

## NAG Library Function Document

### nag\_complex\_sparse\_eigensystem\_option (f12arc)

**Note:** this function uses **optional parameters** to define choices in the problem specification. If you wish to use default settings for all of the optional parameters, then this function need not be called. If, however, you wish to reset some or all of the settings please refer to Section 11 for a detailed description of the specification of the optional parameters.

## 1 Purpose

nag\_complex\_sparse\_eigensystem\_option (f12arc) is an option setting function in a suite of functions consisting of nag\_complex\_sparse\_eigensystem\_init (f12anc), nag\_complex\_sparse\_eigensystem\_iter (f12apc), nag\_complex\_sparse\_eigensystem\_sol (f12aqc), nag\_complex\_sparse\_eigensystem\_option (f12arc) and nag\_complex\_sparse\_eigensystem\_monit (f12asc), for which it may be used to supply individual optional parameters to nag\_complex\_sparse\_eigensystem\_iter (f12apc) and nag\_complex\_sparse\_eigensystem\_sol (f12aqc). nag\_complex\_sparse\_eigensystem\_option (f12arc) is also an option setting function in a suite of functions consisting of nag\_complex\_sparse\_eigensystem\_init (f12anc), nag\_complex\_banded\_eigensystem\_init (f12atc) and nag\_complex\_banded\_eigensystem\_solve (f12auc) for which it may be used to supply individual optional parameters to nag\_complex\_banded\_eigensystem\_solve (f12auc).

The initialization function for the appropriate suite, nag\_complex\_sparse\_eigensystem\_init (f12anc) or nag\_complex\_banded\_eigensystem\_init (f12atc), **must** have been called prior to calling nag\_complex\_sparse\_eigensystem\_option (f12arc).

## 2 Specification

```
#include <nag.h>
#include <nagf12.h>
void nag_complex_sparse_eigensystem_option (const char *str,
    Integer icomm[], Complex comm[], NagError *fail)
```

## 3 Description

nag\_complex\_sparse\_eigensystem\_option (f12arc) may be used to supply values for optional parameters to nag\_complex\_sparse\_eigensystem\_iter (f12apc) and nag\_complex\_sparse\_eigensystem\_sol (f12aqc), or to nag\_complex\_banded\_eigensystem\_solve (f12auc). It is only necessary to call nag\_complex\_sparse\_eigensystem\_option (f12arc) for those arguments whose values are to be different from their default values. One call to nag\_complex\_sparse\_eigensystem\_option (f12arc) sets one argument value.

Each optional parameter is defined by a single character string consisting of one or more items. The items associated with a given option must be separated by spaces, or equals signs [=]. Alphabetic characters may be upper or lower case. The string

```
'Iteration Limit = 500'
```

is an example of a string used to set an optional parameter. For each option the string contains one or more of the following items:

- a mandatory keyword;
- a phrase that qualifies the keyword;
- a number that specifies an Integer or double value. Such numbers may be up to 16 contiguous characters in C's d or g format.

nag\_complex\_sparse\_eigensystem\_option (f12arc) does not have an equivalent function from the ARPACK package which passes options by directly setting values to scalar arguments or to specific elements of array arguments. nag\_complex\_sparse\_eigensystem\_option (f12arc) is intended to make the

passing of options more transparent and follows the same principle as the single option setting functions in Chapter e04 (see `nag_opt_sparse_convex_qp_option_set_string` (e04nsc) for an example).

The setup function `nag_complex_sparse_eigensystem_init` (f12anc) must be called prior to the first call to `nag_complex_sparse_eigensystem_option` (f12arc) or `nag_complex_banded_eigensystem_init` (f12ate), and all calls to `nag_complex_sparse_eigensystem_option` (f12arc) must precede the first call to `nag_complex_sparse_eigensystem_iter` (f12apc) or `nag_complex_banded_eigensystem_solve` (f12auc).

A complete list of optional parameters, their abbreviations, synonyms and default values is given in Section 11.

## 4 References

Lehoucq R B (2001) Implicitly restarted Arnoldi methods and subspace iteration *SIAM Journal on Matrix Analysis and Applications* **23** 551–562

Lehoucq R B and Scott J A (1996) An evaluation of software for computing eigenvalues of sparse nonsymmetric matrices *Preprint MCS-P547-1195* Argonne National Laboratory

Lehoucq R B and Sorensen D C (1996) Deflation techniques for an implicitly restarted Arnoldi iteration *SIAM Journal on Matrix Analysis and Applications* **17** 789–821

Lehoucq R B, Sorensen D C and Yang C (1998) *ARPACK Users' Guide: Solution of Large-scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods* SIAM, Philadelphia

## 5 Arguments

1: **str** – const char \* *Input*

*On entry:* a single valid option string (as described in Section 3 and Section 11).

2: **icomm**[*dim*] – Integer *Communication Array*

**Note:** the dimension, *dim*, of the array **icomm** must be at least `max(1,licomm)` (see `nag_complex_sparse_eigensystem_init` (f12anc)).

*On initial entry:* must remain unchanged following a call to the setup function `nag_complex_sparse_eigensystem_init` (f12anc).

*On exit:* contains data on the current options set.

3: **comm**[*dim*] – Complex *Communication Array*

**Note:** the dimension, *dim*, of the array **comm** must be at least `max(1,licomm)` (see `nag_complex_sparse_eigensystem_init` (f12anc)).

*On initial entry:* must remain unchanged following a call to the setup function `nag_complex_sparse_eigensystem_init` (f12anc).

*On exit:* contains data on the current options set.

4: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_INITIALIZATION**

Either the initialization function has not been called prior to the call of this function or a communication array has become corrupted.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_INVALID\_OPTION**

Ambiguous keyword:  $\langle value \rangle$

Keyword not recognized:  $\langle value \rangle$

Second keyword not recognized:  $\langle value \rangle$

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**7 Accuracy**

Not applicable.

**8 Parallelism and Performance**

`nag_complex_sparse_eigensystem_option` (f12arc) is not threaded in any implementation.

**9 Further Comments**

None.

**10 Example**

This example solves  $Ax = \lambda Bx$  in shifted-inverse mode, where  $A$  and  $B$  are derived from the finite element discretization of the one-dimensional convection-diffusion operator  $\frac{d^2u}{dx^2} + \rho \frac{du}{dx}$  on the interval  $[0, 1]$ , with zero Dirichlet boundary conditions.

**10.1 Program Text**

```
/*
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nag_string.h>
#include <stdio.h>
```

```

#include <naga02.h>
#include <nagf12.h>
#include <nagf16.h>

/* Table of constant values */
static Complex four = { 4., 0. };

static void mv(Integer, Complex *, Complex *);
static void my_zgttrf(Integer, Complex *, Complex *, Complex *,
                      Complex *, Integer *, Integer *);
static void my_zgttrs(Integer, Complex *, Complex *, Complex *,
                      Complex *, Integer *, Complex *);

int main(void)
{
    /* Constants */
    Integer licomm = 140, imon = 0;

    /* Scalars */
    Complex rho, s1, s2, s3, sigma;
    double estnrm, hr, hr1, sr, shs;
    Integer exit_status, info, irevcm, j, lcomm, n, nconv, ncv;
    Integer nev, niter, nshift, nx;
    /* Nag types */
    NagError fail;
    /* Arrays */
    Complex *comm = 0, *eigv = 0, *eigest = 0, *dd = 0, *dl = 0, *du = 0;
    Complex *du2 = 0, *resid = 0, *v = 0;
    Integer *icomm = 0, *ipiv = 0;
    /* Pointers */
    Complex *mx = 0, *x = 0, *y = 0;

    exit_status = 0;
    INIT_FAIL(fail);

    printf("nag_complex_sparse_eigensystem_option (f12arc) Example "
           "Program Results\n");
    /* Skip heading in data file */
#ifndef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

#ifndef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &nx, &nev, &ncv);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[^\n] ", &nx, &nev, &ncv);
#endif
    n = nx * nx;
    lcomm = 3 * n + 3 * ncv * ncv + 5 * ncv + 60;
    /* Allocate memory */
    if (!(comm = NAG_ALLOC(lcomm, Complex)) ||
        !(eigv = NAG_ALLOC(ncv, Complex)) ||
        !(eigest = NAG_ALLOC(ncv, Complex)) ||
        !(dd = NAG_ALLOC(n, Complex)) ||
        !(dl = NAG_ALLOC(n, Complex)) ||
        !(du = NAG_ALLOC(n, Complex)) ||
        !(du2 = NAG_ALLOC(n, Complex)) ||
        !(resid = NAG_ALLOC(n, Complex)) ||
        !(v = NAG_ALLOC(n * ncv, Complex)) ||
        !(icomm = NAG_ALLOC(licomm, Integer)) ||
        !(ipiv = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    /* Initialize communication arrays for problem using
       nag_complex_sparse_eigensystem_init (f12anc). */
    nag_complex_sparse_eigensystem_init(n, nev, ncv, icomm, licomm,

```

```

comm, lcomm, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_complex_sparse_eigensystem_init (f12anc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
}
/* Select the required mode using
   nag_complex_sparse_eigensystem_option (f12arc). */
nag_complex_sparse_eigensystem_option("SHIFTED INVERSE", icomm,
                                       comm, &fail);
/* Select the problem type using
   nag_complex_sparse_eigensystem_option (f12arc). */
nag_complex_sparse_eigensystem_option("GENERALIZED", icomm, comm, &fail);
/* Set values for sigma and rho */
/* Assign to Complex type using nag_complex (a02bac) */
sigma = nag_complex(500.0, 0.0);
rho = nag_complex(10.0, 0.0);
hr1 = (double) (n + 1); /* one/h */
hr = 1.0 / hr1; /* h */
sr = 0.5 * rho.re; /* s */
shs = sigma.re * hr / 6.0; /* sigma*h/6 */
/* Assign to Complex type using nag_complex (a02bac) */
s1 = nag_complex(-hr1 - sr - shs, 0.0); /* -one/h - s -sigma*h/six */
s3 = nag_complex(-hr1 + sr - shs, 0.0); /* -one/h + s -sigma*h/six */
s2 = nag_complex(2.0 * hr1 - 4.0 * shs, 0.0); /* two/h - four*sigma*h/six */

for (j = 0; j <= n - 2; ++j) {
    dl[j] = s1;
    dd[j] = s2;
    du[j] = s3;
}
dd[n - 1] = s2;

my_zgttrf(n, dl, dd, du, du2, ipiv, &info);
irevcm = 0;
REVCOMLOOP:
/* repeated calls to reverse communication routine
   nag_complex_sparse_eigensystem_iter (f12apc). */
nag_complex_sparse_eigensystem_iter(&irevcm, resid, v, &x, &y, &mx,
                                    &nshift, comm, icomm, &fail);
if (irevcm != 5) {
    if (irevcm == -1) {
        /* Perform x <--- OP*x = inv[A-SIGMA*M]*M*x */
        mv(nx, x, y);
        my_zgttrs(n, dl, dd, du, du2, ipiv, y);
    }
    else if (irevcm == 1) {
        /* Perform x <--- OP*x = inv[A-SIGMA*M]*M*x, */
        /* MX stored in mx */
        for (j = 0; j < n; ++j) {
            y[j] = mx[j];
        }
        my_zgttrs(n, dl, dd, du, du2, ipiv, y);
    }
    else if (irevcm == 2) {
        /* Perform y <--- M*x */
        mv(nx, x, y);
    }
    else if (irevcm == 4 && imon == 1) {
        /* If imon=1, get monitoring information using
           nag_complex_sparse_eigensystem_monit (f12asc). */
        nag_complex_sparse_eigensystem_monit(&niter, &nconv, eigv,
                                             eigest, icomm, comm);
        /* Compute 2-norm of Ritz estimates using
           nag_zge_norm (f16uac). */
        nag_zge_norm(Nag_ColMajor, Nag_FrobeniusNorm, nev, 1,
                     eigest, nev, &estnrm, &fail);
        printf("Iteration %3" NAG_IFMT ", ", niter);
        printf(" No. converged = %3" NAG_IFMT ", ", nconv);
        printf(" norm of estimates = %17.8e\n", estnrm);
    }
}

```

```

    }
    goto REVCOMLOOP;
}
if (fail.code == NE_NOERROR) {
    /* Post-Process using nag_complex_sparse_eigensystem_sol
       (f12aqc) to compute eigenvalues/vectors. */
    nag_complex_sparse_eigensystem_sol(&nconv, eigv, v, sigma, resid, v,
                                       comm, icomm, &fail);
    printf("\n The %4" NAG_IFMT " generalized Ritz values closest to "
           "( %7.3f , %7.3f ) are:\n\n", nconv, sigma.re, sigma.im);
    for (j = 0; j <= nconv - 1; ++j) {
        printf("%8" NAG_IFMT "%5s( %12.4f , %12.4f )\n", j + 1, "", 
               eigv[j].re, eigv[j].im);
    }
}
else {
    printf(" Error from nag_complex_sparse_eigensystem_iter (f12apc)."
           "\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
NAG_FREE(comm);
NAG_FREE(eigv);
NAG_FREE(eigest);
NAG_FREE(dd);
NAG_FREE(dl);
NAG_FREE(du);
NAG_FREE(du2);
NAG_FREE(resid);
NAG_FREE(v);
NAG_FREE(icomm);
NAG_FREE(ipiv);

return exit_status;
}

static void mv(Integer nx, Complex *v, Complex *y)
{
    /* Compute the out-of-place matrix vector multiplication Y<---M*X, */
    /* where M is mass matrix formed by using piecewise linear elements */
    /* on [0,1]. */

    /* Scalars */
    Complex hsix, z1;
    Integer j, n;
    /* Function Body */
    n = nx * nx;
    /* Assign to Complex type using nag_complex (a02bac) */
    hsix = nag_complex(1.0 / (6.0 * (double) (n + 1)), 0.0);
    /* y[0] = (four*v[0]+v[1])*(h/six) */
    /* Compute Complex multiply using nag_complex_multiply
       (a02ccc). */
    z1 = nag_complex_multiply(four, v[0]);
    /* Compute Complex addition using nag_complex_add (a02cac). */
    z1 = nag_complex_add(z1, v[1]);
    y[0] = nag_complex_multiply(z1, hsix);
    for (j = 1; j <= n - 2; ++j) {
        /* y[j] = (v[j-1] + four*v[j] + v[j+1])*(h/six) */
        /* Compute Complex multiply using nag_complex_multiply
           (a02ccc). */
        z1 = nag_complex_multiply(four, v[j]);
        /* Compute Complex addition using nag_complex_add (a02cac). */
        z1 = nag_complex_add(v[j - 1], z1);
        z1 = nag_complex_add(z1, v[j + 1]);
        y[j] = nag_complex_multiply(z1, hsix);
    }
    /* y[n-1] = (v[n-2] + four*v[n-1])*(h/six) */
    /* Compute Complex multiply using nag_complex_multiply (a02ccc). */
    z1 = nag_complex_multiply(four, v[n - 1]);
    /* Compute Complex addition using nag_complex_add (a02cac). */
}

```

```

z1 = nag_complex_add(v[n - 2], z1);
y[n - 1] = nag_complex_multiply(z1, hsix);
return;
} /* mv */

static void my_zgttrf(Integer n, Complex dl[], Complex d[],
Complex du[], Complex du2[], Integer ipiv[], Integer *info)
{
    /* A simple C version of the Lapack routine zgttrf with argument
       checking removed */
    /* Scalars */
    Complex temp, fact, z1;
    Integer i;
    /* Function Body */
    *info = 0;
    for (i = 0; i < n; ++i) {
        ipiv[i] = i;
    }
    for (i = 0; i < n - 2; ++i) {
        du2[i] = nag_complex(0.0, 0.0);
    }
    for (i = 0; i < n - 2; ++i) {
        if (fabs(d[i].re) + fabs(d[i].im) >= fabs(dl[i].re) + fabs(dl[i].im)) {
            /* No row interchange required, eliminate dl[i]. */
            if (fabs(d[i].re) + fabs(d[i].im) != 0.0) {
                /* Compute Complex division using nag_complex_divide
                   (a02cdc). */
                fact = nag_complex_divide(dl[i], d[i]);
                dl[i] = fact;
                /* Compute Complex multiply using nag_complex_multiply
                   (a02ccc). */
                fact = nag_complex_multiply(fact, du[i]);
                /* Compute Complex subtraction using
                   nag_complex_subtract (a02cbc). */
                d[i + 1] = nag_complex_subtract(d[i + 1], fact);
            }
        }
        else {
            /* Interchange rows I and I+1, eliminate dl[I] */
            /* Compute Complex division using nag_complex_divide
               (a02cdc). */
            fact = nag_complex_divide(d[i], dl[i]);
            d[i] = dl[i];
            dl[i] = fact;
            temp = du[i];
            du[i] = d[i + 1];
            /* Compute Complex multiply using nag_complex_multiply
               (a02ccc). */
            z1 = nag_complex_multiply(fact, d[i + 1]);
            /* Compute Complex subtraction using nag_complex_subtract
               (a02cbc). */
            d[i + 1] = nag_complex_subtract(temp, z1);
            du2[i] = du[i + 1];
            /* Compute Complex multiply using nag_complex_multiply
               (a02ccc). */
            du[i + 1] = nag_complex_multiply(fact, du[i + 1]);
            /* Perform Complex negation using nag_complex_negate
               (a02cec). */
            du[i + 1] = nag_complex_negate(du[i + 1]);
            ipiv[i] = i + 1;
        }
    }
    if (n > 1) {
        i = n - 2;
        if (fabs(d[i].re) + fabs(d[i].im) >= fabs(dl[i].re) + fabs(dl[i].im)) {
            if (fabs(d[i].re) + fabs(d[i].im) != 0.0) {
                /* Compute Complex division using nag_complex_divide
                   (a02cdc). */
                fact = nag_complex_divide(dl[i], d[i]);
                dl[i] = fact;

```

```

        /* Compute Complex multiply using nag_complex_multiply
           (a02ccc). */
        fact = nag_complex_multiply(fact, du[i]);
        /* Compute Complex subtraction using
           nag_complex_subtract (a02cbc). */
        d[i + 1] = nag_complex_subtract(d[i + 1], fact);
    }
}
else {
    /* Compute Complex division using nag_complex_divide
       (a02cdc). */
    fact = nag_complex_divide(d[i], dl[i]);
    d[i] = dl[i];
    dl[i] = fact;
    temp = du[i];
    du[i] = d[i + 1];
    /* Compute Complex multiply using nag_complex_multiply
       (a02ccc). */
    z1 = nag_complex_multiply(fact, d[i + 1]);
    /* Compute Complex subtraction using nag_complex_subtract
       (a02cbc). */
    d[i + 1] = nag_complex_subtract(temp, z1);
    ipiv[i] = i + 1;
}
}
/* Check for a zero on the diagonal of U. */
for (i = 0; i < n; ++i) {
    if (fabs(d[i].re) + fabs(d[i].im) == 0.0) {
        *info = i;
        goto END;
    }
}
END:
    return;
}

static void my_zgttrs(Integer n, Complex dl[], Complex d[],
                      Complex du[], Complex du2[], Integer ipiv[],
                      Complex b[])
{
    /* A simple C version of the Lapack routine zgttrs with argument
       checking removed, the number of right-hand-sides=1, Trans='N' */
    /* Scalars */
    Complex temp, z1;
    Integer i;
    /* Solve L*x = b. */
    for (i = 0; i < n - 1; ++i) {
        if (ipiv[i] == i) {
            /* b[i+1] = b[i+1] - dl[i]*b[i] */
            /* Compute Complex multiply using nag_complex_multiply
               (a02ccc). */
            temp = nag_complex_multiply(dl[i], b[i]);
            /* Compute Complex subtraction using nag_complex_subtract
               (a02cbc). */
            b[i + 1] = nag_complex_subtract(b[i + 1], temp);
        }
        else {
            temp = b[i];
            b[i] = b[i + 1];
            /* Compute Complex multiply using nag_complex_multiply
               (a02ccc). */
            z1 = nag_complex_multiply(dl[i], b[i]);
            /* Compute Complex subtraction using nag_complex_subtract
               (a02cbc). */
            b[i + 1] = nag_complex_subtract(temp, z1);
        }
    }
    /* Solve U*x = b. */
    /* Compute Complex division using nag_complex_divide (a02cdc). */
    b[n - 1] = nag_complex_divide(b[n - 1], d[n - 1]);
    if (n > 1) {

```

```

/* Compute Complex multiply using nag_complex_multiply
   (a02ccc). */
temp = nag_complex_multiply(du[n - 2], b[n - 1]);
/* Compute Complex subtraction using nag_complex_subtract
   (a02cbc). */
z1 = nag_complex_subtract(b[n - 2], temp);
/* Compute Complex division using nag_complex_divide (a02cdc). */
b[n - 2] = nag_complex_divide(z1, d[n - 2]);
}
for (i = n - 3; i >= 0; --i) {
/* b[i] = (b[i]-du[i]*b[i+1]-du2[i]*b[i+2])/d[i]; */
/* Compute Complex multiply using nag_complex_multiply
   (a02ccc). */
temp = nag_complex_multiply(du[i], b[i + 1]);
z1 = nag_complex_multiply(du2[i], b[i + 2]);
/* Compute Complex addition using nag_complex_add
   (a02cac). */
temp = nag_complex_add(temp, z1);
/* Compute Complex subtraction using nag_complex_subtract
   (a02cbc). */
z1 = nag_complex_subtract(b[i], temp);
/* Compute Complex division using nag_complex_divide
   (a02cdc). */
b[i] = nag_complex_divide(z1, d[i]);
}
return;
}

```

## 10.2 Program Data

```
nag_complex_sparse_eigensystem_option (f12arc) Example Program Data
10  4  20 : Values for nx, nev and ncv
```

## 10.3 Program Results

```
nag_complex_sparse_eigensystem_option (f12arc) Example Program Results
```

The 4 generalized Ritz values closest to ( 500.000 , 0.000 ) are:

1	(	509.9390	,	0.0000	)
2	(	380.9092	,	0.0000	)
3	(	659.1558	,	-0.0000	)
4	(	271.9412	,	-0.0000	)

## 11 Optional Parameters

Several optional parameters for the computational suite functions `nag_complex_sparse_eigensystem_iter` (f12apc) and `nag_complex_sparse_eigensystem_sol` (f12aqc), and for the banded driver `nag_complex_banded_eigensystem_solve` (f12auc), define choices in the problem specification or the algorithm logic. In order to reduce the number of formal arguments of `nag_complex_sparse_eigensystem_iter` (f12apc), `nag_complex_sparse_eigensystem_sol` (f12aqc) and `nag_complex_banded_eigensystem_solve` (f12auc) these optional parameters have associated *default values* that are appropriate for most problems. Therefore, you need only specify those optional parameters whose values are to be different from their default values.

The remainder of this section can be skipped if you wish to use the default values for all optional parameters.

The following is a list of the optional parameters available. A full description of each optional parameter is provided in Section 11.1.

### Advisory

### Defaults

### Exact Shifts

### Generalized

**Initial Residual**  
**Iteration Limit**  
**Largest Imaginary**  
**Largest Magnitude**  
**Largest Real**  
**List**  
**Monitoring**  
**Nolist**  
**Print Level**  
**Random Residual**  
**Regular**  
**Regular Inverse**  
**Shifted Inverse**  
**Smallest Imaginary**  
**Smallest Magnitude**  
**Smallest Real**  
**Standard**  
**Supplied Shifts**  
**Tolerance**  
**Vectors**

Optional parameters may be specified by calling `nag_complex_sparse_eigensystem_option` (f12arc) before a call to `nag_complex_sparse_eigensystem_iter` (f12apc) or `nag_complex_banded_eigensystem_init` (f12atc), but after a corresponding call to `nag_complex_sparse_eigensystem_init` (f12anc) or `nag_complex_banded_eigensystem_solve` (f12auc). One call is necessary for each optional parameter. Any optional parameters you do not specify are set to their default values. Optional parameters you do specify are unaltered by `nag_complex_sparse_eigensystem_iter` (f12apc), `nag_complex_sparse_eigensystem_sol` (f12aqc) and `nag_complex_banded_eigensystem_solve` (f12auc) (unless they define invalid values) and so remain in effect for subsequent calls unless you alter them.

## 11.1 Description of the Optional Parameters

For each option, we give a summary line, a description of the optional parameter and details of constraints.

The summary line contains:

- the keywords, where the minimum abbreviation of each keyword is underlined;
- a parameter value, where the letters *a*, *i* and *r* denote options that take character, integer and real values respectively;
- the default value, where the symbol  $\epsilon$  is a generic notation for ***machine precision*** (see `nag_machine_precision` (X02AJC)).

Keywords and character values are case and white space insensitive.

Optional parameters used to specify files (e.g., **Advisory** and **Monitoring**) have type `Nag_FileID`. This ID value must either be set to 0 (the default value) in which case there will be no output, or will be as returned by a call of `nag_open_file` (x04acc).

**Advisory** Default = 0

(See Section 2.3.1.1 in How to Use the NAG Library and its Documentation for further information on NAG data types.)

Advisory messages are output to `Nag_FileID` **Advisory** during the solution of the problem.

**Defaults**

This special keyword may be used to reset all optional parameters to their default values.

**Exact Shifts**

Default

**Supplied Shifts**

During the Arnoldi iterative process, shifts are applied as part of the implicit restarting scheme. The shift strategy used by default and selected by the optional parameter **Exact Shifts** is strongly recommended over the alternative **Supplied Shifts** and will always be used by nag\_complex\_banded\_eigensystem\_solve (f12auc).

If **Exact Shifts** are used then these are computed internally by the algorithm in the implicit restarting scheme. This strategy is generally effective and cheaper to apply in terms of number of operations than using explicit shifts.

If **Supplied Shifts** are used then, during the Arnoldi iterative process, you must supply shifts through array arguments of nag\_complex\_sparse\_eigensystem\_iter (f12apc) when nag\_complex\_sparse\_eigensystem\_iter (f12apc) returns with **irevcm** = 3; the complex shifts are supplied in **y**. This option should only be used if you are an experienced user since this requires some algorithmic knowledge and because more operations are usually required than for the implicit shift scheme. Details on the use of explicit shifts and further references on shift strategies are available in Lehoucq *et al.* (1998).

**Iteration Limit***i*

Default = 300

The limit on the number of Arnoldi iterations that can be performed before nag\_complex\_sparse\_eigensystem\_iter (f12apc) or nag\_complex\_banded\_eigensystem\_solve (f12auc) exits. If not all requested eigenvalues have converged to within **Tolerance** and the number of Arnoldi iterations has reached this limit then nag\_complex\_sparse\_eigensystem\_iter (f12apc) or nag\_complex\_banded\_eigensystem\_solve (f12auc) exits with an error; nag\_complex\_banded\_eigensystem\_solve (f12auc) returns the number of converged eigenvalues, the converged eigenvalues and, if requested, the corresponding eigenvectors, while nag\_complex\_sparse\_eigensystem\_sol (f12aqc) can be called subsequent to nag\_complex\_sparse\_eigensystem\_iter (f12apc) to do the same.

**Largest Magnitude**

Default

**Largest Imaginary****Largest Real****Smallest Imaginary****Smallest Magnitude****Smallest Real**

The Arnoldi iterative method converges on a number of eigenvalues with given properties. The default is for nag\_complex\_sparse\_eigensystem\_iter (f12apc) or nag\_complex\_banded\_eigensystem\_solve (f12auc) to compute the eigenvalues of largest magnitude using **Largest Magnitude**. Alternatively, eigenvalues may be chosen which have **Largest Real** part, **Largest Imaginary** part, **Smallest Magnitude**, **Smallest Real** part or **Smallest Imaginary** part.

Note that these options select the eigenvalue properties for eigenvalues of OP (and *B* for **Generalized** problems), the linear operator determined by the computational mode and problem type.

**Nolist**

Default

**List**

Normally each optional parameter specification is not printed to **Advisory** as it is supplied. Optional parameter **List** may be used to enable printing and optional parameter **Nolist** may be used to suppress the printing.

**Monitoring**

Default = -1

(See Section 2.3.1.1 in How to Use the NAG Library and its Documentation for further information on NAG data types.)

Unless **Monitoring** is set to  $-1$  (the default), monitoring information is output to Nag\_FileID **Monitoring** during the solution of each problem; this may be the same as **Advisory**. The type of information produced is dependent on the value of **Print Level**, see the description of the optional parameter **Print Level** in this section for details of the information produced. Please see nag\_open\_file (x04acc) to associate a file with a given Nag\_FileID.

<b><u>Print Level</u></b>	<i>i</i>	Default = 0
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This controls the amount of printing produced by nag\_complex\_sparse\_eigensystem\_option (f12arc) as follows.

- = 0      No output except error messages.
- > 0      The set of selected options.
- = 2      Problem and timing statistics on final exit from nag\_complex\_sparse\_eigensystem\_iter (f12apc) or nag\_complex\_banded\_eigensystem\_solve (f12auc).
- $\geq 5$     A single line of summary output at each Arnoldi iteration.
- $\geq 10$    If **Monitoring** is set, then at each iteration, the length and additional steps of the current Arnoldi factorization and the number of converged Ritz values; during re-orthogonalization, the norm of initial/restarted starting vector.
- $\geq 20$    Problem and timing statistics on final exit from nag\_complex\_sparse\_eigensystem\_iter (f12apc). If **Monitoring** is set, then at each iteration, the number of shifts being applied, the eigenvalues and estimates of the Hessenberg matrix  $H$ , the size of the Arnoldi basis, the wanted Ritz values and associated Ritz estimates and the shifts applied; vector norms prior to and following re-orthogonalization.
- $\geq 30$    If **Monitoring** is set, then on final iteration, the norm of the residual; when computing the Schur form, the eigenvalues and Ritz estimates both before and after sorting; for each iteration, the norm of residual for compressed factorization and the compressed upper Hessenberg matrix  $H$ ; during re-orthogonalization, the initial/restarted starting vector; during the Arnoldi iteration loop, a restart is flagged and the number of the residual requiring iterative refinement; while applying shifts, the indices of the shifts being applied.
- $\geq 40$    If **Monitoring** is set, then during the Arnoldi iteration loop, the Arnoldi vector number and norm of the current residual; while applying shifts, key measures of progress and the order of  $H$ ; while computing eigenvalues of  $H$ , the last rows of the Schur and eigenvector matrices; when computing implicit shifts, the eigenvalues and Ritz estimates of  $H$ .
- $\geq 50$    If **Monitoring** is set, then during Arnoldi iteration loop: norms of key components and the active column of  $H$ , norms of residuals during iterative refinement, the final upper Hessenberg matrix  $H$ ; while applying shifts: number of shifts, shift values, block indices, updated matrix  $H$ ; while computing eigenvalues of  $H$ : the matrix  $H$ , the computed eigenvalues and Ritz estimates.

<b><u>Random Residual</u></b>	Default
<b><u>Initial Residual</u></b>	

To begin the Arnoldi iterative process, nag\_complex\_sparse\_eigensystem\_iter (f12apc) and nag\_complex\_banded\_eigensystem\_solve (f12auc) requires an initial residual vector. By default nag\_complex\_sparse\_eigensystem\_iter (f12apc) and nag\_complex\_banded\_eigensystem\_solve (f12auc) provides its own random initial residual vector; this option can also be set using optional parameter **Random Residual**. Alternatively, you can supply an initial residual vector (perhaps from a previous computation) to nag\_complex\_sparse\_eigensystem\_iter (f12apc) and nag\_complex\_banded\_eigensystem\_solve (f12auc) through the array argument **resid**; this option can be set using optional parameter **Initial Residual**.

<b>Regular</b>	Default
<b>Regular Inverse</b>	
<b>Shifted Inverse</b>	

These options define the computational mode which in turn defines the form of operation  $\text{OP}(x)$  to be performed by nag\_complex\_banded\_eigensystem\_solve (f12auc) or when nag\_complex\_sparse\_eigen system\_iter (f12apc) returns with  $\text{irevcm} = -1$  or 1 and the matrix-vector product  $Bx$  when nag\_complex\_sparse\_eigensystem\_iter (f12apc) returns with  $\text{irevcm} = -2$ .

Given a **Standard** eigenvalue problem in the form  $Ax = \lambda x$  then the following modes are available with the appropriate operator  $\text{OP}(x)$ .

<b>Regular</b>	$\text{OP} = A$
<b>Shifted Inverse</b>	$\text{OP} = (A - \sigma I)^{-1}$

Given a **Generalized** eigenvalue problem in the form  $Ax = \lambda Bx$  then the following modes are available with the appropriate operator  $\text{OP}(x)$ .

<b>Regular Inverse</b>	$\text{OP} = B^{-1}A$
<b>Shifted Inverse</b>	$\text{OP} = (A - \sigma B)^{-1}B$

<b>Standard</b>	Default
<b>Generalized</b>	

The problem to be solved is either a standard eigenvalue problem,  $Ax = \lambda x$ , or a generalized eigenvalue problem,  $Ax = \lambda Bx$ . The optional parameter **Standard** should be used when a standard eigenvalue problem is being solved and the optional parameter **Generalized** should be used when a generalized eigenvalue problem is being solved.

<b>Tolerance</b>	$r$	Default = $\epsilon$
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An approximate eigenvalue has deemed to have converged when the corresponding Ritz estimate is within **Tolerance** relative to the magnitude of the eigenvalue.

<b>Vectors</b>	Default = RITZ
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The function nag\_complex\_sparse\_eigensystem\_sol (f12aqc) or nag\_complex\_banded\_eigensystem\_ solve (f12auc) can optionally compute the Schur vectors and/or the eigenvectors corresponding to the converged eigenvalues. To turn off computation of any vectors the option **Vectors** = NONE should be set. To compute only the Schur vectors (at very little extra cost), the option **Vectors** = SCHUR should be set and these will be returned in the array argument **v** of nag\_complex\_sparse\_eigensystem\_sol (f12aqc) or nag\_complex\_banded\_eigensystem\_solve (f12auc). To compute the eigenvectors (Ritz vectors) corresponding to the eigenvalue estimates, the option **Vectors** = RITZ should be set and these will be returned in the array argument **z** of nag\_complex\_sparse\_eigensystem\_sol (f12aqc) or nag\_complex\_banded\_eigensystem\_solve (f12auc), if **z** is set equal to **v** (as in Section 10) then the Schur vectors in **v** are overwritten by the eigenvectors computed by nag\_complex\_sparse\_eigensys tem\_sol (f12aqc) or nag\_complex\_banded\_eigensystem\_solve (f12auc).

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