \nu_2^{\frac{1}{2}\nu_2} \Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \int_0^{f_p} F^{\frac{1}{2}(\nu_1-2)} \left(\nu_2 + \nu_1 F\right)^{-\frac{1}{2}(\nu_1+\nu_2)}\, dF,$$

where $\nu_1, \nu_2 > 0$; $0 \leq f_p < \infty$.

The value of f_p is computed by means of a transformation to a beta distribution, $P_\beta(B \leq \beta : a, b)$:

$$P(F \leq f : \nu_1, \nu_2) = P_\beta\left(B \leq \frac{\nu_1 f}{\nu_1 f + \nu_2} : \nu_1/2, \nu_2/2\right)$$

and using a call to nag_deviates_beta (g01fec).

For very large values of both ν_1 and ν_2, greater than 10^5, a normal approximation is used. If only one of ν_1 or ν_2 is greater than 10^5 then a χ^2 approximation is used; see Abramowitz and Stegun (1972).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Arguments

1: **p** – double *Input*

On entry: p, the lower tail probability from the required F-distribution.

Constraint: $0.0 \leq \mathbf{p} < 1.0$.

2: **df1** – double *Input*

On entry: the degrees of freedom of the numerator variance, ν_1.

Constraint: $\mathbf{df1} > 0.0$.

3: **df2** – double *Input*

On entry: the degrees of freedom of the denominator variance, ν_2.

Constraint: **df2** > 0.0.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

On any of the error conditions listed below except **fail.code** = NE_SOL_NOT_CONV nag_deviates_f_dist (g01fdc) returns 0.0.

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_PROBAB_CLOSE_TO_TAIL

The probability is too close to 0.0 or 1.0. The value of f_p cannot be computed. This will only occur when the large sample approximations are used.

NE_REAL_ARG_GE

On entry, **p** = ⟨*value*⟩.
Constraint: **p** < 1.0.

NE_REAL_ARG_LE

On entry, **df1** = ⟨*value*⟩ and **df2** = ⟨*value*⟩.
Constraint: **df1** > 0.0 and **df2** > 0.0.

NE_REAL_ARG_LT

On entry, **p** = ⟨*value*⟩.
Constraint: **p** ≥ 0.0.

NE_SOL_NOT_CONV

The solution has failed to converge. However, the result should be a reasonable approximation. Alternatively, nag_deviates_beta (g01fec) can be used with a suitable setting of the argument **tol**.

7 Accuracy

The result should be accurate to five significant digits.

8 Parallelism and Performance

nag_deviates_f_dist (g01fdc) is not threaded in any implementation.

9 Further Comments

For higher accuracy nag_deviates_beta (g01fec) can be used along with the transformations given in Section 3.

10 Example

This example reads the lower tail probabilities for several F-distributions, and calculates and prints the corresponding deviates until the end of data is reached.

10.1 Program Text

```
/* nag_deviates_f_dist (g01fdc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
  Integer exit_status = 0;
  double df1, df2, f, p;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_deviates_f_dist (g01fdc) Example Program Results\n");
  printf("     p      df1     df2      f\n\n");
#ifdef _WIN32
  while (scanf_s("%lf %lf %lf", &p, &df1, &df2) != EOF)
#else
  while (scanf("%lf %lf %lf", &p, &df1, &df2) != EOF)
#endif
  {
    /* nag_deviates_f_dist (g01fdc).
     * Deviates for the F-distribution
     */
    f = nag_deviates_f_dist(p, df1, df2, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_deviates_f_dist (g01fdc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
    printf("%8.3f%8.3f%8.3f%8.3f\n", p, df1, df2, f);
  }

END:
  return exit_status;
}
```

10.2 Program Data

```
nag_deviates_f_dist (g01fdc) Example Program Data
0.9837  10.0   25.5
0.9000  1.0    1.0
0.5342  20.25  1.0
```

10.3 Program Results

```
nag_deviates_f_dist (g01fdc) Example Program Results
     p      df1     df2      f

   0.984  10.000  25.500   2.837
   0.900   1.000   1.000  39.863
   0.534  20.250   1.000   2.500
```