NAG Library Function Document

nag_dtbsv (f16pkc)

1 Purpose

nag_dtbsv (f16pkc) solves a system of equations given as a real triangular band matrix.

2 Specification

```
#include <nag.h>
#include <nagf16.h>
```

```
void nag_dtbsv (Nag_OrderType order, Nag_UploType uplo, Nag_TransType trans,
     Nag_DiagType diag, Integer n, Integer k, double alpha,
     const double ab[], Integer pdab, double x[], Integer incx,
     NagError *fail)
```

3 Description

nag_dtbsv (f16pkc) performs one of the matrix-vector operations

$$x \leftarrow \alpha A^{-1}x \quad \text{or} \quad x \leftarrow \alpha A^{-\mathrm{T}}x,$$

where A is an n by n real triangular band matrix with k subdiagonals or superdiagonals, x is an n-element real vector and α is a real scalar. $A^{-\mathrm{T}}$ denotes $\left(A^{\mathrm{T}}\right)^{-1}$ or equivalently $\left(A^{-1}\right)^{\mathrm{T}}$.

No test for singularity or near-singularity of A is included in this function. Such tests must be performed before calling this function.

4 References

Basic Linear Algebra Subprograms Technical (BLAST) Forum (2001) *Basic Linear Algebra Subprograms Technical (BLAST) Forum Standard* University of Tennessee, Knoxville, Tennessee http://www.netlib.org/blas/blast-forum/blas-report.pdf

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **uplo** – Nag_UploType *Input*

On entry: specifies whether A is upper or lower triangular.

uplo = Nag_Upper
A is upper triangular.

uplo = Nag_Lower
A is lower triangular.

Constraint: **uplo** = Nag_Upper or Nag_Lower.

3: **trans** – Nag_TransType *Input*

On entry: specifies the operation to be performed.

trans = Nag_NoTrans
$x \leftarrow \alpha A^{-1} x$.

trans = Nag_Trans or Nag_ConjTrans
$x \leftarrow \alpha A^{-\mathrm{T}} x$.

Constraint: **trans** = Nag_NoTrans, Nag_Trans or Nag_ConjTrans.

4: **diag** – Nag_DiagType *Input*

On entry: specifies whether A has nonunit or unit diagonal elements.

diag = Nag_NonUnitDiag
The diagonal elements are stored explicitly.

diag = Nag_UnitDiag
The diagonal elements are assumed to be 1 and are not referenced.

Constraint: **diag** = Nag_NonUnitDiag or Nag_UnitDiag.

5: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

6: **k** – Integer *Input*

On entry: k, the number of subdiagonals or superdiagonals of the matrix A.

Constraint: $\mathbf{k} \geq 0$.

7: **alpha** – double *Input*

On entry: the scalar α.

8: **ab**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **ab** must be at least $\max(1, \mathbf{pdab} \times \mathbf{n})$.

On entry: the n by n triangular band matrix A.

This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of A_{ij}, depends on the **order** and **uplo** arguments as follows:

if **order** = Nag_ColMajor and **uplo** = Nag_Upper,
A_{ij} is stored in $\mathbf{ab}[k + i - j + (j - 1) \times \mathbf{pdab}]$, for $j = 1, \dots, n$ and $i = \max(1, j - k), \dots, j$;
if **order** = Nag_ColMajor and **uplo** = Nag_Lower,
A_{ij} is stored in $\mathbf{ab}[i - j + (j - 1) \times \mathbf{pdab}]$, for $j = 1, \dots, n$ and $i = j, \dots, \min(n, j + k)$;
if **order** = Nag_RowMajor and **uplo** = Nag_Upper,
A_{ij} is stored in $\mathbf{ab}[j - i + (i - 1) \times \mathbf{pdab}]$, for $i = 1, \dots, n$ and $j = i, \dots, \min(n, i + k)$;
if **order** = Nag_RowMajor and **uplo** = Nag_Lower,
A_{ij} is stored in $\mathbf{ab}[k + j - i + (i - 1) \times \mathbf{pdab}]$, for $i = 1, \dots, n$ and $j = \max(1, i - k), \dots, i$.

If **diag** = Nag_UnitDiag, the diagonal elements of AB are assumed to be 1, and are not referenced.

9: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **ab**.

Constraint: **pdab** $\geq$ **k** $+ 1$.

10: **x**$[dim]$ – double *Input/Output*

Note: the dimension, *dim*, of the array **x** must be at least $\max(1, 1 + (\mathbf{n} - 1)|\mathbf{incx}|)$.

On entry: the right-hand side vector b.

On exit: the solution vector x.

11: **incx** – Integer *Input*

On entry: the increment in the subscripts of **x** between successive elements of x.

Constraint: **incx** $\neq 0$.

12: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **incx** = ⟨*value*⟩.
Constraint: **incx** $\neq 0$.

On entry, **k** = ⟨*value*⟩.
Constraint: **k** ≥ 0.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

NE_INT_2

On entry, **pdab** = ⟨*value*⟩, **k** = ⟨*value*⟩.
Constraint: **pdab** $\geq$ **k** $+ 1$.

NE_INTERNAL_ERROR

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The BLAS standard requires accurate implementations which avoid unnecessary over/underflow (see Section 2.7 of Basic Linear Algebra Subprograms Technical (BLAST) Forum (2001)).

8 Parallelism and Performance

nag_dtbsv (f16pkc) is not threaded in any implementation.

9 Further Comments

None.

10 Example

This example solves the real triangular band system of linear equations $Ax = y$, where A is the 4 by 4 triangular matrix given with one subdiagonal given by

$$A = \begin{pmatrix} -4.16 & & & \\ -2.25 & 4.78 & & \\ & 5.86 & 6.32 & \\ & & -4.82 & 0.16 \end{pmatrix}$$

and where

$$y = (-16.64, -13.78, 13.10, -14.14)^{\mathrm{T}}.$$

A is stored in array **ab** using banded storage format and y is stored in array **x**. nag_dtbsv (f16pkc) returns the solution in **x**.

10.1 Program Text

```
/* nag_dtbsv (f16pkc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf16.h>

int main(void)
{

  /* Scalars */
  double alpha;
  Integer exit_status, i, incx, j, kd, n, pdab, xlen;

  /* Arrays */
  double *ab = 0, *x = 0;
  char nag_enum_arg[40];

  /* Nag Types */
  NagError fail;
  Nag_OrderType order;
  Nag_TransType trans;
  Nag_UploType uplo;
  Nag_DiagType diag;

#ifdef NAG_COLUMN_MAJOR
```

```
#define AB_UPPER(I, J) ab[(J-1)*pdab + kd + I - J]
#define AB_LOWER(I, J) ab[(J-1)*pdab + I - J]
  order = Nag_ColMajor;
#else
#define AB_UPPER(I, J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I, J) ab[(I-1)*pdab + kd + J - I]
  order = Nag_RowMajor;
#endif

  exit_status = 0;
  INIT_FAIL(fail);

  printf("nag_dtbsv (f16pkc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* Read the problem dimensions */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &n, &kd);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &n, &kd);
#endif

  /* Read the uplo storage parameter */
#ifdef _WIN32
  scanf_s("%39s%*[^\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
  scanf("%39s%*[^\n] ", nag_enum_arg);
#endif
  /* nag_enum_name_to_value (x04nac).
   * Converts NAG enum member name to value
   */
  uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
  /* Read the transpose parameter */
#ifdef _WIN32
  scanf_s("%39s%*[^\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
  scanf("%39s%*[^\n] ", nag_enum_arg);
#endif
  /* nag_enum_name_to_value (x04nac), see above. */
  trans = (Nag_TransType) nag_enum_name_to_value(nag_enum_arg);
  /* Read the unit-diagonal parameter */
#ifdef _WIN32
  scanf_s("%39s%*[^\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
  scanf("%39s%*[^\n] ", nag_enum_arg);
#endif
  /* nag_enum_name_to_value (x04nac), see above. */
  diag = (Nag_DiagType) nag_enum_name_to_value(nag_enum_arg);

  /* Read scalar parameters */
#ifdef _WIN32
  scanf_s("%lf%*[^\n] ", &alpha);
#else
  scanf("%lf%*[^\n] ", &alpha);
#endif
  /* Read increment parameter */
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n] ", &incx);
#else
  scanf("%" NAG_IFMT "%*[^\n] ", &incx);
#endif

  pdab = kd + 1;
  xlen = MAX(1, 1 + (n - 1) * ABS(incx));
  if (n > 0) {
    /* Allocate memory */
    if (!(ab = NAG_ALLOC(pdab * n, double)) || !(x = NAG_ALLOC(xlen, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
  }
  else {
    printf("Invalid n\n");
    exit_status = 1;
    return exit_status;
  }

  /* Input matrix AB and vector x */

  if (uplo == Nag_Upper) {
    for (i = 1; i <= n; ++i) {
      if (diag == Nag_NonUnitDiag)
#ifdef _WIN32
        scanf_s("%lf", &AB_UPPER(i, i));
#else
        scanf("%lf", &AB_UPPER(i, i));
#endif
      for (j = i + 1; j <= MIN(i + kd, n); ++j)
#ifdef _WIN32
        scanf_s("%lf", &AB_UPPER(i, j));
#else
        scanf("%lf", &AB_UPPER(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif
  }
  else {
    for (i = 1; i <= n; ++i) {
      for (j = MAX(1, i - kd); j < i; ++j)
#ifdef _WIN32
        scanf_s("%lf", &AB_LOWER(i, j));
#else
        scanf("%lf", &AB_LOWER(i, j));
#endif
      if (diag == Nag_NonUnitDiag)
#ifdef _WIN32
        scanf_s("%lf", &AB_LOWER(i, i));
#else
        scanf("%lf", &AB_LOWER(i, i));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif
  }
  for (i = 0; i < xlen; ++i)
#ifdef _WIN32
    scanf_s("%lf%*[^\n] ", &x[i]);
#else
    scanf("%lf%*[^\n] ", &x[i]);
#endif

  /* nag_dtbsv (f16pkc).
   * Solution of real triangular band system of linear equations.
   *
   */
  nag_dtbsv(order, uplo, trans, diag, n, kd, alpha, ab, pdab, x, incx, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_dtbsv (f16pkc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  /* Print output vector x */
  printf("%s\n", " Solution x:");
  for (i = 0; i < xlen; ++i) {
    printf("%11f\n", x[i]);
  }

END:
  NAG_FREE(ab);
  NAG_FREE(x);

  return exit_status;
}
```

10.2 Program Data

```
nag_dtbsv (f16pkc) Example Program Data
  4  1                        :Values of n and kd
  Nag_Lower                   :Storage of A
  Nag_NoTrans                 :Transpose A?
  Nag_NonUnitDiag             :Unit diagonal elements?
  1.0                         :Value of alpha
  1                           :Value of incx
 -4.16
 -2.25   4.78
         5.86   6.32
               -4.82   0.16   :End of matrix A
-16.64
-13.78
 13.10
-14.14                        :End of vector x
```

10.3 Program Results

```
nag_dtbsv (f16pkc) Example Program Results

 Solution x:
   4.000000
  -1.000000
   3.000000
   2.000000
```